A TREATISE

ON

NAVIGATION AND NAUTICAL ASTRONOMY,

Especially adapted for the Use of Students.

BY

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EIGHTH EDITION.

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TO

HIS GRACE THE DUKE OF SOMERSET, K G.,

VICE-ADMIRAL THE HON SIR FREDERIC WILLIAM GREY, K.C B.,

REAR-ADMIRAL CHARLES EDEN, C.B,

REAR-ADMIRAL CHARLES FREDERICK,

REAR-ADMIRAL THE HON. JAMES ROBERT DRUMMOND, C B,

AND

HUGH CULLING EARDLEY CHILDERS, Esq, M.P.,

THE LORDS COMMISSIONERS FOR EXECUTING THE OFFICE OF LORD HIGH ADMIRAL OF THE UNITED KINGDOM OF GREAT BRITAIN AND IRELAND,

THIS WORK,

DESIGNED TO IMPART A SCIENTIFIC CHARACTER TO THE PROFESSIONAL EDUCATION OF BRITISH SEAMEN,

IS,

WITH THEIR LORDSHIPS' PERMISSION,

RESPECTFULLY INSCRIBED BY

THEIR LORDSHIPS' MOST OBEDIENT SERVANT,

THE EDITOR.

PREFACE TO THE SEVENTH EDITION.

THE work of which this is the seventh edition has once more undergone a careful revision, with the desire to make it keep pace with the best modern practice of the art of navigation. It is essentially a work for students, and intended to teach navigation soundly. It may be thought that the calculations are occasionally too minute or diffuse; but, from long experience as a teacher, the Author is convinced that such minuteness is necessary for the sound instruction of young persons; that by insisting on the same full exhibition of all the figures employed in their own calculations, and the adoption of an orderly arrangement in them, much greater accuracy is induced; and the work of examination is also greatly facilitated. It has not been attempted, therefore, to curtail the processes of calculation, however desirable this may appear to persons employed in navigating ships; the primary object of the work being the full exposition of every detail in the subjects treated of in its pages. And with this view also the "special tables" are reduced to the smallest limit with respect to number, thus compelling the learner to seek what he requires from first principles.

The practical and the theoretical parts of the course are kept in a great measure distinct, each has thus its own completeness; while on the one hand it is hoped that no rule is given without its full complement of reason, on the other, it has not been thought desirable so to interweave them as to break the order of the purely practical course.

And thus, too, the unmathematical reader may attain some of the mere mechanical skill of the computer, without being discouraged by a great array of mathematical symbols.

Of the more ambitious student, for whom the book is designed, some preparation is required in arithmetic, algebra, Euclid, and trigonometry. He is warned also not to expect to find all that he ought to know of nautical science in one little book, and that professedly a school-book.

Although intended for the learner, it is hoped that it may commend itself to the attention of the skilled navigator, and the nautical surveyor, to whom it offers all the exactitude in the astronomical computations and investigations which their professions require; together with some important problems not yet so popular nor so widely known as they deserve to be.

Part I.—Is devoted to navigation, the sailings, the compass, the log, and the chart; and includes some simple rules for the solution of problems in great-circle sailing.

The article on circular-arc sailing contains an important modification of great-circle sailing, to include those instances in which the great-circle track may ascend to latitudes which are too high for ordinary navigation.

Part II.—Contains the practical course of nautical astronomy with such explanations of the subordinate parts of the computations as require only a little knowledge of geometry, and plane trigonometry, or mere verbal explanation. This part contains the method of computing the latitude from the altitude of the pole-star by common logarithms, more directly than by interpolation between the corrections given in the tables of the "Nautical Almanac" for the purpose.

A convenient method of deducing the latitude from altitudes of the sun near noon.

A full explanation of Sumner's method of double altitudes.

Ivory's method of double altitudes perfected by the addition of the corrections for the change of the sun's declination.

The method of clearing the lunar distance from the effects of parallax and refraction, by the use of the ordinary trigonometrical tables; sound in principle, easy, direct, and expeditious in practice, and the least obscure of the many methods which have been proposed for this purpose.

Part III.—Contains the formal demonstrations of those rules which require a knowledge of spherical trigonometry, or higher mathematical knowledge than is involved in the preceding investigations.

PREFACE TO THE EIGHTH EDITION.

It having become necessary to issue another edition of "Riddle's Navigation and Nautical Astronomy," I was requested to undertake the superintendence of these sheets through the press.

Nothing has been added to or taken from the former edition, as it was considered unadvisable to alter in any way the plan of the book; it will, therefore, be unnecessary for me to preface this edition with any further remarks than to say that I have very carefully reworked the examples, and thoroughly revised the text, making, however, alterations only, where, as it seemed to me, the Author's meaning might be a little more fully elucidated.

I hope that my endeavours to render this excellent work still more perfect, will be found advantageous to those engaged in the important duty of teaching navigation and nautical astronomy.

Albert Escott.

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South Street, Greenwich.

PART I.

NAVIGATION.

ELEMENTARY PRINCIPLES, PRACTICAL RULES AND EXAMPLES.

PART I.

NAVIGATION.

ELEMENTARY PRINCIPLES OF NAVIGATION.

THAT the art of navigation may be rightly understood, or practised with advantage, it is necessary that the navigator should be acquainted with the form of the earth, the relative situation of the lines conceived to be drawn upon its surface, and be furnished with correct charts of such parts of it as he may have occasion to visit, as well as with tables, in which the situations of the most remarkable sea-coasts, islands, rocks, shoals, &c, are accurately described; and he must also understand the use and application of such instruments as are necessary to determine the direction in which the ship is steered, and the distance which she sails; and be further possessed of sufficient mathematical skill to deduce, from the data which these instruments furnish, the situation of the ship at any time, and to find the direction and distance of any place to which it may be required that the ship should be taken.

That the earth, in its general figure, is a round body is evident from various considerations. If it were flat, then in clear weather, though distant objects upon its surface might appear small, they would still be within the limit of view; but it is uniformly observed, in every part of the earth, that to whatever quarter a ship sails, after she has proceeded a few miles to sea, she is gradually lost sight of, appearing as it were to sink in the waters, or to be hidden behind their convexity; the lower parts disappearing first, and the higher in succession. Now the figure of the object on which this appearance uniformly takes place must necessarily be round.

In lunar eclipses, which are caused by the moon's passing through the shadow of the earth, it is always observed that the bounding line of the shadow on the face of the moon is a curve line, the earth therefore, which casts that shadow, must be a round body. To these and many other considerations it may be added, that several celebrated navigators, by proceeding forward always, or as nearly as circumstances admitted, in the same direction, have actually arrived at the place which they sailed from, and have thus sailed round, or circumnavigated the earth.

But though the figure of the earth is *nearly* spherical, it is not *exactly* so It revolves round one of its diameters once in a day; and this revolution produces an effect upon its figure which, in nice observations, becomes very apparent. *It is flattened towards the extremities of the axis of rotation*, but so slightly, that in computing the place of a ship, from the distance which she has gone, and the direction in which she has sailed, the earth may be safely considered as a sphere.

The diameter round which it revolves is called the *axis*, and the extremities of that diameter the *poles* of the earth That to which we in Europe are nearest is called the *north pole*, and the other the *south pole.*

Great circles passing through the poles are called *meridians;* the great circle, equidistant from both poles, and which therefore cuts the meridians at right angles, is called the *equator*, the *equinoctial*, or the *line;* and less circles, whose planes are parallel to the plane of the equator, are called *parallels of latitude* The meridian passing over any place is called the meridian of that place; and the portion of a meridian intercepted between a place and the equator is called the *latitude* of that place; and it receives the denomination of *north* or *south*, according as the place is on the north or south side of the equator.

It is customary to call the meridian of some remarkable place the first meridian, and the angle included between the first and any other meridian is called the *longitude* of that other meridian, or of any place over which the meridian passes. And as the angle included between two great circles is measured by the arc which they intercept on another great circle, whose pole is at the point of their intersection, the *longitude of a place* may also be defined to be *the arc of the equator intercepted between the first meridian and the meridian of that place;* and it is considered as *east* or *west*, according as the place is situated towards the east or west of the first meridian. English geographers and seamen refer to the meridian of the Royal Observatory at Greenwich as the first meridian, Frenchmen to that of the Observatory at Paris, &c.

The *difference of latitude* between any two places is an arc of a meridian intercepted between the parallels of latitude on which the places are situated; and the *difference of their longitudes* is the angle at the pole included between their meridians, or the arc of the equator which those meridians intercept.

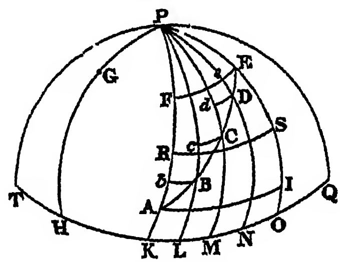
Hence when the latitudes or the longitudes of two places are of the *same* denomination, the difference of their latitudes, or of their longitudes, will be found by subtracting the less from the greater; but when they are of *different* denominations, by taking their sum.

A curve that cuts every meridian which it meets at the same angle, is called a *rhumb line;* the angle which the rhumb line makes with the meridian is called the *course* between any two places through which the rhumb passes; and the arc of a rhumb line intercepted between two places is called their *nautical distance.*

The *meridian distance* which a ship has made is an arc of the

which the ship is, intercepted between the meridian left
eridian arrived at; and the *departure* which a ship makes
upon a rhumb line, *is the sum of all the intermediate meri-
nces, computed on the supposition that the distance is divided
nitely small equal parts.*
'allel of latitude, which is 23° 28′ north of the equator, is
tropic of cancer; and that which is 23° 28′ south of the
called the *tropic of capricorn.* The parallel of latitude,
3° 28′ from the north pole, is called the *arctic circle;* and
is at the same distance from the south pole is called the
ircle. These four circles divide the surface of the earth
parts, called *zones.* The part included between the tropics
ne *torrid* or *burning zone,* from the intense heat produced
tical action of the sun's rays Those included between the
the arctic and antarctic circles are called *frigid* or *frozen*
n the great cold arising from the periodical absence of the
he obliquity with which his rays at all times meet the sur-
ose parts of the earth. The two remaining parts are called
zones, from their enjoying the advantage of an intermediate
ween the extremes of heat and cold which prevail in the
frigid zones.
juator bisects the torrid zone, and also divides the whole
the earth into two equal parts; that in which the north pole
being called the northern hemisphere, and the other the
hemisphere.
espect to the magnitude of the earth, it has been found, by
dmeasurements, to be nearly equal to a sphere of 7916
niles in diameter, or 24,869 miles in circumference. Hence
phical or nautical mile, which is the 21,600th of 360°, is
79 English feet.
e sake of illustration, let us suppose that in the annexed
is the north pole:

Fig. 1.



Q the equator, or a great circle, every part of which is a quad-
ant from P;
P H, P K, &c., great circles passing through P, and of course
he equator at right angles;
B, R S, &c., arcs of smaller circles whose planes are parallel to
of the equator, and therefore cut the meridians at right angles;

A E a curve making equal angles with P K, P L, P M, &c.

Then P H, P K, &c, produced till they meet at the opposite pole, are called meridians;

A I, *b* B, R S, &c, continued round the globe, are called parallels of latitude,

A E is called the rhumb line, passing through A and E;

The length of A E is called the nautical distance from A to E; and the angle *b* A B, or any of its equals, *c* B C, *d* C D, &c, is called the course from A to E.

Let G be the situation of Greenwich, then G H is the latitude of Greenwich, G P its colatitude, A K the latitude of the point A, or of any place on the parallel A I; F K, or its equal E O, is the latitude of F, or of E, or of any place on the parallel F E; F A or E I is the difference of latitude of the points A and E, or of the parallels A I and E F, or of any places on those parallels.

As P G H is the first meridian, the longitude of G, or of any place on the meridian P G H, is nothing.

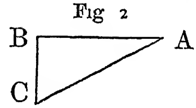
The arc T H, or the angle T P H, which T H measures, is the longitude of the meridian P T, or of any place on that meridian; the arc H K, or the angle H P K, is the longitude of A, or of F, or of any place on the meridian P K; the arc H O, or the angle H P O, is the longitude of the points O, I, S, and E, or of any place on the meridian P O.

The arc K O, which is the difference of H O and H K, is the difference of the longitudes of the meridians P K and P O, or of any two places on those meridians; and T O the sum of T H and H O, is the difference of longitude of the meridians P T and P O.

A rhumb line is a curve which makes an angle of the same magnitude with every meridian which it crosses; it represents the track of a ship while sailing on any one given course; it coincides with the meridian when the course is due N or S, or with a parallel of latitude when the course is due E or W. On any other course but these the rhumb line is a spiral, approaching nearer and nearer to one of the poles at every convolution.

The angle which the rhumb line or ship's track makes with any meridian is called the course, and the length of the rhumb line passed over by the ship is called the nautical distance.

Fig 2



Let C A in the adjoining figure be a straight line equal in length to the curve line A E in the fig. (1), and let it meet the upright or meridian line B C, so that $\angle$ B C A shall be equal to the course, or $\angle b$ A B in figure (1).

And next let the rhumb line in figure (1) be divided into minute equal parts, A B, B C, C D, &c., and through B, C, D let the parallels B *b*, C *c*, &c., be drawn, and it is evident that A *b* + B *c* + C *d* +, &c.,

is always exactly equal to A F, the difference of latitude. Again, as the number of the parts A B, B C, &c., into which the distance is divided is arbitrary, we may suppose them so numerous and minute as to diminish any errors arising from the supposition that A b B, B c C, &c., are plane right angled triangles, to less than any assignable quantities; and next we may suppose these triangles transferred so that their hypothenuses may lie in succession from C to A in fig. (2). It will then be evident that in their new position, A b + B c + C d +, &c, will exactly equal C B. Hence it is seen that C B in fig. (2) is equal to A F in fig. (1), or that C B equals the difference of latitude.

The line A B of the plane triangle is called the *departure*, and is equal to B b + C c + D d +, &c., a principle which may be thus enunciated: *the departure is the sum of all the intermediate meridian distances belonging to the indefinitely small parts of the rhumb line between the place sailed from and the place arrived at.* It is commonly assumed to be equal to the meridian distance R S in the middle latitude between the place sailed from and the place arrived at, or *the departure equals the meridian distance in the middle latitude nearly;* for the sum of the elementary meridian distances B b + C c + D d, &c., is greater than the arc F E of the one, and less than the arc A I of the other of the two parallels between which the ship's track lies; but it must be observed that this is advanced only as an approximation; and that this reasoning fails when the latitudes are on different sides of the equator But even in this case if the distance between the parallels be not very great, the *departure* may be considered as equal to the *difference of longitude.*

The right-angled triangle fig. (2) is, then, a projection of certain lines and angles of the sphere upon a plane; A C is the nautical distance, B C the difference of latitude, A B the departure, and ∠ C is the course of the ship The points C and A are the projected positions of the places between which the ship sails.

It may be further observed, that if B C and B A, or the difference of latitude and departure, are equal, ∠ C, or the course, is half a right angle, and that the course is greater or less than 45°, as the departure is greater or less than the difference of latitude.

When any two of the four quantities which are represented in the triangle A B C, viz., course, distance, difference of latitude, and departure are given, the remaining parts can be computed by the rules of trigonometry. And problems relating to this projected plane triangle constitute what is called plane sailing.

By means of two of the relations furnished by this figure, viz.,

Distance × *cosine of course* = *difference of latitude,*

and *Distance* × *sine of course* = *departure,*

the contents of Tables XVII. and XVIII. are calculated; and these (which are called traverse tables), greatly facilitate the solutions of problems in navigation, especially when the ship frequently changes her course and makes a crooked track or *traverse,* when it is neces-

sary to reduce her northings and southings and her eastings and westings on the several portions of her track to a total difference of latitude and departure. This process is called *resolving a traverse*, or traverse sailing.

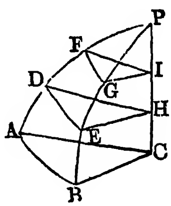
All the great circles of a sphere are equal, the radius of each being equal to the radius of the sphere itself; hence the degrees, &c., of the equator are equal to those of any meridian, the earth being here supposed to be a sphere.

The parallels of latitude being less circles of the sphere, their subdivisions or degrees, minutes, &c., are less than those of the equator, and are less and less the further the parallel is from the equator. A simple formula, however, gives the relation between the magnitudes of an arc of the equator and the corresponding arc of a parallel of latitude, that is, of arcs of these circles intercepted between the same meridians.

Let L represent the difference of longitude or arc of the equator,
M the corresponding arc of a parallel of latitude,
l the latitude of the parallel,
Then the formula referred to is $M = L \cdot \cos l$.

In the accompanying figure, which represents a solid portion of the sphere, P is one of the poles, C the centre, A B an arc of the equator, D E and F G corresponding arcs of two parallels of latitude, D H E and F I G their planes, and I and H their centres.

Fig. 3



$$\begin{array}{llcl} \text{Then} & A B : D E & :: & B C \cdot E H \\ & & :: & \sin B P : \sin E P, \\ & & :: & 1 \quad : \cos E B; \\ \text{or,} & L : M & :: & 1 \quad : \cos l; \\ & & \therefore & M = L \cdot \cos l. \end{array}$$

D E, which is here denoted by M, is sometimes called the meridian distance, and then the rule here proved may be thus enunciated *The meridian distance* is equal to the *difference of longitude* multiplied into the *cosine of the latitude*.

On referring again to fig. (1) it will be seen that it necessarily follows from the principle just demonstrated that

$$R S = K O \times \text{cosine } R K.$$

But R S has been shown to be nearly equal to the departure; K O is

the difference of longitude between the meridians P K and K O, or of the points A and E between which the ship's track lies; and R K is half the sum of the latitudes of the same points; for it is as much greater than one latitude as it is less than the other: it is called the middle latitude. The equation above may therefore now be expressed in words as follow:

The departure is nearly equal to the difference of longitude multiplied into the cosine of the middle latitude.

And this is the fundamental relation in what is called middle latitude sailing. Its importance consists in the simple connection which it gives between the *difference of longitude* and the other elements of plane sailing.

Mercator's chart is a projection of the globe upon a plane surface according to certain mathematical principles In the first place the meridians are represented by parallel straight lines meeting the parallels of latitude, which are also represented as straight lines, at right angles.

Thus the distances between the meridians are made the same in all latitudes, and everywhere equal to the difference of longitude; but this is not the case upon the globe; for it has been shown that the

$$\textit{meridian distance} = \textit{difference of long.} \times \textit{cosine of the lat.}$$

or, which is the same thing,

$$\textit{difference of long.} = \textit{meridian distance} \times \textit{secant of the lat.}$$

Therefore when the meridian distances are made equal to the differences of longitude, as they are in Mercator's projection of the globe, they are *increased* in the ratio of the secant of the latitude.

This being established as a necessary consequence of the mere straightening of the meridians and parallels, and arranging them at right angles to each other, the next peculiarity to be noticed is the mode of dividing the meridians for the representation of the latitudes.

The divisions of the meridian for latitude are made to depend on the following principle: *The minute divisions of the meridian in the neighbourhood of any parallel of latitude shall be increased in the same ratio as the parallel itself is.* But the parallels are *increased* in the ratio of the secants of their latitude; and therefore the divisions of the meridians must be increased in this ratio. For example, $1'$ of the meridian at latitude 60° north or south upon Mercator's chart must be twice as long as $1'$ of the meridian close to the equator where the latitude is 0°; for the secant of $0^\circ = 1$, and the secant of $60^\circ = 2$.

To find the length to which any latitude is extended by this proceeding, we must add the lengths of all the lines which represent the minutes of that latitude together, thus—

$1'$ in lat. $1'$ is represented by $1' \times \sec 1'$
$1'$ in lat. $2'$,, ,, $1' \times \sec 2'$
$1'$ in lat. $3'$,, ,, $1' \times \sec 3'$
$1'$ in lat. $4'$,, ,, $1' \times \sec 4'$ &c, &c.

therefore the line which represents any latitude containing n minutes is equal to the line which represents $1'$ on the equator multiplied by

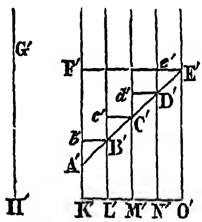
$$\text{Sec } 1' + \sec 2' + \sec 3' + \sec 4' + \ldots\ldots \sec n$$

And the sum of all these natural secants reckoned as miles is tabulated for every latitude under the name of meridional parts.

Table XIX. is a table of this kind, but it is adapted to the spheroid whose compression $= \frac{1}{321}$. (See notes)

That which constitutes the chief value of Mercator's chart for the purposes of navigation is, *that the angle which a straight line joining any two places on the chart makes with the meridians, is equal to that which the rhumb line joining the same two places on the globe makes with the meridians.*

Fig 4



Let A' and E' represent the projected places of the points A and E of the globular figure (1), then $A'F'$ is the projected or meridional difference of latitude, and $F'E'$ or $K'O'$ the difference of longitude.

$A'F'$ is composed of the elementary parts of the proper difference of latitude increased in the ratio of the secant of the latitudes, and is called the meridional difference of latitude.

The intermediate meridian distances are increased in the same ratio, and as each is thus made equal to the corresponding difference of longitude, their sum is equal to the whole difference of longitude $F'E'$; which therefore exceeds the sum of the intermediate meridian distances on the globe, in the same ratio that the sum of the meridional differences of latitude exceeds the proper difference of latitude; but the intermediate meridian distances on the globe are collectively equal to the departure, consequently,

$$\frac{\textit{Difference of longitude.}}{\textit{Meridional difference of latitude.}} = \frac{\textit{Departure};}{\textit{Difference of latitude.}}$$

But the latter fraction = tangent of the course.

$$\frac{\textit{Difference of longitude.}}{\textit{Meridional difference of latitude.}} = \textit{tangent of the course.}$$

But the difference of longitude $= F'E'$, and the meridional difference of latitude $= A'F'$, and $\frac{F'E'}{A'F'} = \text{tangent} \angle F'A'E'$, and therefore $\angle F'A'E' =$ the course, which establishes the principle that a

straight line drawn through any two points on a Mercator's chart shows, by the angle it makes with the meridians intersected by it, the course from the one place to the other.

The meridional difference of latitude is found by taking the sum of the meridional parts (Table XIX.) corresponding to the two latitudes, when one is north and the other south, and the difference when they are both north or both south.

1. For example, let it be required to find the meridional difference of latitude between 35° 40′ N and 20° 45′ N.

35° 40′ N meridional parts		2281
20 45 N „	„	1266
Meridional difference of latitude .	. miles	1015

2. Again, between latitudes 3° 51′ N and 5° 57′ S.

3° 51′ N meridional parts		230
5 57 S „	„	355
Meridional difference of latitude .	. miles	585

If the middle latitude is defined to be that intermediate latitude at which the distance between the meridians is exactly equal to the departure, then we shall have this equation accurately.

Departure = difference of longitude × cosine of the middle latitude.

We have also by fig. (2),

$$\textit{Tangent course} = \frac{\textit{Departure}}{\textit{Difference of latitude}}$$

$$\text{and} \quad \therefore \quad = \frac{\textit{Difference of longitude} \times \textit{cosine middle lat.}}{\textit{Difference of latitude.}}$$

And, again, it has been shown that

$$\textit{Tangent course} = \frac{\textit{Difference of longitude}}{\textit{Meridional difference of latitude.}}$$

Equating these two values of tangent course, and reducing, we arrive at the formula by which the correct middle latitude can be computed, viz.,

$$\textit{Cosine of middle latitude} = \frac{\textit{Difference of latitude}}{\textit{Meridional difference of latitude.}}$$

The mean middle latitude, which is found by taking half the sum or difference of the latitude, according as they are on the same or different sides of the equator, is generally used instead of this correct middle latitude. The difference between them is often considerable; and when the difference of latitude is large, it is desirable to use Mercator's sailing only.

Let it, for example, be required to find the mean and correct middle latitudes between 57° 45′ S and 43° 45′ S. The work stands thus—

	57° 45′ S	. . .	4248
	43 45 S	. . .	2910
D Lat = 840 =	14 0	. . .	1338 = M D Lat
D Lat 840		. . . . .	12·924279 log + 10
−M D Lat 1338		. . . . .	−3·126456
51° 7′		. . . cos	9·797823

Therefore the correct middle latitude, or that which should be employed, is 51° 7′.

Now by taking half the sum of the latitudes the mean latitude is 50° 45′, and the difference, 22′, is what must be added to the mean to obtain the true or corrected middle latitude.

The greater the difference of latitude, the greater the error in using the mean middle latitude, and hence the precept which directs the employment of Mercator's sailing in such cases.

The difference between the mean and correct middle latitude varies not as the difference of latitude, but very nearly as its square, so that doubling the difference of latitude will give four times the error.

A table might be given of the corrections to be applied in all cases, but the middle-latitude method thus becomes Mercator's sailing disguised, and therefore no advantage would be gained by doing so.

Still when the difference of latitude is small, or the ship sails nearly on a parallel of latitude, the middle-latitude method is to be preferred.

To draw a Mercator's Chart.

Having drawn at the bottom of the paper a line to represent the lowest parallel on the chart, and divided and subdivided it as may be thought convenient for degrees, &c., of longitude, let this line be considered as a *general line of measures*, and draw perpendiculars at the extremities of it to represent the extreme meridians. Then, to obtain the proper length of the degrees in the different parts of the meridian, proceed as follows:—

Take from Table XIX. the meridional parts for the latitude at which it is proposed the chart should commence, and also for each successive degree of latitude intended to be contained in it.* Then find the meridional differences of latitude between the latitude of the lowest parallel, and every other one intended to be contained in the chart, and dividing them by 60, for degrees, &c., take the quotients

* If the degrees be very small in the intended drawing, or the chart be a general chart, including a great extent of the earth's surface on a small scale, every fifth or tenth degree will be sufficient

from the bottom line, or the line of measures, and apply them from the bottom on the extreme meridians, and the degrees of latitude will thus be determined; and for practical purposes they may be further subdivided in a manner similar to the degrees of longitude, but theoretically the degrees should be divided in the same way as the chart. The meridians and parallels of latitude being then drawn at such intervals as may be judged convenient, the situation of every remarkable place, rock, island, shoal, &c., comprised within the space represented by the chart, may be laid down from its known latitude and longitude; and a compass (or more than one, if the chart be large) being inserted in any convenient situation, to determine the relative bearings of the different points on the chart, the coasts, sands, rocks, &c., may be drawn and shaded, and the chart will be completed.

To find the latitude and longitude of any place on the Chart.

Take the perpendicular distance of the given place from any convenient parallel, and from the point where that parallel cuts the graduated meridian, apply the distance upon the meridian, and the same direction that the place lies with respect to the parallel, at the point to which the distance reaches will be the latitude of the place.

In the same way, if the perpendicular distance of a place from any meridian be taken and applied from the place where that meridian cuts the divided parallel, the longitude of the place will be determined

From the latitude and longitude of a place to find its situation on the Chart.

Lay the edge of a scale over the parallel of the given latitude; measure on one of the graduated parallels the distance of any convenient meridian from the given longitude, and apply this distance along the edge of the scale from the place where the meridian measured from cuts the scale, in the same direction that the longitude lies from the meridian, and the point to which the distance reaches will be the required situation of the place.

To find the course between two places in the Chart.

Lay the edge of a scale over both the places, and applying a parallel ruler to the edge of it, move the two parts of the ruler in succession till the edge of one of them passes through the centre of a compass, and that edge will, on the compass, indicate the course.

Or draw a line in pencil along the edge of the scale, cutting any meridian, and the angle included between the line and the meridian will be the course.

Or, having laid the scale over the places as before, place one foot of a pair of compasses in the centre of any convenient compass on

the chart, and with the other take the nearest distance to the edge of the scale. Then carry both points of the compasses forward, keeping one of them by the edge of the scale, and the imaginary line which joins them perpendicular to the edge; and the line which the other point describes from the centre of the compass will indicate the course.

It can, however, be most conveniently done by placing a little semicircular protractor against the edge of the scale as it lies over the places, and so that the centre of the protractor may be upon one of the meridians marked upon the chart, for the number of degrees between the edge of the scale and the meridian shows at once the course of the ship.

To find a ship's place by the bearing of two known places or headlands.

Place a ruler over one of the headlands on the chart, and apply a protractor to its edge, keeping the centre of it upon one of the meridians; now turn the scale about until the degrees between the meridian and the edge of the scale, as indicated upon the protractor, correspond with the given bearing, and draw a line along the edge of the scale.

Repeat this process with the other headland, and where the lines intersect is the place of the ship Or, having drawn a line to represent a meridian, and the lines of bearing in their proper relative positions upon a piece of tracing-paper, lay the paper upon the chart so that the meridian line shall be parallel to the meridians on the chart, and so that the lines of bearing shall at the same time pass over the two points which were observed, and the intersection of them will then fall on the place whence the bearings were taken.

Upon a known coast such a method as this, which can be constantly practised, is most valuable, while its simplicity renders the neglect of it quite inexcusable.

To lay down an island, rock, or headland from observed bearings.

Let the bearing be taken, and after running for some time, let another bearing be taken. Now marking the positions of the ship at the times of observation, and drawing the lines indicated by the observed bearings, their intersection is the place of the observed object.

Such a method as this may be employed in sketching an *unknown* coast as a ship passes along.

The distance of the objects may be easily computed, for the lines of direction from the ship, and the distance run by the ship between the observations, form a triangle whose angles are known from the course of the ship and the observed bearings; and one side is also known, viz., the distance run by the ship between the observations.

To find the distance between two places on the Chart.

If the places are in the same longitude, find the latitude of each, and if their latitudes are of the same denominations, take their *difference;* otherwise, their *sum* for the distance of the places.

If they are in the same latitude, or on the same parallel, take half their distance, and apply it on the graduated meridian on both sides of the parallel on which the places are situated, and the degrees of the meridian intercepted between the two points to which the distance reaches will be the distance of the two places *nearly.*

But if the places differ both in latitude and longitude, lay a scale over them both, and, taking half their distance, apply it both upwards and downwards on the graduated meridian from the middle parallel between the two places, and the degrees of the meridian intercepted between the points to which the distance reaches will be the distance of the places *nearly.*

If half the distance be too great to be conveniently measured at once, one-fourth or one-eighth of it may be taken and applied upwards and downwards from the middle parallel as before, and the intercepted degrees of the meridian will be one-half or one-fourth of the required distance *nearly.*

The distance may be found by the following method, which is exact in principle, and is in fact only an abridged solution of the question by construction, on the principles of Mercator's sailing:—

Find the difference of latitude between the two places, and take it in the compasses from the graduated parallel. Then having laid a scale over the two places, slide one foot of the compasses along its edge till the other, in sweeping, just touches a parallel of latitude; and the distance from the point where that parallel cuts the scale to the point of the compasses by the edge of the scale, applied to the graduated parallel, will show the distance of the two places. But this method should not be used when the course is very large.

From the course and the distance which a ship has run from a known place, to determine her situation on the Chart.

Lay the edge of a scale over the given place in the direction of the ship's course, and take the distance, reduced to degrees, &c., from that part of the graduated meridian opposite the place on which the ship has been sailing, and this distance, applied from the given place along the edge of the scale, will determine the situation of the ship.

Of Great Circle Sailing.

Having now discussed in a popular way the ordinary principles of Navigation, it is necessary to say a word with respect to *great circle sailing*, which has risen into rather more importance than in former times through the extensive use of steam vessels both for commerce and war.

Great circle sailing is so called because by it it is attempted to keep a ship as nearly as possible upon the great circle which passes over the two places between which a voyage is made. The object of this is to shorten the distance, the arc of a great circle being the shortest distance between two points upon a sphere.

With a little manipulation with a common globe the two places may both be brought to the wooden horizon, which is itself a great circle, and this is an excellent method for observing how the great circle track lies between them

It will then be seen that the great circle does not meet all the meridians at the same angle, and this renders it necessary to change the course from time to time, in order that the ship may avail herself of the shorter track thus offered to her.

A knowledge of spherical trigonometry is necessary for a thorough understanding of this subject; but the beginner who has not yet sufficient mathematical knowledge for this, will, nevertheless, find little difficulty in solving problems in great circle sailing by the rules which are given for them. The demonstration of these rules will also be found in the *third section* of this work, which is devoted to the mathematical discussion of the rules given in the first and second portions.

PROBLEMS.

1. *To find the difference of latitude, or the difference of longitude, between any two places whose latitudes and longitudes are given.*

RULE. When the given latitudes are of the same denomination, take their difference; and when they are of contrary denominations, add them together; and the sum, or the remainder, is the difference of latitude.

The difference of longitude is found in the same manner, observing, however, that the difference of longitude signifies the *less* arc of the equator intercepted between the two meridians; and therefore, when the longitudes are of different denominations, and their sum exceeds 180°, the sum must be subtracted from 360° to find the difference of longitude.

The difference of latitude which a ship makes is called *north* or *south*, and is marked N or S according as the ship has sailed on a northerly or southerly course. Hence a north difference of latitude must be considered as increasing a north latitude, and as diminishing a south one, and a south difference of latitude as tending to increase a south latitude, and to diminish a north one.

Examples.

1. Required the difference of latitude and difference of longitude between the Lizard Point and the Peak of Pico?

	Latitudes		Longitudes
Lizard Point . .	49° 58′ N		5° 11′ W
Pico	38 26 N		28 28 W
	11 32 S		23 17 W
Diff lat .	692 miles	Diff long . .	1397 miles.

2. Required the difference of latitude and difference of longitude between Halifax and the Cape of Good Hope?

	Latitudes		Longitudes
Halifax	44° 40′ N		63° 38′ W
Cape.	34 23 S		18 24 E
	79 3 S		82 2 E
Diff lat. .	4743 miles.	Diff. long. . .	4922 miles.

3. Required the difference of latitude and difference of longitude between a place A in lat. 55° 58′ S; long. 67° 11′ W, and another place B in lat. 43° 37′ S; long 146° 49′ E?

	Latitudes		Longitudes
A.	55° 58′ S	A	67° 11′ W
B.	43 37 S	B	146 49 E
Diff. lat. ..	12 21 N	.	214. 0 E
			360 0
		Diff. long . .	146 0 W

Or, in nautical miles, difference of latitude = 741 and difference of longitude = 8760 miles.

EXAMPLES FOR EXERCISE.

Required the difference of latitude and difference of longitude between A and B in the following cases:—

Lat. A	Lat B	D Lat	Long A	Long B	D Long
° ′	° ′		° ′	° ′	
11 57 S	18 54 N	1851 N	49 14 E	72 56 E	1422 E
5 55 N	12 43 N	408 N	80 43 E	53 18 E	1645 W
32 0 N	51 25 N	1165 N	80 57 W	9 29 W	4288 E
6 14 S	4 56 N	670 N	12 40 E	52 18 W	3898 W
36 47 N	44 24 N	457 N	3 4 E	8 54 E	350 E

PROBLEM 2.

To find the latitude and longitude at which a ship has arrived, when those of the place which she left, and the difference of latitude and longitude which she has made, are given.

RULE. If the latitude left and the difference of latitude are of the same denomination, add them together; but if they are of different denominations, take their difference; and the sum or the remainder is the latitude arrived at, and of the same denomination with the greater.

Remark. As no place can be further distant from the equator than the *poles,* the latitude cannot exceed 90°.

The longitude arrived at is found in the same manner as the latitude; but as the longitude is reckoned both *east* and *west,* if the longitude left and the difference of longitude are of the same denomination, and their sum exceeds 180°, the difference between the sum and 360° is the longitude arrived at, and of a contrary denomination to the longitude left.

EXAMPLES.

1. If a ship sail from Cape Finisterre towards the south-west till her difference of latitude is 140, and her difference of longitude 118 miles, required her latitude and longitude in?

Cape Finisterre lat. . .	42° 54′ N	Long. left	9° 16′ W
Diff lat. 140 . . .	2 20 S	Diff. long 118. . .	1 58 W
Lat in	40 34 N	Long. in . . .	11 14 W

2. If a ship sail from latitude 50° 18′ S, and longitude 178° 21′ E towards the S E till her difference of latitude is 638 and her difference of longitude 400 miles, required her latitude and longitude in?

Lat left	50° 18′ S	Long left	178° 21′ E
Diff lat. 638 . .	10 38 S	Diff. long 400 . .	6 40 E
Lat. in	60 56 S		185 1
			360 0
		Long. in	174 59 W

Examples for Exercise.

In the following examples the latitude and longitude arrived at are required:—

	Lat left	Long left.	Diff lat. Miles	Diff long Miles	Answer Lat in	Answer Long in
1.	48° 2′ S	16° 34′ W	149 N	218 E	45° 33′ S	12° 56′ W
2	55 18 N	2 18 E	80 N	162 W	56 38 N	0 24 W
3	48 30 N	30 6 W	175 S	260 W	45 35 N	34 26 W
4	0 0	0 0	238 N	141 W	3 58 N	2 21 W
5	64 2 N	3 13 W	304 S	158 E	58 58 N	0 35 W
6.	39 37 S	28 17 E	112 S	300 E	41 29 S	33 17 E

Problem 3.

To know in what quarter of the horizon the course between any two places lies.

Rule. If the place bound to has a greater north latitude, or less south latitude, than the place to be sailed from, the course will be northerly; otherwise it will be southerly. And if the place bound to has greater east longitude, or less west longitude, than the place to be sailed from, the course will be easterly; otherwise it will be westerly. These directions combined will indicate the quarter in which the course lies.

Examples.

In what quarter of the horizon will the course lie from latitude 28° N longitude 16° W to latitude 35° N longitude 2° W?

Here the place bound to has greater north latitude than the place to be sailed from; the course, therefore, is *northerly;* and as the place bound to has less west longitude than the place to be sailed from, the course is also easterly. The course is therefore between the *north* and *east*, or in the *north-east* quarter of the horizon.

In what quarter of the compass will the course lie in sailing from the first to the second of each of the following places?

1. From Aberdeen to Rotterdam? Answer, in the SE quarter.
2. From the Lizard to Halifax? Answer, in the NE quarter.
3. From the Cape of Good Hope (Africa) to Van Diemen's Land? Answer, in the SE quarter.
4. From Cape Horn to St. Helena? Answer, in the NE quarter.
5. From Lisbon to Cape Farewell (Greenland)? Answer, in the NW quarter.
6. From the Cape of Good Hope (Africa) to Rio Janeiro? Answer, in the NW quarter.

THE COMPASS.

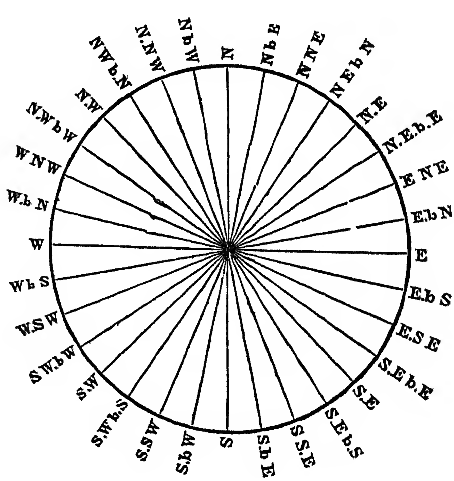
The course of a ship, or the angle which the line on which she sails makes with the meridian, is determined by an instrument called THE COMPASS; which consists of a circular card suspended horizontally on a point, and having a magnetized bar of hardened steel, called THE NEEDLE, for one of its diameters

The circumference of the card is generally divided into thirty-two equal parts, called *points;* and each of those divisions is again subdivided into four parts, called *quarter points.*

A point of the compass being therefore the 32nd part of the circumference of a circle, is equal to $11° 15'$; but in some compasses, for delicate observations, particularly those called azimuth compasses, the rim of the card is divided into degrees.

The magnetized needle has the peculiar property of pointing always in a particular direction, generally not far from the direction of the meridian.

That point of the card which coincides with the northerly end of the needle is called the magnetic *north,* and the opposite point the magnetic *south;* and, looking towards the north end of the needle, the middle point on the right, between the north and south, is called the *east,* and the opposite point the *west.* These four are called *cardinal points,* and the others are named according to their situation with respect to these cardinal points, as in the annexed figure.



The following table shows the degrees, &c., corresponding to each quarter point of the compass:—

Points				Points.			
0	0°	0′	0″	4¼	47°	48′	45″
0¼	2	48	45	4½	50	37	30
0½	5	37	30	4¾	53	26	15
0¾	8	26	15	5	56	15	0
1	11	15	0	5¼	59	3	45
1¼	14	3	45	5½	61	52	30
1½	16	52	30	5¾	64	41	15
1¾	19	41	15	6	67	30	0
2	22	30	0	6¼	70	18	45
2¼	25	18	45	6½	73	7	30
2½	28	7	30	6¾	75	56	15
2¾	30	56	15	7	78	45	0
3	33	45	0	7¼	81	33	45
3¼	36	33	45	7½	84	22	30
3½	39	22	30	7¾	87	11	15
3¾	42	11	15	8	90	0	0
4	45	0	0				

The points and quarter points in any course are usually reckoned from the north and south points, thus N by E is one point to the right of N, and NW by W five points to the left of N; SE is four points left of S; SSW ¼ W, two and a quarter to the right of S. The word *right* being used to indicate the same direction as that in which the hands of a watch move over the dial, and *left* the contrary direction.

The situation of the needle with respect to the meridian is not the same at every place, nor is it always the same at the same place. At present at London the north end of the needle points about 23½° towards the west of the true north point of the horizon, but at the North Cape it points only about 1° towards the west, while in some parts of Davis's Straits its direction is more than 6½ points towards the west, and near Cape Horn it points about 22° towards the east of the true north.

Again, in the year 1580, the direction of the needle, at London, was about one point towards the *east* of the north, while, as has been already observed, it at present points about 23½° towards the *west*. But in the West Indies, for a very long period, the deviation of the needle has undergone but a very trifling variation.

Delicate observations appear to indicate that it is again at London retrograding towards the east; and Mr. Barlow, in his valuable "Essay on Magnetic Attractions," observes that all the phenomena attending the progressive change of the needle's deviation from the meridian may be accounted for by conceiving the magnetic pole to revolve in a parallel of latitude from east to west; but that every place appears to have its individual pole.

The deviation of the compass, or, as it is called, the *variation of the compass*, may however be determined (as will afterwards be shown) by astronomical observations; the points of the horizon, which correspond to the several points of the compass, may therefore easily be found by allowing for the variation, when it is known.

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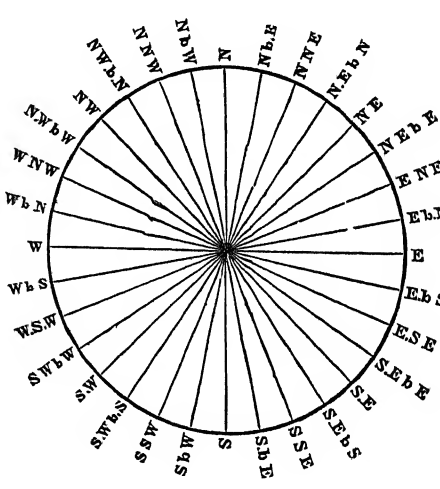

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0½	5	37	30
0¾	8	26	15
1	11	15	0
1¼	14	3	45
1½	16	52	30
1¾	19	41	15
2	22	30	0
2¼	25	18	45
2½	28	7	30
2¾	30	56	15
3	33	45	0
3¼	36	33	45
3½	39	22	30
3¾	42	11	15
4	45	0	0

Points			
4¼	47°	48′	45″
4½	50	37	30
4¾	53	26	15
5	56	15	0
5¼	59	3	45
5½	61	52	30
5¾	64	41	15
6	67	30	0
6¼	70	18	45
6½	73	7	30
6¾	75	56	15
7	78	45	0
7¼	81	33	45
7½	84	22	30
7¾	87	11	15
8	90	0	0

The points and quarter points in any course are usually reckoned from the north and south points, thus N by E is one point to the right of N, and NW by W five points to the left of N; SE is four points left of S; SSW ¼ W, two and a quarter to the right of S. The word *right* being used to indicate the same direction as that in which the hands of a watch move over the dial, and *left* the contrary direction.

The situation of the needle with respect to the meridian is not the same at every place, nor is it always the same at the same place. At present at London the north end of the needle points about 23½° towards the west of the true north point of the horizon, but at the North Cape it points only about 1° towards the west, while in some parts of Davis's Straits its direction is more than 6½ points towards the west, and near Cape Horn it points about 22° towards the east of the true north.

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The deviation of the compass, or, as it is called, the *variation of the compass*, may however be determined (as will afterwards be shown) by astronomical observations; the points of the horizon, which correspond to the several points of the compass, may therefore easily be found by allowing for the variation, when it is known.

Thus, if it be found that the north end of the needle points to the NNW point of the horizon, the compass is then said to have two points westerly variation, and the NNE point of the compass will coincide with the meridian, the east point of the compass with the ENE point of the horizon, &c.

But if the north point of the compass points to the NE by N point of the horizon, the compass is said to have three points easterly variation, and the NW by N point of the compass will coincide with the meridian, the east point of the compass with the SE by E point of the horizon, &c. If, therefore, a ship is steered NW by a compass which has two points westerly variation, the angle which her way makes with the true meridian will be six points, or the ship's true course will be WNW.

Hence, when the compass course is given to determine the true course, allow the variation, if it be westerly, to the *left* of the compass course; and, if easterly, to the *right* of the compass course. On the contrary, when the true course is known, and the corresponding course is required by a compass whose variation is given, allow the variation, when it is west, to the *right* of the true course, and when east to the *left*.

When a needle which is balanced horizontally on a point is magnetized, it not only acquires the property of pointing in a particular horizontal direction, but it *loses its balance*, or becomes inclined to the horizon; and it requires an additional weight to be applied to the elevated end of the needle to restore it to its horizontal position. This inclination of the needle to the horizon is called the *dip*; and as it is different in different situations, a magnetized needle which is horizontal in one place may not be horizontal in another. The weight, therefore, which is a counterpoise to the dip in one place may not be so in another; and on this account needles properly fitted up for mariners' compasses have a sliding weight applied to them, to adjust them to the horizontal position at any time.

The needle, with its apparatus, is generally placed in a brass case, which being slung in gimbals, the card is always at liberty to assume a horizontal position; and in the inside of the case there are two black vertical lines, which with respect to the card are diametrically opposite to each other. The imaginary horizontal line joining these two vertical ones ought to be exactly in the vertical plane cutting the ship from stem to stern; and the point of the card which coincides with the vertical line towards the stem of the vessel indicates the direction of the ship's head, or shows her apparent course by the compass.

No iron whatever should be allowed to be near the compass. Indeed, the whole mass of iron in a ship often exerts a perceptible influence on the direction of the needle, and the change of direction thus caused is named the *local deviation*, and its amount varies according to the situation of the ship's head with respect to the magnetic meridian.

If the iron were uniformly distributed in the ship, the effect of the local attraction on the needle would be nothing when the course is on

the magnetic meridian, and greatest when the compass course is *east* or *west*; and in practice the deviation might be taken as equal to the greatest deviation multiplied by the sine of the compass course.

But in general there is nothing like uniformity in the distribution of the masses of iron on ship-board, and the deviation produced by local attraction is generally found experimentally thus:—

A compass being sent to a distant place, visible from the ship, and out of the reach of local attraction, the bearing from each other of the compass so sent and the binnacle compass are taken simultaneously, as the ship's head is warped round to each point of the compass, and the difference between the bearing of the shore compass from the ship and the opposite bearing to that observed on shore towards the ship's compass, is the local deviation.

The directions of the ship's head having been taken by the compass in the ship, are therefore affected by the local attraction, and the apparent *compass bearing* of the ship's head differs from *the true magnetic bearing* by the amount of local deviation due to the position of the ship.

For instance, when the ship is apparently lying with her head east, supposing the local deviation to be one point easterly, or the north end of the needle to be drawn one point to the eastward, the east point of the compass card will be drawn to E b S, or the direction of the ship's head will be really magnetic E b S.

The results should not be finally tabulated until the *true magnetic* bearing of the ship's head at each observation is found as above, this being the proper *argument* for the tables of local deviation.

In ships of war, where the guns and other masses of iron present a large attracting surface, and in iron ships, the effect of this local attraction on the needle is very considerable, and instances occur when it even amounts to two points.*

The general effect, north of the magnetic equator, is to draw the north point of the needle towards the east on easterly courses, and to the west on westerly courses, a result which may be represented by the action of the south pole of a magnet placed somewhere in the fore part of the ship. The reverse of this is generally the case in the southern hemisphere, so that if the local deviation upon every compass course be obtained and tabulated at one place, it by no means follows that such a table will represent the deviation at another place. And although several mechanical methods for reducing or destroying the effect of the mass of iron in the ship have been devised, the injudicious application of them may produce errors of even greater amount, and therefore it is an imperative duty of the navigator frequently to determine the errors of his compasses by astronomical observations.

It has been found that the *local deviation* varies with many other circumstances besides the position of the ship's head. In steam-ships, which can raise and lower their funnels, the deviation is different with the funnel up and down; and Commander Walker has shown by ob-

* See "Practical Illustrations of the Necessity for Ascertaining the Deviations of the Compass," by the late Captain Edward J Johnson, R N., F R S.

servations on board Her Majesty's iron brig 'Recruit,' that the heeling of the vessel affected the deviations to a very considerable amount. And by observations on board Her Majesty's iron steam-vessel 'Bloodhound,' by Captain Johnson, it was found that when she was heeled 8° to port, the deviations on the north and south points were increased by about 4°, that on the east and west points remaining nearly the same as when the ship was on an even keel.

For further information on this important subject the reader is referred to the work before mentioned of the late Captain Johnson; "The Magnetism of Ships and the Mariner's Compass," by Commander Walker; and "Rudimentary Magnetism" (Weale's series), by Sir W. Snow Harris, &c.

An ingenious and very simple instrument for determining the true course of a ship by astronomical observations, a recent invention of Lieut. Friend, R.N., F.R.S., and called by him the Pelorus,* deserves to be more extensively known, for the many useful purposes to which it can be employed in practical navigation.

PROBLEMS.

Problem I.

When the true course, the variation, and local deviation are given, to find the compass course.

Rule. When the variation is west, allow it to the right; and when east, to the left of the true course

The rule for allowing the deviation is the same.

r is written for right; l for left; r S for right of south; r N for right of north, &c.

Examples.

1. The true course from the Lizard to St. Mary's is SW $\frac{3}{4}$ W, and the variation $2\frac{3}{4}$ points W, and supposing the local deviation to amount to 8° W, what is the compass course?

SW $\frac{3}{4}$ W =	$4\frac{3}{4}$ pts.	.	r S
Var. =	$2\frac{3}{4}$.	. .	r
	$7\frac{1}{2}$ =	84° 23′	r S
	Dev	8 0	r
Compass course		92 23	r S

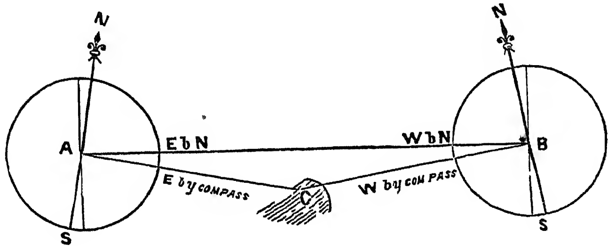
This 92° 23′ r S, being subtracted from 180°, leaves 87° 37′ l N, or the required compass course is N 87° 37′ W, or W $\frac{1}{4}$ N nearly.

* Manufactured by Messrs. Browning, of the Minories.

2. Let it be required to find the course by compass in the same ship, back again from St. Mary's to the Lizard: the true course being NE $\frac{3}{4}$ E, variation $2\frac{3}{4}$ points W, the local deviation will be about the same amount as before, but to the eastward, say 7° 45′ E.

NE $\frac{3}{4}$ E	. . .	$4\frac{3}{4}$ *r* N	
Var	. . .	$2\frac{3}{4}$ *r*	
		$7\frac{1}{2}$ =	84° 23′ *r* N
		Dev	7 45 *l*
			76 38 *r* N

The course by compass is therefore N 76° 38′ E, or about ENE $\frac{3}{4}$ E, whereas the course by compass in the opposite direction was W $\frac{1}{4}$ N, and had there been no local deviation, the compass courses would have been $7\frac{1}{2}$ pts. *r* N and $7\frac{1}{2}$ *r* S, that is. E $\frac{1}{2}$ N and W $\frac{1}{2}$ S, which are diametrically opposite to each other. This may be illustrated by the annexed diagram:—



Let A and B represent two places on the same parallel; a ship at A with her head towards B will have the north end of the needle of her compass drawn eastward in north magnetic latitude; suppose one point to be the amount of deviation, in this case her course to B will be E by N. Again at B, with her head towards A, the north end of the needle will be drawn to the westward as indicated by the *fleur-de-lis*, and her course to A will be about W by N: and if, not knowing the existence of local deviation, she were to steer east from A, or west from B, she would be in danger of running on the reef C, to the southward both of A and B.

Problem 2.

When the course by the compass is given, with the variation and local deviation, to find the true course.

Rule. When the variation is west, allow it to the *left*, and when east, to the *right* of the compass course. The rule for allowing the local deviation is the same.

Example.

If a ship steer by compass NNW $\frac{1}{2}$ W, when the variation is 23° E, and the local deviation for the position of the ship's head 3° $20'$ W, what is the true course?

NNW $\frac{1}{2}$ W =	28°	$7'$	*l* N
Var =	23		*r*
	5	7	*l* N
Dev.	3	20	*l*
	8	27	*l* N

Therefore the required true course is N 8° $27'$ W.

LEEWAY.

The anglê included between the direction of the fore-and-aft line of a ship, and that in which she moves through the water, is called the *leeway*.

When the wind is on the right-hand side of a ship, she is said to be on the *starboard* tack; and when on the left-hand side, she is said to be on the *port* tack; and when she sails as near the wind as she will lie, she is said to be *close-hauled*. Few large vessels will lie within less than six points of the wind, though small ones will sometimes lie within about five points, or even less; but, under such circumstances, the real course of a ship is seldom precisely in the direction of her head; for a considerable portion of the force of the wind is then exerted in driving her to leeward, and hence her course through the water is in general found to be to the leeward of that on which she is steered by the compass. Therefore, to determine the point towards which a ship is actually moving, the leeway must be allowed *from the wind*, or towards the *right* of her apparent course, when she is on the *port* tack; but towards the *left* when she is on the *starboard* tack.

Example.

If a ship's course by compass be N b E $\frac{1}{2}$ E, and she make $2\frac{1}{4}$ points leeway, with the wind at NW $\frac{1}{2}$ W, required her true course? The variation of the compass being 17° $45'$ W, and the local deviation 5° E.

Variation . .	17° 45′ *l*	
Deviation . .	5 0 *r*	
	12 45 *l*	

Compass course .	16° 52′ *r* N	
	12 45 *l*	
Apparent course	4 7 *r* N	
Leeway $2\frac{1}{4}$ points	25 19 *r*	
True course . .	29 26 *r* N	

Therefore the true course is N 29° 26′ E.

Examples for Exercise.

Compass Course	Variation	Deviation	Tack	Leeway.	True Course.
SSW $\frac{3}{4}$ W .	17° W . .	5° W	Port . .	$2\frac{1}{2}$ pts .	S 37° 3′ W
NW b W .	25 W . .	$7\frac{1}{2}$ W	Starboard	3 . .	S 57 30 W
E b N . .	32 E . .	12 E	Starboard	$1\frac{3}{4}$ . .	S 76 56 E
WSW $\frac{1}{2}$ W .	15 E . .	15 W	Port . .	2 . .	N 84 23 W
E $\frac{1}{2}$ S .	21 W . .	4 W	Port .	$2\frac{1}{2}$ . .	S 81 15 E
SW b S . .	25 E .	10 W	Starboard	$1\frac{1}{2}$ . .	S 31 53 W
SSE . .	2 pts W .	5 E	Port . .	3 . .	S 6 15 E
W b N .	$3\frac{1}{2}$ pts E	17 W	Starboard	$3\frac{1}{4}$ . .	S 87 4 W
ESE .	4 pts W .	20 E	Port . .	2 . .	S 70 0 E
NNE $\frac{1}{2}$ E .	$1\frac{1}{2}$ pts W .	2 E	Port .	$2\frac{1}{2}$ . .	N 41 22 E
NE . .	2 pts E .	5 E	Starboard	3 . .	N 38 45 E
NW b W .	$1\frac{1}{4}$ pts W .	11 W	Starboard	$1\frac{3}{4}$.	S 79 0 W

THE LOG.

A ship's rate of sailing is estimated by heaving into the sea a piece of wood called The Log, so loaded with lead that it will just swim. The log is then conceived to remain stationary in the water, and a line is attached to it, called the Log-line, which at its other end is wound round a reel. The reel being turned, the part of the line that is withdrawn from it by the log in a given time is the distance which the ship runs from the log during that interval; and hence, by proportion, her distance for any other time may be obtained, while her rate of sailing continues the same.

The log is made in the form of a sector of a circle, and the lead with which it is loaded is applied to the arc; the central point is therefore vertical in the water.

The line is so attached to it that the flat side of the log is kept towards the ship, that the resistance of the water against the face of the log may prevent it, as much as possible, from being dragged after the ship by the weight of the line or the friction of the reel.

The time which is usually occupied in determining a ship's rate is half a minute, and the experiment for the purpose is generally made at the end of every hour, but in common merchantmen at the end of every second hour. As the time of operating is half a minute, or the 120th part of an hour, if the line were divided into 120ths of a

nautical mile, whatever number of those parts a ship might run in half a minute, she would, at the same rate of sailing, run exactly a like number of miles in an hour. The 120th part of a mile is by seamen called a *knot*, and the knot is generally subdivided into smaller parts, called *fathoms*. Sometimes (and it is the most convenient method of division) the knot is divided into ten parts; more frequently perhaps into eight; but in either case the subdivision is called a *fathom*. In ships where no great accuracy in navigation is attempted, the knot is subdivided into four parts, and sometimes only into two.

We shall, however, consider a fathom as the *tenth* part of a knot; and as a nautical mile (p. 5) is 6079 feet, the 120th part of this, or the length of a knot, will be 50 66 feet, or nearly 50 feet 8 inches. Hence a fathom ought to be 5 feet and eight-tenths of an inch nearly. In practice, however, 50 feet is generally considered as sufficient for the length of a knot, for the log is always in some degree drawn towards the ship, and therefore the distance given by a correct line is always less than the true distance. The operation for estimating the rate is called by seamen *heaving the log*.

The time is measured by a sand-glass, which ought of course to run out in 30 seconds. A quantity of the line, called the *stray line*, is allowed to run out before the glass is turned, that the log may be without the reach of the ship's wake. When the glass is run out, the knots, and parts of a knot between the ship and the mark at the end of the stray line, indicate the distance which the ship has run from the log in the interval of time measured by the sand-glass; hence her hourly rate of sailing is known.

The time which the sand-glass takes in running out, and the length of the knots of the log-line, should frequently be examined; for the time by the sand-glass is materially affected by the state of the atmosphere, and the log-line is liable to contract from the action of the water; and it may happen that the whole line, or different parts of it, may accidentally be stretched.

If either the glass or the line, or both, be found erroneous, the error must be ascertained, and the true distance may then be found by a simple formula, which may be thus investigated:—

1. Let K and D represent the correct length of the knot, and the true distance, and k and d the erroneous knot, and erroneous distance found with it. Now, if the knots used be too short, the number run out in half a minute will be greater than it should be, and the estimated distance will be too great in the same proportion; or, stating this in general terms, the distance found will be inversely as the length of the knots—

$$\therefore\ K : k :: d : D,$$

and consequently

$$D = \frac{k.d}{K}$$

Hence this rule for correcting an erroneous length of knot.

Multiply the estimated distance (d) by the measured length of the knot in feet (k), divide the product by the proper length in feet ($K = 50$), and the quotient will be the corrected distance.

2. If the sand-glass runs out too quickly, then as the line will not be allowed to run out so long as it ought, the estimated distance will be too small, and when the sand-glass runs out too slowly, it will be too great.

Hence the distance estimated by an erroneous sand-glass will be proportional to the number of seconds the sand-glass takes to run out, or if T and t represent the number of seconds the glass should take, and the number of seconds it actually takes to run out—

$$t : T :: d : D$$

Hence the true distance $D = \dfrac{T \cdot d}{t}$

Therefore multiply the distance corrected for any error in the length of the knot by the number of seconds the sand-glass should take to run out ($T = 30^s$), and divide the product by the number it really takes, and the result will be the distance corrected for both errors.

Examples for Exercise.

In the following examples the true distance is required:—

	Dist by Log Miles	Length of Knots Feet	Seconds by Glass	Answer True distance
1.	87	48	26	96
2.	218	53	31	224
3.	146	46	28	144
4.	159	52	27	184
5.	46	51	28	50
6.	102	48	28	105

PLANE SAILING.

In plane sailing the earth is considered as a plane, the meridians as parallel straight lines, and the parallels of latitude as lines cutting the meridians at right angles. And though it is not strictly correct to consider any part of the earth's surface as a plane, yet, when the operations to be performed are confined within the distance of a few miles, no material error will arise from considering them as performed on a plane surface And, as we have already seen, in all questions where the *nautical distance, difference of latitude, departure and course* are the objects of consideration, the results will be the same, whether the lines be considered as curves drawn on the surface of the globe, or as equal straight lines drawn on a plane.

In all maps and charts, and constructions, when it is not otherwise stated, it is customary to consider the top of the page as the north, the lower part as the south, the right side as the east, and the left as the west. The meridians, therefore, in any construction, will be represented by vertical lines, and the parallels of latitude, by horizontal ones.

Hence, in constructing a figure for the solution of any case in plane sailing, the difference of latitude will be represented by a vertical line, the departure by a horizontal one, and the distance by the hypothenusal line, which forms, with the difference of latitude and departure, a right-angled triangle; and the course will be the angle included between the difference of latitude and distance.

With this understanding, the solution of any case that can arise from varying the data in plane sailing will present no difficulty.

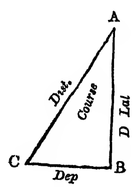
When the course is given, with the distance, difference of latitude or departure, the questions can be readily solved by Tables XVII. and XVIII. The course being found at the top of the page when *less* than half a right angle, and at the bottom when more than half a right angle, taking care also to look for the names of the columns at the same end of the page as the course is found at.

With a little more trouble those problems in which the course is required may be also solved by these tables.

When the distances, &c., are over 300 they may be divided, taking care to multiply the results by the number used in the division.

Examples.

1. If a ship sail from Cape St. Vincent S W ½ S 148 miles, required her latitude in, and the departure which she has made?



Let A represent Cape St. Vincent, A B the meridian, B C the parallel arrived at, to the southward of the Cape, and let C be the place arrived at to the westward of the Cape. Then A C is the distance, A B the difference of latitude, B C the departure, and angle A the course.

Angle A and A C are given quantities, A B and B C are required.

AB = AC. cos A . . and . . BC = AC. sin A.

AC 148	2·170262	AC 148	2·170262
Cos ∠ A 3½ points .	9·888185	Sin ∠ A 3½ points .	9 802359
AB 114·4	2·058447	BC 93·89	1 972621

Thus the ship is 114·4 miles to the southward of the Cape, or the difference of latitude = 1° 54′·4 S.

Latitude of Cape St Vincent	37° 3′ N
Difference of latitude	1 54 S
Latitude in	35° 9′ N

2. If a ship sail from the Cape of Good Hope southwestward till she arrive in latitude 36° 34′ S, and it be found that upon the whole she has made 75 miles of departure, required the course and distance which she has made?

Latitude of Cape of Good Hope . . .	34° 23′ S
Latitude in	36 34 S
Difference of latitude	2° 11′ S = 131 miles.

(See fig. to Example 1.) Here A B represents the difference of latitude 131 miles south, and B C the departure 75 miles west.

$$\text{And } \frac{\mathrm{B\,C}}{\mathrm{A\,B}} = \tan \angle \mathrm{A}; \text{ and A B. sec} \angle \mathrm{A} = \mathrm{A\,C};$$

$$\text{Or } \frac{\textit{Dep.}}{\textit{Diff. Lat.}} = \textit{tan course, and Diff. lat.} \times \textit{sec course} = \textit{distance.}$$

Dep. 75	1·875061	Diff lat 131 . . .	2·117271
Diff. lat. 131 . . .	2·117271	Course 29° 47′ sec	10·061525
Course 29° 47′ . tan	9·757790	Dist 150·9	2·178796

Hence the course is S 29° 47′ W, and the distance 150·9 miles.

Examples for Exercise.

1. If a ship sail from Oporto, latitude 41° 9′ N, NW ¼ W 315 miles, required her departure, and the latitude she has arrived at?

Answer, dep. 233·4 miles W, and lat. 44° 41′ N.

2. If a ship sail from lat. 55° 1′ N, SE by S, till her departure is 45 miles, required the distance she has sailed, and her latitude?

Answer, dist. 81 miles, and lat. 53° 54′ N.

3. A ship from lat. 36° 12′ N, sails southwestward till she arrives in lat. 35° 1′ N, having made 76 miles of departure, required her course and distance?

Answer, course S 46° 57′ W, and dist. 104 miles.

4. A ship from lat. 40° 5′ N sails SW ½ S till she arrives in lat. 36° 7′ N, required her distance and departure?

Answer, dist. 307·9 miles, and dep. 195·3 miles W.

5. A ship from Funchal, latitude 32° 37′ N, sails between the south and west, till her diff. lat. is 114, and her dep. 97 miles, required her course, distance, and latitude?

Answer, course S 40° 24′ W, dist. 149·7 miles, and lat. 30° 43′ N.

6. A ship sails from the Cape of Good Hope, latitude 34° 23′ S, southeastward till she arrives in lat. 40° 10′ S, having run 700 miles, required her course and departure?

Answer, course S 60° 17′ E, and dep. 608·0 miles E.

7. If a ship sail NNW $\frac{3}{4}$ W, 123 miles, from lat. 18° 2′ N, required her departure and latitude arrived at?

Answer, lat. 19° 47′ N, and dep. 63·2 miles W.

8. If a ship from Halifax, latitude 44° 40′ N, sail SE $\frac{1}{2}$ E till her dep. is 128 miles, required her latitude and distance?

Answer, lat. 42° 55′ N, and distance 165·6 miles.

9. If a ship sail from Cape Finisterre, latitude 42° 54′ N, SSW $\frac{3}{4}$ W, 234 miles, required her latitude and departure?

Answer, lat. 39° 33′ N, and dep. 120·3 miles W.

10. If a ship from lat. 50° 16′ N, sail southeastward till her distance is 137, and her departure 112 miles, required her course and latitude come to?

Answer, course S 54° 50′ E, and lat. 48° 57′ N.

11. If a ship sail NW by W $\frac{1}{2}$ W from lat. 30° 14′ N till her departure is 204 miles, required her latitude, and the distance she has sailed?

Answer, distance 231·4 miles, and lat. 32° 3′ N.

12. A ship from lat. 12° 17′ N, sails NE $\frac{1}{4}$ N 201 miles, required her latitude and departure?

Answer, lat. 14° 46′ N, and dep. 135 miles E.

13. A ship sails from the North Cape, in Lapland, in latitude 71° 10′ N, 200 miles, and it is then found that she is to the westward of the Cape, and 125 miles south of it, required her course, latitude, and departure?

Answer, course S 51° 19′ W, lat. 69° 5′ N, dep. 156 1.

14. A ship from the east point of St. Mary, Azores, latitude 36° 58′ N, sails ENE $\frac{1}{2}$ E till she arrives in the latitude of Lisbon, 38° 42′ N, required her distance and departure?

Answer, dist. 358·3, and dep. 342·8.

15. A ship leaving Charleston light, latitude 32° 43′ N, sails northeastward 128 miles, and is then by observation found 39 miles north of the light, required her course, latitude, and departure?

Answer, lat. 33° 22′ N, course N 72° 16′ E, and dep. 122.

16. A ship sails from Cape St. Roque, latitude 5° 25′ S, NE $\frac{1}{2}$ N, 7 miles an hour, from 3 P.M. till 10 A.M., required her distance, departure, and latitude arrived at?

Answer, lat. 3° 42′ S, dep. 84·38, and dist. 133.

17. A ship from lat. 41° 2′ N sails NNW $\frac{3}{4}$ W, $5\frac{1}{2}$ miles per hour for $2\frac{1}{2}$ days, required her distance, departure, and latitude arrived at?

Answer, lat. 45° 45′ N, dep. 169 7, and dist. 330.

18. If a ship sail from lat. 48° 27′ S, SW by W, 7 miles an hour, in what time will she reach the parallel of 50° S?

Answer, 23·914 hours.

19. If a ship sail from Cape Horn, latitude 55° 58′ S, due south 121 miles, and then due west 121 miles, required her course, distance, and latitude?

Answer, course SW, dist. 171·1, lat. 57° 59′ S.

TRAVERSE SAILING.

When a ship is obliged to sail on different courses, the crooked line which she describes is called a traverse, and the method of finding a single course and distance, which would have brought the ship to the same place, is called *resolving a traverse.*

A traverse is resolved by finding the difference of latitude and departure corresponding to each course and distance, and entering them in a table, of which the form will be found in the first of the following examples; taking care, when the ship steers *southward*, to enter the difference of latitude in the column marked S; but in the column marked N, when the course is *northward.* In like manner, when the course is *easterly* or *westerly*, the departure must be entered in the column marked E, or W, accordingly.

Thus, when the course is SE by S, the difference of latitude must be entered in the column S, and the departure in the column E; when the course is W $\frac{1}{4}$ N, the difference of latitude must be entered in the column N, and the departure in the column W; when the course is exactly E, W, N, or S, the whole distance will, of course, be entered in the corresponding column, E, W, N, or S. Then the difference between the sum of the numbers in the column marked N, and the sum of those in the column marked S, will be the whole difference of latitude, and of the same denomination with the greater sum; and in like manner the difference between the sum of the numbers in the columns marked E and W will be the whole departure, and of the same denomination as the greater sum.

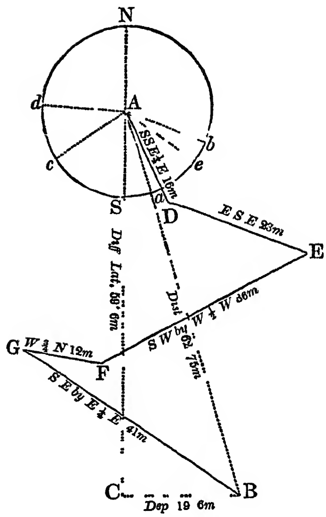
The difference of latitude and departure corresponding to each course and distance are to be taken from Table XVII. when the course is given in points, and from Table XVIII. when it is given in degrees.

Having then obtained the whole difference of latitude and departure which the ship has made, the corresponding course and distance may be computed by "plane sailing."

Examples.

1. A ship from Cape Clear sails SSE $\frac{1}{4}$ E 16, ESE 23, SW by W $\frac{1}{2}$ W 36, W $\frac{3}{4}$ N 12, and SE by E $\frac{1}{4}$ E 41 miles, required the equivalent course and distance, and the latitude of the place which the ship has arrived at?

Fig. 5.



Take A for the place sailed from, and draw the vertical line N A S C representing the meridian. Then A D E F G B may represent the track of the ship, B the place arrived at, A B the distance made good; and B C being drawn perpendicular to the meridian N C, will represent the departure, A C the difference of latitude, and ∠ C A B the course made good.

TRAVERSE TABLE.

Courses.	Dist.	Difference of Latitude		Departure.	
		N	S	E.	W
SSE ¾ E	16	.	14 5	6 8	..
ESE	23	.	8·8	21 3	..
SW by W ½ W	36	..	17·0	..	31·8
W ¾ N	12	1 8	..	..	11·9
SE by E ½ E	41	..	21·1	35·2	..
		1 8	61·4	63·3	43·7
		..	1·8	43·7	
Course S 18° E, dist. 63 m.			59 6	19·6	

Lat. left	. . . . .	51° 25′ N
Diff. lat.	. . . . .	1 0 S
Lat. in	. . .	50 25 N

$$Tan\ course = \frac{dep.}{d\ lat},\qquad Dist = diff\ lat. \times sec\ course$$

Dep	19·6	1·292256	D lat 59 6		1·775246
D lat	59 6	1 775246	Course 18° 12′	sec	10 022289
Course S 18° 12′ E tan		9·517010	Dist. 62·74		1·797535

therefore the course is S 18° 12′ E, and the distance 62·74 miles.

2. If a ship sail from latitude 28° 46′ S, on the following compass courses, viz, SW ¾ W 62, S by W 16, W ¾ S 40, SW ¾ W 29, S by E 30, and S ¾ E 14 miles, required her latitude and the course and distance made good; the variation of the compass being 21½° W?

The variation being given in degrees, the courses may be expressed in degrees, and then corrected for the variation as follows—

1 SW ¾ W = 53½° r S
Var. . = 21½ l
S 32 W

2 S by W = 11¼° r S
21½ l
S 10¼ E

3 W ¾ S = 81½° r S
21½ l
S 60 W

4 SW ¾ W as the first.

5. S by E = 11¼° l S
21½ l
S 32¾ E

6 S ¾ E = 8½° l S
21½ l
S 30 E

Courses	Dist	Difference of Latitude.		Departure.	
		N.	S	E	W.
S 32° W	62	..	52 6	..	32·9
S 10 E	16	..	15 8	2 8	..
S 60 W	40	..	20 0	.	34 6
S 32 W	29	.	24 6	.	15·4
S 33 E	30	..	25 2	16·3	..
S 30 E	14	..	12·1	7·0	..
			6,0)15,0·3	26·1	82·9
			D lat 2° 30′·3 S		26·1
				Dep	56·8 W

Lat. from .	28° 46·0′	S
D lat. . .	2 30 3	S
Lat. in . .	31 16·3	S

$$Tan\ course = \frac{Dep}{Diff.\ lat.}\qquad Dist = Diff.\ lat. \times sec\ course$$

Dep 56·8	11·754348	Diff. lat 150·3		2·176959
Diff lat. 150·3	2·176959	Course 20° 42′	sec	10·028982
Course S 20° 42′ W tan	9 577389	Dist. 160·6		2·205941

EXAMPLES FOR EXERCISE.

1. A ship sails from Cape Clear, in latitude $51^\circ 25'$ N, S by W 23, WSW 40, SW $\frac{3}{4}$ W 18, W $\frac{1}{2}$ N 28, S by E 12, and SSE $\frac{3}{4}$ E 16 miles, required her course, distance, and latitude come to?

Answer, course S $45^\circ 47'$ W, dist 102·4, and lat. $50^\circ 14'$ N.

2 If a ship sail from Porto Sancto in latitude $33^\circ 3'$ N, ENE 18, NE $\frac{1}{2}$ E 40, N $\frac{1}{4}$ E 13, NW $\frac{3}{4}$ W 16, and NE $\frac{1}{2}$ N 23 miles, required her course, distance, and latitude?

Answer, course N $34^\circ 30'$ E, dist 88·1, and lat. $34^\circ 16'$ N.

3. A ship from lat. $41^\circ 12'$ N sails SW b W 21, SW $\frac{1}{2}$ S 31, WSW $\frac{1}{2}$ S 16, S $\frac{3}{4}$ E 18, SW $\frac{1}{4}$ W 14, and W $\frac{1}{2}$ N 30 miles, required her course, distance, and latitude arrived at?

Answer, course S $52^\circ 49'$ W, dist. 111·7, and lat. $40^\circ 5'$ N.

4. If a ship sail from lat. $44^\circ 16'$ N the following true courses and distances, viz. :—N 38° W 50 miles, N 47° W 57 miles, N 10° E 47·8 miles, N 33° E 49·5 miles, and N 55° E 101 miles, required her course and distance made good, and latitude in?

Answer, course N $7^\circ 20'$ E, dist. 226·6, lat. $48^\circ 0' 48''$ N.

5. If a ship sail from Halifax, latitude $44^\circ 40'$ N, E $\frac{1}{2}$ S 23, SE b E 30, E b N 45, and NE $\frac{3}{4}$ N 25 miles, required her latitude in, and course and distance made good?

Ans., lat. in $44^\circ 50'$ N, course N $84^\circ 42'$ E, and dist. 107·2 miles.

6. Yesterday, on leaving the Lizard, latitude $49^\circ 58'$ N, the land bore from us NE 18 miles, since that time we have sailed WSW 14, SW by W 26, SW $\frac{1}{2}$ S 37, SSW $\frac{1}{4}$ W 29, and W 15 miles, required our present latitude, and the course and distance which we have made?

Ans, lat. in $48^\circ 31'$ N, course S $48^\circ 20'$ W, and dist. 131·3 miles.

7. On leaving the Cape of Good Hope for St. Helena, we took our departure from Cape Town, latitude $33^\circ 56'$ S, bearing SE b S 12 miles; after running NW 36, and NW b W 140 miles, required our latitude in, and the course and distance which we have made?

Ans., lat. in $32^\circ 3'$ S, course N $52^\circ 41'$ W, and dist. 187 miles.

PARALLEL SAILING.

In the questions under the heads of plane sailing and traverse sailing nothing has been said of the change of longitude which the ship always makes whenever she sails in any direction excepting on the true meridian.

In order, however, to determine the ship's position, it is necessary to know both how much her latitude, and how much her longitude is changed, since her position was last determined.

We will, however, first consider the simple case of a ship sailing on a parallel of latitude, by which her longitude is changed but her latitude remains unaltered.

It has been demonstrated at page 8 that the

$$\textit{Meridian distance} = \textit{difference of longitude} \times \textit{cosine latitude}. \qquad (1)$$

where the *meridian distance* denotes the arc of the parallel between the meridian sailed from and the meridian arrived at.

If the difference of longitude is sought, we have

$$\textit{Difference of longitude} = \frac{\textit{Meridian distance.}}{\textit{Cosine latitude.}} \qquad (2)$$

Again, if the latitude is required,

$$\textit{Cosine latitude} = \frac{\textit{Meridian distance.}}{\textit{Difference of longitude.}} \qquad (3)$$

By means of these relations the following questions may be solved.

EXAMPLES.

1. A degree of a great circle is about $69 \cdot 3$ English statute miles; how long is a degree of the parallel of London in latitude $51^\circ\ 29'$ N?

Let the difference of longitude $= L = 69 \cdot 3$ miles,
And the meridian distance $= M = 1^\circ$ of the parallel.
Then $M = L$ *cos latitude.*

L 69·3	. . . .	1·840733
Lat 51° 29′	. . cos	9 794308
M 43 16	. . . .	1 635041

The answer is therefore $43 \cdot 16$ miles.

2. If a ship sail on the parallel of 60°, and change her longitude 2°, required the meridian distance? Answer, 60 miles.

3. If a ship sail east from Cape Race 212 miles, required her longitude? Latitude of Cape Race 46° 40′ N, and longitude 53° 3′ W.
Answer, longitude 47° 54′ W.

4. Two places are both in latitude 50° 12′, and their difference of longitude is 34° 48′, required their distance from each other?
Answer, 1336 miles.

5 Where will the distance run by a ship on a parallel of latitude be one-third of the difference of longitude she makes?
Answer, in latitude 70° 31′ 44″.

6. One place is carried by the earth's rotation 278·6 miles per hour faster than another, and the difference of their latitudes is 20°, what are their latitudes?
Answer, 53° and 73°.

7. One place is carried twice as fast as another place by the rotation of the earth, and their difference of latitude is 20°, required their latitudes?
Answer, 52° 7′ and 72° 7′.

8. If the still air at A were transferred to B, β degrees further north, it would cause a wind of b miles per hour; and if transferred to C, β degrees further south, it would cause a wind in the contrary direction of c miles per hour; find the latitude of A when the ratio of b to c is 5 to 4 and $\beta = 20°$.
Answer, latitude 57° 47′.

MIDDLE-LATITUDE SAILING.

Whenever a ship sails in any direction, excepting on a meridian, or at right angles to the meridians, both her latitude and longitude are changed; and whenever the difference of longitude made on an oblique course comes under consideration, the principle mentioned at page 9 may be employed, viz.,

$$\textit{Departure} = \textit{difference of longitude} \times \textit{cosine middle lat.} \quad (1)$$

From this equation we have also

$$\textit{Difference of longitude} = \textit{departure} \times \textit{secant middle lat.} \quad (2)$$

Or, since by plane sailing,

$$\textit{Departure} = \textit{distance} \times \textit{sine course.}$$

Therefore

$$\textit{Difference of longitude} = \textit{dist.} \times \textit{sin. course} \times \textit{secant mid. lat.} \quad (3)$$

By plane sailing, also,

$$\textit{Departure} = \textit{difference of latitude} \times \textit{tangent course.}$$

And equating this to the second side of equation (1) and dividing by the difference of latitude,

$$\textit{Tangent course} = \frac{\textit{Difference of longitude} \times \textit{cosine middle lat.}}{\textit{Difference of latitude.}} \qquad (4)$$

And these equations (1), (2), (3), (4) are the principal of those which belong to middle-latitude sailing.

Examples.

1. Required the latitude and longitude arrived at after sailing from latitude $32^\circ\ 30'$ N, longitude $25^\circ\ 24'$ W, NW by W, 212 miles.

Diff. lat. = Dist. × cos course

(1.)		
Dist. 212		2·326336
Course 5 pts. .	cos	9·744739
D. lat. 117·8 . .		2·071175

Dep. = Dist. × sin course.

(2.)		
Dist 212		2·326336
Course 5 pts.	sin	9·919846
Dep. 176 3 . .		2·246182

(3.)		
Lat. from . .	$32^\circ\ 30'$	N
Diff. lat. 117·8	1 57·8	N
Lat. in . . .	34 27·8	N

(4)		
Lat from . . .	$32^\circ\ 30'$	N
Lat in	34 27·8	N
	2)66 57 8	
Middle lat. . .	33 28 9	

(5.) *Diff. long. = Dep. × sec mid lat.*

Dep. 176 3		2·246182
Mid. lat $33^\circ\ 29'$	sec	10·078810
Diff. long 211·3 .		2 324992

(6)		
Long from . .	$25^\circ\ 24'$	W
Diff. long 211 3	3 31 3	W
Long. in . .	28 55 3	W

Therefore the answer to the question is, latitude in $34^\circ\ 27'\cdot 8$ N, longitude in $28^\circ\ 55'\cdot 3$ W.

2. Required the true course and distance from A to B, the latitude of A $52^\circ\ 34'$ N, the longitude $41^\circ\ 18'$ W, and the latitude of B $51^\circ\ 20'$ N, and longitude $40^\circ\ 57'$ W?

Lat. A	$52^\circ\ 34'$ N
Lat B	51 20 N
Diff. lat.	1 14 = 74 S
	2)103 54
Mid. lat	51 57

Long A	$41^\circ\ 18'$ W
Long B	40 57 W
Diff long	21 E

(1.) $Dep = Diff\ long. \times \cos mid\ lat$

Diff long 21	. .	1·322219
Mid lat 51° 57′ cos		9 789827
Dep. 129 4	. .	1·112046

(2.) $Tan\ course = \dfrac{Dep}{Diff\ lat}$

Dep 129 4	. .	1·112046
Diff lat 74	. .	1 869232
Course 9° 55′ tan	.	9·242814

$Dist = Diff\ lat. \times sec\ course$

Diff lat 74	. . .	1·869232
Course, sec	. . .	·006538
Dist. 75·12	=	1·875770

Therefore the course is S 9° 55′ E, and the distance 75·12.

The difference of longitude in the first question, and the course in the second question, may be computed directly by means of equations (3) and (4), and the work stands thus:—

D. long. in 1st Question.

Dist. 212	. . . . .	2·326336
Course 5 points	. sin	9 919846
Mid lat. 33° 29′	. sec	10·078810
D. long 211·3	. .	2·324992

Course in 2nd Question.

D long 21	. .	1·322219
Mid. lat. 51° 57′	cos	9·789827
		11 112046
D. lat. 74 . . .	—	1 869232
Course S 9° 55′ E tan		9·242814

It will be noticed that the tabulations are a little more concise.

Examples.

In these questions the latitude and longitude of A, and the course and distance sailed, are given, and the latitude and longitude of B are required.

A, Place Sailed from, B, Place Arrived at						
	Lat. A	Long A.	True Course	Dist in Miles	Lat B	Long B
	° ′	° ′			° ′	° ′
1	39 30 S	74 20 E	SW b W	210	41 26 7 S	70 30 5 E
2	46 24 N	47 15 W	NE ½ E	270	49 15 3 N	42 4 1 W
3	51 10 S	168 37 E	WNW	415	48 31·2 S	158 42·5 E
4	22 18 S	57 28 E	E by S	317	23 19 8 S	63 5 3 E
5	23 25 S	13 35 W	E	255	23 25 0 S	8 57·1 W
6	20 5 N	154 17 W	ESE ½ E	333	18 28 3 N	148 39·4 W
7	53 30 N	11 46 W	N 80° W	432	54 45·0 N	23 52 0 W
8	0 56 N	29 34 W	S 47° E	168	0 58·6 S	27 31·1 W

In the following questions the compass course and the distance from A to B are required :—

	Lat A	Long A	Lat B	Long B	Variation	Course	Dist in Miles
	° ′	° ′	° ′	° ′	°	° ′	
1	49 35 N	24 50 W	43 37 N	20 17 W	27 W	S 0 39 E	404·2
2	45 16 S	13 46 E	43 10 S	12 22 W	15 W	N. 68 36 W	1130
3	33 20 S	82 45 W	35 21 S	85 15 W	11 ½ E	S. 34 10 W	173·2
4	59 26 N	43 10 W	50 0 N	53 33 W	17 E	S. 15 27 W	670·7

MERCATOR'S SAILING.

Middle-latitude sailing should only be used when the change of latitude is small, or where the course exceeds five points; for when the distance run on a course nearer the meridian is considerable, then the error in the principle of this method may greatly vitiate the results.

In this case, and always when the difference of latitude is considerable, recourse must be had to *Mercator's Sailing*, and then the following are the rules and formulæ to be employed :—

$$\textit{Difference of longitude} = \textit{meridional diff. lat.} \times \textit{tan course.}$$

$$\textit{Tangent course} = \frac{\textit{Difference of longitude}}{\textit{Meridional difference latitude.}}$$

1. *To find the difference of longitude on a given course.*—Add the logarithm of the meridional difference of latitude to the logarithm tangent of the course, and the sum is the logarithm of the difference of longitude.

2. *To find the course between two known places.*—Subtract the logarithm of the meridional difference of latitude from the logarithm of the difference of longitude, and the remainder is the logarithm tangent of the course.

Example 1. Required the course and distance from the east point of St. Michael's, Azores, to the Start ?

	Lat	Tab. (XIX)		Long.
Start	50° 13′ N	3478	. . . .	3° 38′ W
St Michael's . .	37 48 N	2440	. . . .	25 10 W
Diff. lat. = 745 m	12 25 N	1038	Mer d. lat Diff long.	21 32 = 1292 m.

$$\text{Tan course} = \frac{\text{Diff. long}}{\text{Mer diff lat}} \qquad \text{Dist.} = \text{Diff lat} \times \text{sec course.}$$

Diff long 1292 . . .	3 111263	Diff lat. 745	2 872156
Mer diff lat 1038 . .	3 016197	Course 51° 13′ . .	sec 10 203164
Course N 51° 13′ E tan	10·095066	Dist. 1189	3·075320

Thus the true course on the rhumb line is N 51° 13′ E, and the distance 1189 miles.

2. If a ship sail from Cape Finisterre SE by E 1200 miles, what will be the latitude and longitude arrived at?

(1) *Diff lat = Dist cos course*			(3) *Diff long = M diff. lat tan course.*	
Dist. 1200 . . .	3 079181		M diff lat 838. .	2·923244
Course 5 pts . .	9 744739		Course 5 pts. . .	10 175107
Diff lat 666·7 S	2 823920		Diff long 1254 E	3·098351
(2) Cape Finisterre lat	42° 54′ 0 N	2840	(4) Long. C F.	9° 16′ W
Diff lat 666·7 =	11 6·7 S		Diff long. 1254 =	20 54 E
Lat arrived at . .	31 47·3 N	2002	Long. in . . .	11 38 E
	Mer diff lat.	838		

therefore the latitude arrived at is 31° 47′·3 N, and the longitude 11° 38′ E.

Examples for Exercise.

1. If a ship from Lisbon, (latitude 38° 42′·5 N, longitude 9° 8′ W,) sail WSW ½ W 168 miles, required her latitude and longitude in?
Answer, lat. 37° 54′ N, long. 12° 33′ W.

2. If a ship sail from Cape Race, (latitude 46° 40′ N, longitude 53° 3′ W,) SSE ¾ E 216 miles, required her latitude and longitude in?
Answer, lat. 43° 35′ N, long. 50° 26′ W.

3. A ship sails from latitude 40° 12′ N, longitude 18° 3′ W, NE b N ½ N 248, what are her present latitude and longitude?
Answer, lat. 43° 51′ N, long. 15° 26′ W.

4. A ship having reached latitude 43° 51′ N, and longitude 15° 26′ W, as in Question 3, what are the compass course and distance to the Lizard in latitude 49° 58′ N, and longitude 5° 11′ W, the variation of the compass being 27° W?
Answer, compass course N 75° 53′ E, dist. 558 miles.

5. A ship from Cape Horn, in latitude 55° 58′ S, sails to latitude 60° 10′ S, making 3° 38′ difference of longitude westward, what has been her course and distance?
Answer, course S 24° 36′ W, dist. 277 miles.

6. Required the true course and distance from lat. 53° 18′ N, long. 0° 55′ E, to the Naze, latitude 57° 58′ N, longitude 7° 3′ E?
Answer, course N 36° 38′ E, dist. 349 miles.

7. If a ship leave Cape Clear, latitude 51° 25′ N, longitude 9° 29′ W, bearing NE ¾ N 16 miles and sail SW ¾ S 150 miles, what will be the latitude and longitude arrived at?

Answer, lat 49° 12′ N, long 12° 3′ W.

8. Required the course and distance from lat. 49° 12′ N, long. 12° 4′ W, to St. Mary's, Azores, in lat 36° 58′ N, long 25° 12′ W?

Answer, course S 38° 3′ W, dist. 932 miles.

TO FIND THE DIFFERENCE OF LONGITUDE MADE ON A TRAVERSE.

In the following questions the ship has sailed on several courses in succession. Having found the difference of latitude and departure as in traverse sailing, apply the difference of latitude to the latitude left, and find the latitude arrived at

The difference of longitude can then be found either by middle latitude or Mercator's sailing

This method of finding the difference of longitude made upon a traverse is not strictly correct; and it is sometimes desirable, especially in high latitudes, to find the difference of longitude made upon each course separately, and then taking the difference between the sum of those east and the sum of those west, the result will be the required change of longitude.

To do this the *latitude in* at the end of each course must be found, and this is done by applying the first difference of latitude in the traverse table to the latitude sailed from, the next difference of latitude to the latitude last found, and so on. Then find the middle latitude between that sailed from and the *latitude in* at the end of the first course; again between the latitude at the end of the first and second, second and third, third and fourth courses, &c., after which compute the successive differences of longitude from this equation—

Difference of longitude = Departure × sec middle latitude.

The departures to which this rule is to be applied must be taken from the traverse table in succession as they stand there, observing that the difference of longitude is east or west as the course is eastward or westward.

Or, by Mercator's sailing, having found the meridional difference of latitude from the latitude sailed from to that at the end of the first course, from this latitude to that at the end of the second, and so on, compute the successive differences of longitude from this relation—

Difference of longitude = Meridional diff. lat. × tan course.

Or they may be found by Table XVIII., by taking the nearest degree of middle latitude as a course at the top or bottom of the page,

and the departure in the column marked *lat.*, the difference of longitude will be found in the column marked *dist.*; noting that the names of the columns must be looked for at the bottom when the course is found at the bottom, and at the top when the course is at the top of the page.

Or with the course itself at the top or bottom of the page in Table XVIII., and the meridional difference of latitude in the column marked *lat.*, the difference of longitude will be found in the column marked *dep.*

Example.

A ship in latitude 66° 14′ N, longitude 3° 12′ E, is bound for Archangel; after sailing NNE ½ E 46, NE ½ E 28, N ¾ W 52, NE by E ¼ E 57, and ESE 24 miles, required her course and distance to the North Cape?

TRAVERSE TABLE.

Courses	Dist.	Difference of Latitude		Departure	
		N	S	E	W
NNE ½ E	46	40·6	..	21·7	..
NE ½ E	28	17·8	..	21·6	..
N ¾ W	52	51·4	..	..	7·6
NE b E ¼ E	57	29·3	..	48·9	..
ESE	24	..	9·2	22·2	..
		139·1	9·2	114·4	7·6
		9·2		7·6	
Course N 39° ½ E, dist. 168 m		129·9	..	106·8	..

Lat left.	66° 14′ N	Mer parts . .	5339	Long left .	3° 12′ E
Diff. lat.	2 10 N			Diff. long. .	4 37 E
Lat. in .	68 24 N	. . . . . .	5676	Long. in .	7 49 E
	2)134 38	Mer. diff. lat. .	337		
Mid lat.	67 19				

With the diff. lat. 129·9, and departure 106·8, the course is found in Table XVIII. to be about 39½°, and the distance 168 miles. With this course, and mer. diff. lat. 337 in the latitude column, the diff. long. is found in the departure column to be about 277 miles.

Or with the middle lat. 67° 19′ as a course, and the departure 106 8 in the lat. column, the diff. long. is found in the distance column to be nearly 277 miles.

To find the diff. long. made on each course separately by middle-latitude sailing.

	Successive Lat.	Mid Lat.	Departure	Diff Long E	Diff Long W.
Lat left.	66° 14′				
at the end 1st Course.	66 55	66° 34′	21·7 E	54·6	
2nd	67 13	67 4	21·6 E	54·4	
3rd	68 04	67 38	7·6 W		20·0
4th	68 33	68 18	48·9 E	132·3	
5th	68 24	68 28	22·2 E	60 5	
Long left		3° 12′ E		301·8	
Diff long		4 42 E		20·0	
Long in		7 54 E		281·8	

With each middle latitude, and the corresponding departure extracted from the previously given traverse table, the difference of longitude is found and entered in the column which is of the same name with the departure, and the difference between the change of longitude made towards the east and that made towards the west is 281·8; the whole difference of longitude made on the traverse, differing about 5 miles from that found from the difference of latitude and departure made on the whole traverse

To find the difference of longitude made on each course separately by Mercator's sailing.

Courses.	Successive Lat	Mer Parts	Mer diff Lat	Diff Long E.	Diff Long W.
	66° 14′	5339			
NNE ½ E	66 55	5441	102	54 5	
NE ½ E	67 13	5488	47	57·2	
N ¾ W	68 4	5622	134	..	19· 9
NE b E ¾ E	68 33	5700	78	130·2	
ESE	68 24	5676	24	58·2	
Long, left		3° 12′ E		300·1	
Diff long		4 40 E		19·9	
Long in		7 52 E		280 2	

With each course, and the meridional difference of latitude made on that course, the difference of longitude is found and entered in the columns marked E or W, according as the course has been easterly or westerly, and the difference between the sum of the numbers in the E and W columns is 280·2, the total difference of longitude.

Hence the latitude in is 68° 24′, and the longitude computed from the result of the whole traverse is 7° 49′; but by computing the diff. long. for each course separately, the long is 7° 52′ east.

To find the course and distance from the ship to the North Cape.

Ship's place lat.	68° 24′ N	Mer. parts .	5676	Long	7° 52′ E
N Cape . .	71 10 N	. . . . .	6156		25 51′ E
Diff lat 166 =	2 46 N	Mer. diff lat	480	Diff. long.	17 59 = 1079

$$\textit{Tan course} = \frac{\textit{Diff long}}{\textit{Mer diff lat}} \qquad \textit{Dist.} = \textit{Diff lat.} \times \textit{sec course.}$$

Diff long. 1079	. 3·033021		Diff lat 166 . . .	2·220108
Mer. diff lat 480	. 2 681241		Course 66° 1′ sec .	. 10·390971
Course 66° 1′ tan.	. 10 351780		Dist 408 4 . . .	2·611079

Hence the course to the North Cape is N 66° 1′ E, distant 408·4 miles.

Examples for Exercise.

1. If a ship sail from the Naze, latitude 57° 58′ N, longitude 7° 3′ E, on the following true courses, WNW 24, NW ½ W 16, SSW 31, S ¼ E 12, and SW ¾ S 20 miles, required her latitude and longitude? Answer, lat. 57° 21′ N, and long. 5° 15′ E.

2. If a ship sail from Funchal, in latitude 32° 37′ N, and longitude 16° 56′ W, on the following true courses, S 59 W 46, SW ½ W 15, and S ¼ W 24 miles, required her latitude and longitude?

Answer, lat. 31° 5′ N, and long 18° 5′ W.

3. If a ship sail from the Cape of Good Hope, in latitude 34° 23′ S, and longitude 18° 24′ E, on the following true courses, NW 25, N ½ W 21, NNE ¼ E 35, NW ¾ W 40, and N b E 18 miles, required her latitude and longitude?

Answer, lat. 32° 31′ S, and long. 17° 44′ E.

4. If a ship sail from Fort Royal, in the Island of Grenada, in latitude 12° 3′ N, and longitude 61° 48′ W, on the following true courses, NW 46, NNW ¾ W 50, NW b W 70, and W ½ N 21 miles, required her latitude and longitude?

Answer, lat. 13° 59′ N, long 64° 9′ W.

ON A SEA JOURNAL.

A journal is a register of occurrences that take place on board a ship, either in harbour or during a voyage, and it ought to contain a particular detail of everything relative to the navigation of the ship, as the courses, winds, currents, &c, that her situation may be known at any instant at which it may be required.

On commencing a voyage, the true course to the first place which is expected to be seen is either taken from a chart or computed; and thence, from the variation, the compass or steering course is found; and the ship is kept as near that course as the wind and other circumstances will admit When the ship leaves the land, the bearing of some known place is taken by the compass, and its distance is in general estimated by the eye, which seamen soon acquire considerable skill in doing. This is called *taking the departure.* But the distance may be otherwise determined by taking the bearing of an object at

two different times, carefully noting the course and distance run in the interval. For the sine of the change in the object's bearing is to the distance run by the ship in the interval, as the sine of the angle included between the ship's course and the bearing of the object at the first observation is to its distance at the second observation.

But whether the distance of the place from which the departure is taken be estimated or computed, the opposite point to that on which it bears is considered as the first course, and its distance as the first distance sailed from the place; and the other courses and distances made during the day being determined by the compass and the log, they are severally written in chalk, on a painted black board, called the *log-board*, of which the general form will be found below; and afterwards copied into a book, similarly ruled, called the *log-book*. The courses are either corrected for leeway before they are put down on the *log-board*, or the leeway is marked in the proper column opposite the course to which it belongs. The setting and drift of currents, and the estimated effect of the swell of the sea, are also inserted in the column of remarks; and this column, besides, must contain an account of every occurrence deemed of importance.

Then the several courses in the log-book being corrected, as in the preceding problems, for leeway and variation, and the distances on each course summed up and entered in a traverse table, care being taken to introduce the effect of currents and the swell of the sea, when any exist, as separate courses and distances, the latitude and longitude arrived at are determined by the methods which have already been explained, under the different heads of practical Navigation.

The computation of the ship's place from the reckoning is always made at noon, and the operation is called working a *day's work*. An abstract of the result is inserted in the log-book, containing the true course and distance which the ship on the whole has made during the day; the latitude and longitude as deduced from the reckoning, with those also which are obtained from observations; and the bearing and distance of the port, or of the nearest land that lies in the ship's way.

When the variation of the compass is given in degrees, it will be found convenient first to correct the courses in the traverse-table for leeway only, and, with these courses and distances, to find the difference of latitude and departure, and thence the *compass course* and the distance made during the day. The resulting course being then corrected for variation, with it and the distance already found, the *true* difference of latitude and departure may be readily obtained; and the calculation for the difference of longitude may then be made in the usual manner.

In hard-blowing weather, with a contrary wind and a high sea, it is impossible to gain any advantage by sailing, and the object is then to avoid, as much as possible, being driven back. To effect this object it is usual to lie-to under no more sail than is necessary to prevent the violent rolling which the ship would otherwise acquire. The tiller

being put over to leeward, brings the ship's head round towards the wind, which then having little power on her sails, she loses her way through the water, and the directive power of the rudder consequently ceases; her head falls off from the wind, the sail which she has set fills again, and gives her fresh way through the water, which, acting on the rudder, brings her head about to the wind, and she thus comes up and falls off alternately. In such cases the middle point between those on which she comes up and falls off is taken as her apparent course, and the leeway and variation being allowed from that point, the result is entered as a course in the traverse-table, with the estimated drift of the ship through the water as a distance.

But notwithstanding all the care that can be taken in keeping a sea-reckoning, the place of the ship as deduced from it must always be considered as only *approximately* determined. It is often extremely difficult to arrive at any considerable certainty respecting the precise course and distance which a ship has actually made. Different rates of sailing between the times of heaving the log, want of care in steering, and sometimes also the great difficulty in steering steadily; sudden squalls, incorrect allowances for leeway and variation, and many other circumstances, conspire to render the ship's place, as deduced from the common reckoning, very uncertain. No opportunity ought therefore to be lost to determine her place by celestial observations; and the sea-reckoning should chiefly be considered as a means of estimating her situation *nearly* in the interval between such observations.

The log is dated according to the civil reckoning of time, and the place of the ship therefore, computed at noon, is for the middle of the civil day: the courses or distances in the afternoon, or P.M. of the preceding day, being connected with those made in the morning, or A.M. of the current day, in computing the change of her place from the last noon.

When the place of the ship has been determined by observation, her place by the reckoning ought to be disregarded, and the future reckoning should be carried forward from her place found by observation, till other observations give a new point of reference. But some mariners keep a separate account of the ship's place, by the reckoning and by observation, during the whole voyage; which, as they conceive, enables them to judge of the total effect of currents and other generally operating causes of error on the reckoning.

EXAMPLES.

1. April 26th, 1858, at noon, a point of land in latitude 36° 35′ S and longitude 110° 25′ W, bore by compass ENE ¾ E, distant 18 miles (the ship's head being SE by S, by compass, deviation 0), afterwards sailed as by the following log account; required the latitude and longitude in on April 27th, at noon?

H.	K	$\frac{1}{10}$ths	Courses.	Wind.	Lee-way	Dev^n^.	Remarks.
					Points	° ′	
1	2	6	SW ¾ W	S b E ¼ E	2¼	7 27W	P.M.
2	3	5					
3	4	3					
4	3	2	WSW ½ W	S b W	2½	8 20W	
5	4	4					
6	2	3					Variation of compass 1¾ pts. E.
7	2	5	WNW ½ W	SW	2	8 10W	
8	3	3					
9	4	1					
10	5	4					
11	4	2					
12	4	4					
1	3	2	NW ½ W	WSW ½ W	2½	4 50W	A.M.
2	4	1					
3	5	4					
4	3	2					A current set the ship the last 8 hours by compass E ½ S 2 miles an hour.
5	3	2	W b S	S ½ W	1½	8 20W	
6	4	1					
7	4	2					
8	3	4					
9	3	4	SW	S b E	2½	7 0W	
10	5	2					
11	2	6					
12	3	5					

1. True courses, and distances in miles, N 84° W, 18·0; N 89° W, 10·4; N 67° W, 9·9; N 39° W, 23·9; N 8° W, 15·9; N 73° W, 14·9; S 86° W, 14·7; S 65° E, 16. Latitude in 35° 58′ S; longitude in 111° 51′ W. Course and distance made good, N 62° W, 78 miles.

E

2. May 20th, 1858, at noon, a point of land in latitude 47° 11′ N, and longitude by account 3° 12′ W, bore by compass ENE ¼ E, distant 16 miles (the ship's head being S by compass, deviation 3° W), afterwards sailed as by the following log account; required the latitude and longitude in on May 21st, at noon?

H.	K.	$\frac{1}{8}$ths	Courses.	Wind.	Lee-way	Devn.	Remarks.
					Points.	° ′	
1	2	5	ENE ¾ E	N ½ E	2	10 0E	P.M.
2	3	3					
3	4	1					
4	3	2					
5	3	5	WNW	N ¼ E	1½	6 50W	
6	4	2					Variation of the compass 3¼ pts. W.
7	3	6					
8	3	7					
9	4	2	SSW ¼ W	West	2½	5 0W	
10	4	1					
11	3	6					
12	3	2					
1	4	1	NNW ¼ W	West	1¾	1 40W	A.M.
2	3	2					
3	4	1					
4	4	1					
5	4	2					A current set the ship the last 3 hours by compass SSW 2 miles an hour.
6	3	6	SE ¾ E	SSW	1½	3 40E	
7	3	7					
8	4	1					
9	2	5	SW ¾ W	S b E ¼ E	2¾	7 27W	
10	4	2					
11	4	1					
12	5	2					

2. True courses, and distances in miles, S 31° W, 16·0; N 72° E, 13·1; S 52° W, 15·0; S 44° E, 15·1; N 44° W, 19·7; N 77° E, 11·4; S 40° W, 16·0; S 14° E, 6·0. Latitude in 46° 40′ N; longitude in 3° 24′ W. Course and distance made good, S 15° W, 32·2 miles.

3. June 5th, 1858, at noon, a point of land in latitude 56° 59′ N, and longitude by account 75° 33′ W, bore by compass NE by E, distant 19 miles (the ship's head being N by compass, deviation 2° 45′ E), afterwards sailed as by the following log account; required the latitude and longitude in on June 6th, at noon?

H.	K.	$\frac{1}{10}$ths	Courses.	Wind.	Lee-way	Dev^n.	Remarks.
					Points	° ′	
1	4	9	N b W	W b N	$1\frac{1}{2}$	1 10E	
2	4	4					
3	4	2					
4	4	7					
5	2	0					Variation of the compass $1\frac{1}{2}$ pts. W.
6	2	4	SSW $\frac{3}{4}$ W	W b N	$2\frac{1}{4}$	6 7W	
7	2	3					
8	3	1					
9	3	3					
10	2	5	NNE $\frac{3}{4}$ E	NW $\frac{3}{4}$ N	2	9 0E	
11	3	2					
12	2	4					
1	3	0	W b N	NNW	$2\frac{1}{4}$	8 10W	
2	2	5					
3	2	1					
4	3	2					A current set the ship the last 5 hours WNW (by compass) 3 miles an hour.
5	4	1	SE $\frac{3}{4}$ E	SW $\frac{1}{4}$ W	0	3 40E	
6	5	3					
7	2	6					
8	4	1					
9	3	2					
10	3	2	S $\frac{1}{2}$ W	WSW	$2\frac{1}{2}$	3 30W	
11	3	2					
12	1	6					

3. True courses, and distances in miles, S 42° W, 19·0; N 10° W, 20·2; S 17° E, 11·1; N 46° E, 8·1; S 51° W, 10·8; S 67° E, 19·3; S 43° E, 8·0; N 84° W, 15·0. Latitude in 56° 41′ N; longitude in 75° 47′ W. Course and distance made good, S 22° W, 19·2 miles.

4. July 3rd, 1858, at noon, a point of land in latitude 55° 5′ S and longitude 67° 10′ W, bore by compass NE ½ E, distant 3 miles (the ship's head being N by compass, deviation 2° 45′ E), afterwards sailed as by the following log account; required the latitude and longitude in on July 4th, at noon?

H.	K	$\frac{1}{10}$ths	Courses.	Wind.	Lee-way	Devn.	Remarks
					Points	° ′	
1	3	4	N ¼ W	NW b W ¾ W	3¼	2 45 E	
2	3	7					
3	4	2					
4	3	9	W b S	NW b N	2½	8 20 W	
5	2	8					Variation of the compass 3¾ pts. W.
6	2	9					
7	3	7					
8	2	9	NW ¾ W	NNE ¼ E	2¾	5 40 W	
9	2	8					
10	2	5					
11	1	7					
12	1	5	E	SSE ½ E	2½	8 50 E	
1	2	1					
2	1	5					
3	3	1					
4	3	2	SW b S	SE ½ S	2¾	6 7 W	A current set the ship the last 5 hours by compass E by S 3½ miles an hour.
5	3	9					
6	3	8					
7	2	7					
8	3	4	N b E	ENE	3¾	4 57 E	
9	4	3					
10	4	3					
11	4	6					
12	3	4					

4. True courses, and distances in miles, S 11° W, 13·0; N 6° W, 11·3; S 13·3; S 48° W, 9·9; N 29° E, 8·2; S 16° W, 13·6; N 68° W, 20·0; N 59° E, 17·5. Latitude in 55° 16′ S; longitude in 67° 35′ W. Course and distance made good, S 53° W, 18 miles.

GREAT CIRCLE SAILING.

1. It has already been mentioned that the distance run by the ship upon the rhumb line is not the shortest distance; the adoption of this line as the guide from place to place has arisen from the circumstance that the course is uniform throughout the track, and from the great simplicity with which the course can be found by the aid of a Mercator's Chart. The uniformity of the course on the rhumb line, and the straight line which represents the rhumb line upon the Chart, are apt to give an idea of directness which does not really belong to this manner of sailing.

2. The shortest distance between two places is the arc of the great circle which passes through them, and therefore it would in all cases be better to sail upon this arc than upon the rhumb line.

3. Now, a great circle passing through any two given places will meet the equator at two opposite points; and the points midway between them will be those which are most remote from the equator, or those which have the greatest N or S latitude.

4. Of the two places between which the arc of the great circle lies, that which is in the highest latitude is denoted in the following rules by the letter B, and the other by A.

5. Of the two points furthest from the equator mentioned in paragraph 3, that one which lies nearest to B is called the Vertex, and is denoted by V.

6. The point V may or may not fall between the given places. But if they are on the same parallel of latitude, it will be exactly half-way between them.

7. The equality of the course is not a feature of the great circle track, and therefore it is necessary from time to time to change the direction in which the ship is steered, and the method of finding where and how much the course must be changed constitutes *Great Circle Sailing*.

8. And in the following rules it is divided into these five problems:—

1. The computation of the shortest distance.
2. To find the latitude of the vertex V.
3. To find the longitude of the vertex V.
4. To find a succession of points upon the great circle.
5. To compute the course and distance from point to point.

1. *Direct computation of distance.*

1. Add log. cos latitude A, log. cos latitude B, and twice log. sin $\frac{1}{2}$ diff. long., and after rejecting 20 from the index, half the sum is the log. cos of an angle x.

Formula, $\cos x = \sqrt{(\cos lat.\ A \times \cos lat.\ B \times \sin^2 \frac{1}{2} D\ long.)}$

2. Take the sum and difference of x and $\frac{1}{2}$ diff. lat.

3. Add together the log. sines of this sum and difference, and half the sum is the log. cos $\frac{1}{2}$ distance.

Formula, $\cos \frac{1}{2} distance = \sqrt{\{\sin (x + \frac{1}{2} D\ lat.) \cdot \sin (x - \frac{1}{2} D\ lat.)\}}$

2. *To find the latitude of the vertex.*

1. Add log. cos latitude A, log. cos latitude B, and log. sin diff. long. together, and subtract log. sin distance, the remainder with its index diminished by 10 is the log. cos of the latitude of the vertex.

Formula, $\cos lat.\ V = \dfrac{\cos lat.\ A \times \cos lat.\ B \times \sin diff.\ long.}{\sin dist.}$

3. *To find the longitude of the vertex.*

1. Add log. tan latitude A, to log. cot of the latitude of the vertex, and after rejecting 10 from the index, the sum is log. cos of the difference of longitude between A and the vertex.

Formula, $\cos diff.\ long. = \tan lat.\ A. \times \cot lat.\ V.$

2. This difference of longitude is east or west of A, according as B is east or west of A; and allowing it upon the longitude of A, the longitude of the vertex is found.

If this computed difference of longitude is less than the difference of longitude between A and B, the vertex lies between them, otherwise the vertex is east or west of both B and A, according as B is east or west of A.

Examples.

1. Required the distance on a great circle from the Cape of Good Hope to Swan River?

	Lat			Long.
Cape of Good Hope . .	34° 0′ S . .	B		18° 20′ E
Swan River	32 3 S . .	A		115 45 E
Diff lat	1 57	Diff long .		97 25
Half diff. lat	58·5	Half diff. long.		48 42·5

Half diff. long 48° 42′ 5	. . . . . .	sin	9·875848
			2
			19 751696
Lat. A 32° 3′	. . . .	cos	9·928183
Lat. B 34 0	. . . .	cos	9·918574
			2)19·598453
x . . . . 50° 57′·5	. . .	cos	9·799226
Half diff. lat. . . . 58·5	. . .		
Sum . . . . . . 51 56·0	. . .	sin	9·896137
Diff. . . . . . . 49 59·0	. . .	sin	9·884148
			2)19·780285
Half distance. . . 39° 3′·5	. . .	cos	9 890142
2			
78 7			
60			
Distance . . . . 4687 miles.			

2. Required the greatest south latitude reached on this voyage?

Latitude of A	32° 3′	. . .	cos	9·928183
Latitude of B	34 0	. . .	cos	9·918574
Difference of longitude . .	97 25	. . .	sin	9·996351
				29·843108
Distance	78 7	. . .	sin	9·990591
Latitude of the vertex V .	44° 35′·5	. .	cos	9·852517

3. Required the ship's longitude when in the highest latitude?

Latitude of A 32° 3′ S	. . . . . .	tan	9·796632
Latitude of V 44 35·5	. . . . . .	cot	10·006191
50° 35′	. . . . . .	cos	9·802823

∴ 50° 35′ = the difference of longitude between A and V, west because B is west of A.

Longitude of A	115° 45′ E	
Difference of longitude	50 35 W	
Longitude of vertex V	65 10 E	

To find a succession of points on the great circle between two places A and B.

1. GENERAL RULE. To the log. cosine of the difference of longitude between the vertex and any other point on the great circle, add the log. tangent of the latitude of the vertex, and the sum, rejecting 10 from the index, is the log. tangent of the latitude of the other point.

Tangent latitude P = cosine difference of longitude × tangent lat. V.

EXAMPLE. The longitudes, 30°, 50°, 70°, 90°, E, lie between the longitudes of the Cape of Good Hope and Swan River, required the ship's latitude when she is in each of these longitudes while sailing on a great circle between these places?

The longitude of the vertex has been found to be 65° 10′ E, and therefore the differences of longitude between the vertex and each of the given longitudes are found thus—

Long V	65° 10′ E	65° 10′ E	65° 10′ E	65° 10′ E
Long	30 0 E	50 0 E	70 0 E	90 0 E
Diff. long.	35 10	15 10	4 50	24 50

And then the required latitudes are computed as follows—

Diff. long. 35° 10′	cos 9·912477		Diff long. 15° 10′	cos 9·984603	
Lat. V 44 35·5	tan 9·993809			9 993809	
38 52	tan 9·906286		43 35	tan 9·978412	

Diff. long 4° 50′	cos 9·998453	Diff long 24° 50′	cos 9·957863
Lat. V 44 35·5	tan 9 993809		9 993809
44 29	tan 9·992262	41 49	tan 9·951672

Therefore the following points, including the vertex, lie upon the great circle—

	C	D	V	E	F
Lat.	38° 52′ S	43° 35′ S	44° 35 5 S	44° 29′ S	41° 49′ S
Long	30 0 E	50 0 E	65 10 E	70 0 E	90 0 E

The places C, D, V, E, and F can now be found upon the chart, and the track neatly traced through them with a pencil; this will show what islands, rocks, &c., lie on the way.

Nothing now remains but to find the course from the Cape to C, from C to D, from D to E, and so on, until the voyage is completed by sailing from G to Swan River. And these courses may be computed by middle-latitude sailing.

Cape of Good Hope	34° 0′ S	18° 20′ E
C	38 52 S	30 0 E
Diff. latitude	4 52 = 292 S	11 40 E
	2)72 52	
Middle latitude	36 26	Difference of longitude 700 E

For the course.

Diff. long. 700	2·845098
Middle lat. 36° 26 cos	9·905552
	12·750650
Diff. lat 292	2·465383
Course S 62° 36′ E tan	10·285267

For the distance.

Diff lat. 292	2·465383
Course 62° 36′. sec	337054
Distance 634·5	2·802437

Thus on the first run the true course is S 62° 36′ E, and dist. 634·5 miles. In the same way the course and distance is found from C to D, D to V, &c.

to C	S 62° 36′ E	634·5
C to D	S 72 36 E	946·4
D to V	S 84 43 E	657
V to E	N 88 12 E	206 9
E to F	N 79 39 E	890 6
and F to Swan River	N 64 37 E	1367 0
	Sum	4702·4

We may now inquire how much has been saved by thus sailing from point to point of the great circle, by finding the distance which would have to be sailed on the rhumb line, and since the Cape and Swan River differ less than 2° in latitude, this may be done by middle-latitude sailing.

	° ′	° ′
Cape of Good Hope	34 0 S	18 20 E
Swan River	32 3 S	115 45 E
Diff lat 117 miles	1 57 N	97 25 E
		60
	2)66 3	Diff. long 5845
Middle latitude	33 1·5	

For the course.

Diff long 5845		3·766785
Mid. lat. 33° 1′·5	cos	9·923468
		13·690253
Diff. lat 117		2 068186
Course N 88° 38′ E	tan	11·622067

For the distance

Diff lat. 117		2·068186
Course 88° 38′	sec	11·622501
Dist. 4905·5 miles		3·690687

Therefore the distance on the rhumb line is 4905·5, and subtracting the whole distance found by the other method,

On the rhumb line	4905·5
From point to point of the great circle	4702·4
Difference	203·1

Thus 203 miles have been saved. It will be seen that the whole distance on the great circle, 4687 miles, is less than either of these, but by increasing the number of points between the places, the sum of the distances from point to point of the great circle may be made to agree still more closely with the distance on the great circle. The last run of 1364 miles, for instance, might have been divided into two distances of about 700 miles each, by assuming another longitude of 100° E between the longitude 90° E and Swan River.

Second Method.

To find the longitude of the vertex.

The longitude of the vertex may be computed directly from the latitudes and longitudes of the places between which the voyage is made; and independently of the distance between them, as follows:—

1. Find the *difference of latitude;* call this D.

2. Take the *sum* of the latitudes when they are both N or both S, and the *difference* when one is N and the other S; call this the *sum of latitudes*, and denote it by S.

3. Find *half the difference of longitude*, and denote it by the letter L.

4. Find the *middle longitude*, by taking half the sum of the longitudes when they are both E or both W, or half their difference when one is E and the other W; denote this by the letter M. It is of the same denomination (east or west) as the greater longitude.

5. Compute X by means of this formula:—

$$tan\ X = cot\ L \,.\, cosec\ S \,.\, sin\ D,$$

   and consider X as a difference of longitude *east* or *west*, according as the place B is *east* or *west* of A (paragraph (4), page 53).

6. X is now to be applied to the middle longitude M, adding them together when they are both east or both west, or taking their difference when one is east and the other west. The sum or difference is the longitude of the vertex, of the same denomination as the greater.

For example, from the preceding question—

		° ′
B's lat.		34 0 S
A's lat.		32 3 S
Diff lat	D	1 57
Sum of lats	S	66 3

		° ′
B's long		18 20 E
A's long		115 45 E
Diff long.		97 25
Half diff long	L	48 42·5
Sum of long.		134 5
Half sum or middle longitude	M	67 2·5 E.

To compute X.

	° ′			
L	48 42·5	. . .	cot	9·943625
S	66 3	. . .	cosec	10·039101
D	1 57	. . .	sin	8·531828
X	1 52·5	. . .	tan	8·514554

X is now to be considered as a difference of longitude *west*, because the place B is *west* of A; and it must be subtracted from the middle longitude M, which is east.

	° ′
M	67 2·5 E
X	1 52·5 W

Longitude of the vertex - 65 10 E, agreeing exactly with that which was found by the former method. The vertex is or is not between the given places according as X is less or not less than half the difference of longitude L; and when X = 0, the vertex is midway between them.

To find the latitude of the vertex.

1. Find the difference of longitude between the place A and the vertex.

2. The latitude of the vertex may then be computed from this formula:—

$$\textit{Cot (lat. V)} = \textit{cos (D long. A} - \textit{V)} \times \textit{cot (lat. A)}.$$

		° ′		
Longitude of A	. .	115 45 E		
Longitude of V	. .	65 10 E		
D long	. . . . .	50 35	cos	9 802743
Lat. of A	. . . .	32 3	cot	10 203368
Lat. of V	. . . .	44 35·5	cot	10·006111

Also agreeing with the result of the former calculation.

Examples.

1. Required the distance, and the latitude and longitude of the vertex of the great circle, between San Francisco (lat. 37° 48′ N, long. 122° 8′ W) and Owhyhee (lat. 20° 17′ N, long. 155° 59′ W)?

Answer, distance 2047 miles, lat. of vertex 42° 35′ N, long. of vertex 89° 41 W.

2. Required the distance, and the latitude and longitude of the vertex on the great circle, between Lisbon (lat. 38° 42′ N, long. 9° 9′ W) and Bermuda (lat. 32° 22′ N, long. 64° 30′ W)?

Answer, distance 2688 miles, lat. of vertex 39° 41′ N, long. of vertex 24° 17′·5 W.

3. Required the distance, and the latitude and longitude of the vertex on the great circle, between Sierra Leone (lat. 8° 30′ N, long. 13° 18′ W) and Trinidad (lat. 10° 39′ N, long. 61° 34′ W)?

Answer, distance 2856 miles, lat. of vertex 10° 48′ N, long. of vertex 51° 50′ W.

The following article, in continuation of this subject, contains new and simple methods of modifying the Great Circle track, where the latitude into which the ship would be led is so high as to render the navigation dangerous; to say that this article has been furnished by the Rev. Mr. Fisher, late Chaplain and Principal of the Greenwich Hospital Schools, will be a sufficient guarantee of its value and sound practical character, and the kindness with which it has been contributed is here gratefully acknowledged.

CIRCULAR ARC SAILING.

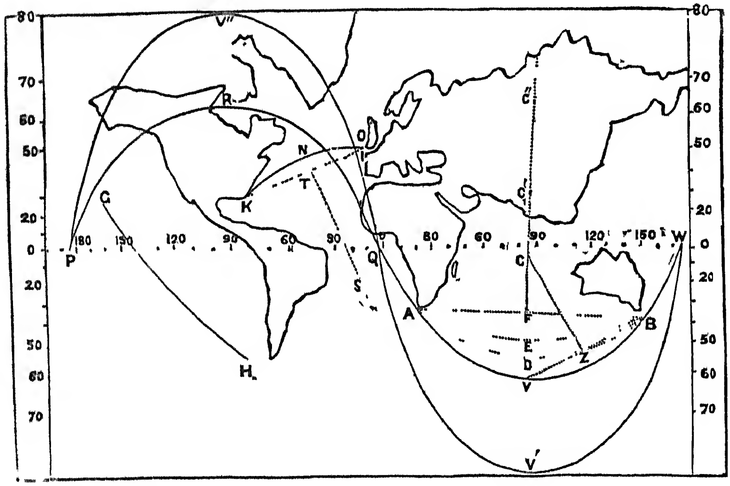
A Graphical mode of representing "GREAT CIRCLE ROUTES," *and also* "CIRCULAR ROUTES," *adapted to different Maximum Latitudes, upon a Mercator's Chart.*—By the REV. GEO. FISHER, M.A., F.R.S.

§ 1. A Great Circle passing through any two places upon the earth's surface, may be marked or traced upon a terrestrial globe, by elevating or depressing the polar axis, and at the same time turning the globe round until the places coincide with the upper edge of the wooden horizon: in this position of the globe the upper edge of the wooden horizon represents the great circle, which can then be marked upon the globe. This is, in fact, the strict practical solution of the problem of "Great Circle Sailing." It exhibits:—1st. The various places through which the great circle passes in both hemispheres. 2ndly. The distance between the two places, which is shown by the number of degrees intercepted between them, upon the wooden horizon. 3rdly. The latitude and longitude of that point of the great circle which reaches the highest latitude, which point, as before stated, is called the "Vertex."

§ 2. Having thus traced upon the globe the great circle passing through the two given places, we may next proceed to take from the globe the latitudes and longitudes of as many points or places situated upon the circle as may be thought sufficient, and mark the corresponding positions upon a Mercator's Chart. Then connecting the points, by drawing a curved line through them by hand, we obtain a

graphical representation of a Great Circle upon the chart, without any computation, and sufficiently accurate for nautical purposes, provided the globe is not less than about 8 inches in diameter, and accurately made.

§ 3. The nature of this curve is not sufficiently elementary to be explained here, but its general form is represented upon the small Mercator's Chart here annexed.



The curve PRQ AVB shows the direction of the great circle which passes through the points A and B, which represent the Cape of Good Hope and the south part of Van Diemen's Land. Although in sailing from one place to another the navigator is only concerned with that portion of the great circle upon which he means to sail, the curve is, nevertheless, continued through both hemispheres upon the chart, in order that its general form may be the better comprehended. The northern and southern portions of this curve are obviously equal and similar to each other; and the curve cuts the equator at two points P and Q, at a distance of 180 degrees of longitude from each other, and at an angle, which at the point of intersection, or contrary flexure, is equal to the latitude of the vertex, or the inclination of the planes of the Great Circle and Equator. In like manner the curve PV″ QV′ represents another great circle when projected upon the chart; the highest point of latitude V″ or V′ being 80 degrees. The arc KNI represents the portion of the great circle which passes through Charles-Town and the Land's-End. Also GH, that which passes between the Sandwich Islands and Cape Horn.

§ 4. The near approximation to a circular form of the portion of the curve which is nearest to the vertex affords a very easy and simple mode of delineating upon a Mercator's Chart a ship's route between two places thus situated.

Prob. A. To represent upon a Mercator's Chart the great circle route between two given places which do not differ considerably in latitude.

1. *Determine the latitude and longitude of the vertex; either by computation or by a globe.*

2 *Through the three points, viz., the two given places and the vertex, describe the arc of a circle. This arc will represent very nearly the Great Circle route.*

3 *Upon this arc the geographical positions of any number of points can be determined at pleasure to a degree of accuracy which will depend upon the scale of the chart, which known points can be sailed towards in succession by either using Middle Latitude, or Mercator's Sailings.*

We shall put this method to the test by applying it to the determination of an approximate great circle route between the Cape of Good Hope in lat. 34° 22′ S, long. 18° 26′ E, and the south part of Van Diemen's Land, in lat. 42° 54′ S, long. 147° 26′ E, which places are represented upon the small chart by A and B. The latitude of the vertex V is 62° S, and long. 87° E; and the true distance between the places on a great circle, computed by spherical trigonometry, is 5388 miles.

Bisect the line (AB), joining the two places by a perpendicular of indefinite length (FC″). On this line find the point C, from which, as a centre, describe the circular arc AVB: this arc is the route required. Suppose the intermediate points upon this arc to be 12 in number, and to be situated on the meridians of 30°, 40°, 50°, &c., East longitude respectively. The following are the geographical positions of these points, taken from a small chart upon a scale of 180 degrees of longitude to a foot The courses and distances between the points are computed by means of Mercator's Sailing:—

Circular Route			Maximum Lat. 62°.	
Places.	Lat	Long.	Courses.	Distances.
	S. ° ′	E. ° ′	° ′	
Cape of Good Hope . .	34 22	18 26	S 34 38 E	..
1st intermediate point	47 0	30 0	48 13	921·2
2nd " "	52 45	40 0	57 3	517·8
3rd " "	56 30	50 0	64 54	413·7
4th " "	59 0	60 0	68 12	353·6
5th " "	61 0	70 0	78 8	323·1
6th " "	62 0	80 0	E.	291·8
7th " "	62 0	90 0	N 78 8 E	281·7
8th " "	61 0	100 0	78 30	291·8
9th " "	60 0	110 0	64 17	300·9
10th " "	57 30	120 0	58 8	345·7
11th " "	54 0	130 0	51 10	397·8
12th " "	49 0	140 0	40 14	478 5
Van Diemen's Land .	42 54	147 26	..	479 4
Whole distance by this method				5397 1

The distance by this approximate great circle in the foregoing example exceeds the true distance upon a great circle computed by means of spherical trigonometry by 9 miles only, a difference obviously too small to be taken into account; and which difference is, moreover, exaggerated by the very limited scale of the chart used in determining the latitudes of the intermediate points, which may be 20′ or 30′ from the truth.

§ 5. A serious difficulty occurs in the application of great circle sailing to navigation in high latitudes (where this method of sailing might otherwise be employed to the greatest advantage), arising from the dangers and obstructions which frequently occur in the vicinity of ice. In the example last given, it appears that in sailing from the Cape to Van Diemen's Land upon the arc of a great circle (which is represented upon the small chart by the arc AVB), the greatest latitude to which the circle reaches is 62° S, which is a higher latitude than most navigators would wish to attain.

If, in order to meet this difficulty in the above case, the latitude of the vertex be limited to 50° S, it will be found that the corresponding great circle will not reach from the Cape to a greater distance than the South-Western Coast of Australia, or the *span* of the arc is not sufficient, the remaining part of the distance must therefore be either completed by other methods, or by adopting an additional great circle to effect the purpose.

This consideration, as well as that which arises from frequent unavoidable departures from *previously assigned routes*, which must frequently occur, particularly to sailing vessels, with contrary winds, errors of reckoning from want of satisfactory observations, and the loss of time in endeavouring to regain those routes, render it desirable to employ an easy method of delineating upon a Mercator's Chart a route of regular and continuous curvature through the whole extent of the distance to be sailed through, such as can be effected and applied immediately whenever the ship's position is determined, without reference to any previously determined route; at the same time to combine to a certain degree the advantages of great circle sailing, depending upon the maximum latitude intended to reach. The mode of effecting this is one of equal simplicity to that of representing the great circle route.

Prob. B. To represent upon a Mercator's Chart a circular route between two given places, the point of highest latitude being between the two places, and which point has a less latitude than that of the vertex of the corresponding great circle.

1. *Describe an arc of a circle upon the chart with such a radius that it may pass through the two given places, and reach the highest latitude previously determined upon. This arc will represent the circular route.*

2. *Upon this arc, determine by the chart the geographical positions of the points, and proceed as before.*

We shall now apply this method to the case before mentioned, viz., in sailing between the Cape of Good Hope and Van Diemen's Land, and suppose the highest latitude to be limited to 55° S.

Join A and B, and draw the bisecting perpendicular FC″ of indefinite length as before. On this line find the point C′, from which as a centre, describe the circular arc ADB, that shall pass through A and B, and just reach to the parallel of latitude of 55° S. Take, as before, 12 intermediate points upon this arc, on the meridians of 30, 40, &c., respectively. In the first of the following Tables the geographical positions of these points are given (determined upon the same chart as before), and also the courses and distances by Mercator's sailing.

The second of these Tables gives the positions, courses, and distances, upon the circular arc A E B, of which the maximum latitude is only 50°. The centre from which the arc A E B is described is C″, the radius, therefore, is longer than that which belongs to the arc A D B.

It appears, therefore, that the distance with the maximum latitude 55° S, exceeds the distance upon a Great Circle (which is 5388 miles) by 69 miles; and that the arc A E B, on which the maximum latitude is 50°, is greater than the distance upon the Great Circle by 190·7 miles.

TABLE I.

Circular Route . . . . Maximum Lat. 55°.				
Places.	Lat. S	Long E	Courses	Distances.
	° ′	° ′	° ′	
Cape of Good Hope .	34 22	18 26	S 48 2 E	..
1st intermediate point	42 30	30 0	54 41	729 8
2nd " "	47 30	40 0	64 12	519·0
3rd " "	50 40	50 0	72 5	436·6
4th " "	52 40	60 0	77 25	390·1
5th " "	54 0	70 0	80 16	367·2
6th " "	55 0	80 0	E.	354·7
7th " "	55 0	90 0	N 88 33 E	344·1
8th " "	54 50	100 0	81 50	354·4
9th " "	54 0	110 0	74 27	352·3
10th " "	52 20	120 0	69 32	373·0
11th " "	50 0	130 0	60 41	400·4
12th " "	46 15	140 0	57 42	459 5
Van Diemen's Land .	42 54	147 26	..	376·1
Whole distance				5457·2

TABLE II

CIRCULAR ROUTE . . . . Maximum Lat. 50°.				
Places	Lat S	Long E.	Courses	Distances.
	° ′	° ′	° ′	
Cape of Good Hope .	34 22	18 26	..	.
1st intermediate point	40 30	30 0	S 56 14 E	662 1
2nd " "	43 5	40 0	70 52	473·0
3rd " "	46 0	50 0	67 42	461·2
4th " "	47 55	60 0	74 21	426·3
5th " "	49 0	70 0	80 43	403 7
6th " "	50 0	80 0	81 17	396 0
7th " "	50 0	90 0	E	385·7
8th " "	50 0	100 0	E	385·7
9th " "	49 30	110 0	N 85 37 E	392·5
10th " "	48 35	120 0	82 2	396·9
11th " "	47 0	130 0	76 46	415·0
12th " "	44 30	140 0	70 17	444·6
Van Diemen's Land .	42 54	147 26	73 24	336·0
Whole distance				5578·7

The following is an abstract of the foregoing results :—

Methods	Maximum Latitude.	Distance in Nautical Miles.	Excess above Great Circle
	°		
1. By Great Circle Sailing	62	5388·0	..
2. " Circular Arc A V B	62	5397·0	9 0
3. " " A D B	55	5457 2	69 2
4. " " A E B	50	5578·7	190·7
5. Rhumb Line A B	(S. 85° 9′ E.)	6052·0	664·0

It is plain, from these results, that by an indefinite extension of the radii of the circular arcs from A to B, both the curvature of the arcs and the maximum latitudes will diminish and the arcs themselves approach the straight line A B as their limit.

The whole distance sailed upon any one of these circular arcs has an intermediate value between that by great circle sailing and that by Mercator's sailing. The shortest route, however, which is represented on the surface of the globe by an arc of a great circle appears, when represented upon a Mercator's Chart, to have the greatest extent of arc; on the other hand, the least distance upon the chart, which is that by Mercator's sailing, and represented by the straight line A B, is the longest route, and a spiral line upon the globe. This apparent anomaly arises from the impossibility of representing in their proper proportions upon a plane or flat surface, such as a chart, any considerable extent of lines or surface of the globe, and also from the nature of Mercator's projection itself.

In the foregoing examples, the geographical positions of a number of points upon the circular arcs were determined from the chart, and the total distances computed, in order to exhibit the results due to different maximum latitudes. But in the practical application of this method, it is necessary only to assume *one* point at a convenient distance in advance upon the arc from the place of departure. Then determine its position upon the chart, and shape a course towards it, either by protraction from the chart itself, or by middle latitude, or Mercator's sailing; and should the ship's position, by subsequent observation, be found to deviate from the circular arc previously determined, instead of endeavouring to obtain a position upon it, another circular arc can be described upon the chart to extend from the ship's position thus determined to the place sailed for, and another point taken upon it as before. By repeating this very simple process when required, the whole distance will be completed.

In those localities, as shown by the chart, where the projected great circle differs but little from a straight line, the application of the great circle method of sailing offers no advantage over that of Mercator, since both methods then become the same. This is the case within the tropics, and in those cases in which the latitude of the vertex does not exceed the limits of about 40 degrees from the equator.—G. F.

PART II.

NAUTICAL ASTRONOMY.

PRACTICAL RULES AND EXAMPLES.

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PART II.

NAUTICAL ASTRONOMY.

INTRODUCTION, DEFINITIONS, AND PREPARATORY PROBLEMS.

THE vast spherical vault in which the stars, the sun, and moon appear to us to be placed, we call the heavens, or celestial concave; and although the sun, the moon, the planets, and the fixed stars are at various distances from us, yet they appear to us all equally remote; nor are we much called upon to correct this illusion of the senses in the practical application of astronomy at present under discussion.

Another illusion is that apparent rotation of the heavens which results from the actual rotation of the earth upon its axis; but as the relative positions of the observer and the heavens are the same, whether the earth rotate from west towards east, or the heavens rotate about the same axis from east to west, the apparent diurnal rotation of the heavens will frequently be spoken of as a real, instead of an apparent motion.

So, also, the annual revolution of the earth about the sun causes the sun to appear to move among the stars in a great circle, and the direction of the motion may be described as from west through south, eastward. This also is often spoken of as a real motion of the sun.

The axis of the earth produced to the heavens points out the celestial poles, or those points round which the apparent diurnal revolution of the celestial sphere is performed; and the plane of the terrestrial equator produced to the heavens is called the *celestial equator.*

Circles from pole to pole of the heavens, intersecting the equator at right angles, are called *circles of declination*, or *hour circles:* one of these is supposed to pass through every celestial object, and to revolve with it in daily course; and that one which passes through the zenith, or point directly over the observer's head, is called the *meridian of the observer.*

Less circles of the sphere, parallel to the celestial equator, are called *parallels of declination;* these are the paths, also, of the heavenly bodies, in their apparent daily revolutions, each running on its particular parallel like a bead along a string.

The ecliptic is that great circle in the heavens which the sun's

centre appears to describe annually among the stars in a direction best defined, as from west through the south, eastward, the motion being viewed from the north side of the plane; it meets the equator in two points, called the *equinoctial points*, from the circumstance, that when the sun arrives at either of them, his path for that day is bisected by the horizon of every place, and, thus, the day and the night are everywhere of equal length.

The plane of the terrestrial equator does not coincide with the plane of the earth's orbit round the sun; hence, the apparent path of the sun, or the *ecliptic*, is inclined to the celestial equator, and the angle of inclination, which is about 23° 28′, is called the *obliquity of the ecliptic*.

That point at which the sun crosses from the south to the north side of the equator is called the first point of Aries; Aries being the first of twelve parts, called *signs*, into which astronomers divide the ecliptic.

The names of these divisions, and their distinguishing characters, are as follow:—

Aries.	Taurus.	Gemini.	Cancer.	Leo.	Virgo.
♈	♉	♊	♋	♌	♍
Libra	Scorpio	Sagittarius	Capricornus	Aquarius.	Pisces.
♎	♏	♐	♑	♒	♓

From the attraction of the sun and moon on the protuberant parts of the earth about the equator, the equinoctial points move westward along the ecliptic about 50″ in a year; this motion is called the *precession of the equinoxes*.

Circles perpendicular to the ecliptic, and meeting in its poles, are called *circles of celestial latitude*.

The *latitude* of a celestial body is the arc of a circle of latitude intercepted between the object and the ecliptic.

The *longitude* of a celestial body is the arc of the ecliptic between the circle of latitude passing over the body, and the first point of Aries, estimated in the order of the signs.

The *declination* of a celestial object is the arc of a circle of declination between the object and the equator.

The *right ascension* of a celestial object is the arc of the equator, or angle at the pole, included between the circle of declination which passes over the object and that which is drawn to the first point of Aries, and is estimated eastward, or in the order of the signs.

The *right ascension of the meridian* is the angle at the pole included by the meridian of the observer and the meridian passing over the first point of Aries, reckoned eastward, in the order of the signs.

The *sensible horizon* is a plane conceived to touch the earth at the point where the observer is situated, and meeting the celestial concave in a great circle.

The *rational horizon*, a great circle of the heavens, whose plane is

parallel to the sensible horizon, and passes through the centre of the earth.

When the term *horizon* only is used, it generally signifies the rational horizon.

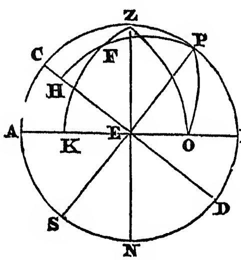
The *zenith* is that point of the heavens which is directly over the observer's head, and the *nadir* is the opposite point; they are the poles of the horizon.

Great circles perpendicular to the horizon, and which, therefore, meet in the zenith, are called *vertical circles, azimuth circles, or circles of altitude*.

Less circles, parallel to the horizon, are called *parallels of altitude.*

In the adjoining figure, which is a projection of the sphere on the plane of the meridian of the observer,—

Fig. 6.



The circle A Z P B N is the meridian; Z, the zenith; N, the nadir; P, S, the north and south poles.

A B is the horizon, which here appears as a straight line, the spectator being supposed to look in the direction of the plane of this circle, as at the edge of a circular card held horizontally before his eyes.

B is the north, and A the south point of the horizon.

All the circles which meet in P are circles of declination, or hour circles.

C D is the celestial equator, and, like all great circles which pass through E, appears in this projection as a right line.

The point E, which may be considered as the east or west point of the horizon, is 90° distant from every point of the meridian, and is therefore a pole of that circle; and all great circles which pass through E are at right angles to the meridian, and appear in this projection as right lines; Z E N, the vertical circle at right angles to the meridian, is called the *prime vertical;* P S, the hour circle at right angles to the meridian, is called the *six o'clock hour circle.*

Since P C = Z B = 90°, if Z P be omitted, Z C = P B; but Z C measures the latitude of the place whose zenith is Z, therefore P B, or *the altitude of the elevated pole, is equal to the latitude of the observer.*

Z P is the colatitude.

Let F represent any celestial object, then F H is its *declination;* F P, its *polar distance;* F K, its *true altitude;* F Z, its *zenith distance.*

The angle Z P F is the *hour angle,* or *meridian distance;* F Z C, its *azimuth* from the south in north latitude, or from the north in south latitude.

F Z P, its azimuth from the north in north latitude, or from the south in south latitude.

The arc C H also measures the hour angle, and A K or K B the azimuth.

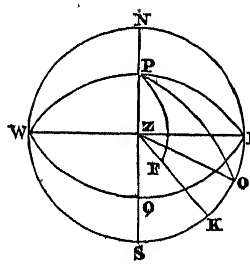
If an object rise or set at O, E O or the angle E Z O is called its *amplitude,* and is reckoned from the east point when the object is rising, and from the west when setting, and lies towards the north or south of those points, according as the declination is north or south.

We might have used, for our illustrations, another picture or projection of the circles of the sphere, viz., that in which the observer is supposed to look perpendicularly from the zenith upon the plane of the horizon.

In this the horizon appears as a circle, and the zenith as its centre, and all the circles which pass through the zenith—for example, the meridian and vertical circles—as straight lines, diameters of the horizon.

The principal advantage of this projection is that it exhibits at one view the whole hemisphere which is above the horizon.

Fig 7.



In this figure the circle represents the horizon, and the reader must suppose himself to be looking down upon it from the zenith Z; N Z S will then represent the meridian of the place whose zenith is Z, P the elevated pole, N P the elevation of the pole, equal to the latitude of the place of observation.

W Z E, the vertical circle at right angles to the meridian, called the *prime vertical,* cutting the horizon in the east and west points, E, W.

The points, E, W, are the poles of the meridian N P S, for they are 90° distant from every point in that circle. All circles, therefore, which cut the meridian at right angles, meet in these points.

N, S, are the poles of the circle W Z E.

The *zenith* Z, and its opposite point, the *nadir*, are the poles of the horizon.

W P E is the *six o'clock hour circle.*

W Q E is the *equator.*

If an object be at F, F K is its *altitude*, F P its *polar distance*, S Z F its *azimuth*, Z P F the *hour angle or meridian distance*, and if an object rise at O, O E is its *amplitude.*

TIME AND ITS MEASURES.

That instant of time at which the sun appears upon the meridian of the observer, is called *apparent noon;* and the interval between two successive noons is called an *apparent solar day.**

Were the sun to move eastward in the equator, as he now appears to do in the ecliptic, and were the spaces thus passed over each day uniform, then the intervals from noon to noon would evidently be perfectly uniform.

But as the sun neither moves in the equator, nor are the spaces moved over in the ecliptic uniform each day, he does not return to the meridian after equal intervals of time, and therefore the *apparent solar days* are of unequal length.

In order to avoid the inconveniences attendant upon these inequalities, the astronomer compensates them by the following hypotheses:—

First, when the sun arrives at that point of the ecliptic where he is nearest to the earth, and his progress along the ecliptic most rapid, an imaginary star is supposed to set out with him, and to move along the ecliptic at the average rate at which the sun moves; at first the star will be outrun by the sun, but as his pace relaxes, the star will again overtake him, and they will arrive together at the opposite point from which they set out, or at that point where the sun is furthest from the earth, and the velocity in the ecliptic the least. Again from this point the star will get at first in advance of the sun, but the sun's rate of motion increasing, they arrive together at the point from which they started. If, now, the transits of this supposed star be substituted for the transits of the sun, the unequal motion of the sun is corrected.

In the next place, this star in its progress round the ecliptic must pass through the first point of Aries; and at this instant another imaginary body, called the *mean sun*, is supposed to set out, and, travelling in the equator eastward at the same rate as the star, they meet again at the first point of Libra, each having passed through 180° at the same rate: they are also on the same meridian together at the solstices.

If, now, the transits of the *mean sun* be substituted for those of the star, both causes of the inequality of the solar days are compensated.

* The student should always bear in mind that we speak of the sun as a point, unless otherwise stated.

The *mean solar day* is the interval between two *mean noons*, or transits of the *mean sun* over the meridian.

The difference between *mean time* and *apparent time* at any instant is called the *equation of time*.

The equation of time consists of two parts, one depending on the *inequality of the sun's motion in the ecliptic*; this vanishes when the earth is nearest to and furthest from the sun; and the other part *depending upon the obliquity of the ecliptic*; this part vanishes at the equinoxes and solstices.

The beginning and ending of the mean solar day differ but little from those of the apparent solar day, being sometimes a little earlier and sometimes a little later. And this near agreement is of no small importance; for while the progress of the sun is the most obvious and natural measure of time, the mean solar day is its standard measure in civil affairs.

The interval between two successive transits of the first point of Aries over the same meridian, is called a *sidereal day*.

In order to compare this portion of time with the length of a mean solar day, we will consider what takes place at the time the mean sun is setting out from the first point of Aries on his annual course round the equator. At this time the same meridian passes over the mean sun and over the first point of Aries; and after the lapse of a sidereal day, during which the earth has made one complete revolution, the meridian again passes through the first point of Aries; but in this interval the mean sun has advanced eastward along the equator, and the meridian has, therefore, also to advance from west towards east (for this is the direction of its rotation) so much beyond a complete revolution in order to come up to it.

Thus, the mean solar day is longer than the sidereal day, and as each day is divided into 24 hours, it is evident that in 24 hours of sidereal time less than 24 hours of mean solar time will have elapsed; in fact, 24 hours of *sidereal time* are equivalent to 23 h. 56 m. 4·0906 s. of *mean time*.

Again, 24 hours of *mean time* are equivalent to 24 h. 3 m. 56·5554 s. of *sidereal time*.

Tables founded on these relations are given near the end of the "Nautical Almanac," under the title of Tables of Time Equivalents, by which intervals of mean or sidereal time are readily expressed in parts of sidereal or mean time respectively.

Example I.—What is the interval of sidereal time corresponding to an interval of 5 h. 27 m. 56·3 s. of mean time?

Mean Time		Sidereal Time
5 h.	is equivalent to	5 h 0 m 49·2824 s.
27 m.	"	27 4·4354
56 s	"	56·1533
·3	"	3008
5 h 27 m 56 3 s	. . . . .	5 h 28 m. 50·1719 s.

Example 2.—What is the interval of mean time corresponding to 4 h. 25 m. 34·37 s. of sidereal time?

Sidereal Time		Mean Time
4 h. is equivalent to		3 h 59 m 20 6818 s.
25 m	"	24 55 9044
34 s.	"	33 9079
·37	"	3690
4 h 25 m. 34·37 s.		4 h. 24 m. 50·8631 s

If a watch measuring intervals of mean time correctly were set at 12 at the time of the mean sun's transit over the meridian, then when the watch showed one hour, the circle of declination passing over the mean sun and revolving with it would make with the meridian of the observer an angle at the pole of 15° to the westward, for the whole revolution takes places in 24 hours. Hence appears a connection between the angular meridian distance of the mean sun and the measure of mean time.

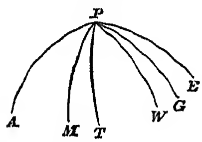
Mean time, then, is measured by the angle at the pole between the meridian of the observer and the circle of declination of the *mean sun* at the rate of one hour to every 15°.

Similar reasoning shows that apparent time from noon is measured by the westerly meridian distance of the *sun*—and sidereal time by the westerly meridian distance of the *first point of Aries*, which is the same thing as if we had said the angle at the pole eastward from the hour circle of the first point of Aries to the meridian of the observer, or what is still the same, the right ascension of the meridian.*

Hence the *sidereal time*, and the *right ascension of the meridian*, are synonymous expressions.

And hence also, when any celestial object is on the meridian, the *sidereal time* is equal to that object's *right ascension*.

Fig. 8.



To illustrate these definitions, let

PA be the meridian of ♈, or first point of Aries.
PG be the meridian of Greenwich.
PE a meridian in east longitude.
PW a meridian in west longitude.
PM the hour circle of the mean sun.
PT the hour circle of the sun.

Then MPG, TPG, and APG, which are respectively the westerly meridian distance of the *mean sun*, the *sun*, and the *first*

* The *sidereal time* at Greenwich *mean noon* of each day, is given in the "Nautical Almanac."

point of *Aries*, measure the *mean*, *apparent*, and *sidereal time* at Greenwich.

M P E, T P E, and A P E, measure the corresponding times at the eastward meridian, and M P W, T P W, and A P W, those at the westward meridian at the same instant, or for the same positions of M, T, and A. Therefore, since W P G and W P E are the longitude of the meridians P W and P E, it is seen that the *difference between the estimated times at different places at the same instant, is a measure of their difference of longitude* (15° being considered still equivalent to one hour).

We also see that the largest measure of time, and, therefore, the latest hour, belongs to the most eastward place.

For example, if two observers, at different places, both having the means of noting local time exactly, observed the same remarkable phenomenon, the explosion of a brilliant meteor, for instance, and that at one place it occurred at five, and at the other at three in the afternoon, then is their difference of longitude two hours or thirty degrees, and he who saw it at five is to the eastward of the other.

Reverting to the figure, the angles to the eastward of P A, namely, A P M, A P T, A P W, &c., are the right ascensions of M, T, W, &c., that is, of the mean sun, the sun, the meridian P W, &c.

A P W, which measures the *sidereal time* on the meridian P W, is also the *right ascension* of that meridian. When any object is on the meridian, its right ascension coincides with that of the meridian, and, therefore, with the *sidereal time*.

The polar angle M P T, subtended by the mean sun and true sun, is the *equation of time*.

In civil reckoning, noon is the middle of the day, but in astronomical reckoning, it is the beginning of the day. The civil day commencing at midnight, and the astronomical day at the following noon.

The hours of the day are also reckoned in astronomy without intermission, from 0 to 24. Thus April 25th, 2 hours A.M., or 2 o'clock in the morning, is the same as April 24th, 14 hours, astronomical reckoning.

April 25th, 3 hours P.M., is also April 25th, 3 hours, in astronomical reckoning.

The astronomical date agrees with the civil date in the afternoon, but in the morning it is found by increasing the hours by 12, and subtracting one from the days of the month.

To prevent confusion, hours, minutes, and seconds of time, are distinguished by the letters h. m. s., written after or over them, and degrees, minutes, and seconds, of arc or angle, by the marks ° ′ ″.

PROBLEMS

PROBLEM 1.—*To convert arc into time.* As 15° correspond to one hour,

1° will correspond to 4 m.
1′ ,, ,, 4 s.
1″ ,, ,, 4 t. (thirds.)

Therefore, multiply the degrees, minutes, and seconds by 4, and the result will be minutes, seconds, and thirds of time.

Example.—Convert 15° 47′ 55″ into its equivalent time.

	°	′	″
	15	47	55
			4
	63 m.	11 s	40 t
or, 1 h	3 m	11 s	40 t.

PROBLEM 2.—*To convert time into arc.*

First express the time in minutes and seconds only, and then divide by four.

Example.—Convert 4 h. 25 m. 30 s. into arc or longitude.

Given time in minutes, &c.		265 m	30 s.
Divided by 4	66°	22′	30″

PROBLEM 3.—*To find the Greenwich date; the time at any other place and the longitude being given.*

As the difference of time at any instant at different places is measured by the longitude in time, and is latest at the eastward place: add the longitude in time to the date at the place, when the longitude is west, or subtract it from that date when the longitude is east, the result is the Greenwich date.

Example 1.—What is the Greenwich date when it is 17 h. 35 m. 45 s. past noon on March 25th, in 122° 13′ W longitude?

	d	h	m.	s
122° 13′ is equivalent to		8	8	52
And the given date is March	25	17	35	45
∴ The Greenwich date is March	26	1	44	37

Example 2.—What is the Greenwich date when the sun is on the meridian of a place in 75° 44′ E longitude, on April 21?

The sun being on the meridian, it is apparent noon.

		d.	h	m.	s	
∴ The given date is. . .	April	21	0	0	0	App. noon.
Longitude in time . .			5	2	56	
Greenwich date . . .	April	20	18	57	4	App. time.

In the first of these questions, the added longitude advances the day of the month; and in the second, the hours of the longitude to be subtracted are taken from a borrowed day or 24 hours, thus making the days of the month at Greenwich one less than at the place.

A bad habit prevails in writing dates, of separating the month and day from the hours, minutes, and seconds. The day of the month should always precede the minor divisions of time which give the precise instant of the day intended.

Problem 4.—*To take out the right ascension of the mean sun for a given mean Greenwich date.*

1. With the given day enter the "Nautical Almanac," page II. of the month, and take out the sidereal time, which is the right ascension of the mean sun at Greenwich mean noon.

2. Enter the Table in the "Nautical Almanac," entitled, *Table for converting intervals of mean solar time into equivalent intervals of sidereal time*, with the hours, minutes, and seconds of the Greenwich date successively, under the headings—"Hours, minutes, seconds of mean time," and take out the difference between the hours, minutes, and seconds, &c., so found, and the time which stands opposite to them, under the heading, "Equivalents of sidereal time."

3. Write these differences (which are the increase of the mean sun's right ascension for those portions of time) under the right ascension at mean noon, and add all up as they stand. The sum is the right ascension of the mean sun at the given mean *Greenwich date.*

Example.—Required the right ascension of the mean sun for the *mean* Greenwich date, 1855, Oct. 5th, 13 h. 54 m. 25·6 s.

		h	m	s
Right ascension of the mean sun at Greenwich mean noon, October 5 . . .		12	54	23·22
Acceleration for	13 h. . . .		2	8·13
	54 m. . . .			8·87
	25 s. . . .			·07
	·6 . . .			·00
Right ascension of mean sun at the given date		12	56	40·29

CORRECTIONS OF OBSERVED ALTITUDES

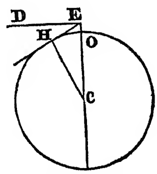
The altitude of a heavenly body above the sea horizon having been measured with a sextant, certain corrections are to be applied to it, to reduce it to the true altitude or elevation above the rational horizon.

These corrections are *dip, index, error, refraction, semidiameter,* and *parallax.*

DIP.

The dip is the depression of the sea horizon, below the level of the observer's eye, or below the level of the sensible horizon.

Fig 9.



Thus, if OE represent an observer standing on the earth at O, $\angle$ D E H is the dip, which is to be subtracted from every altitude above the sea horizon, to find the altitude above the horizontal plane D E.

$$\text{Let } OC = r, \text{ and } OE = h, \text{ then } CE = r + h, \text{ and}$$
$$CH = CO = r,$$
$$\text{also } \angle DEH + \angle HEC = 90^\circ,$$
$$\text{and } \angle HEC + \angle HCE = 90^\circ,$$
$$\therefore \angle DEH + \angle HEC = \angle HEC + \angle HCE,$$
$$\text{omitting } \angle HEC \text{ from each,}$$
$$\therefore \angle DEH = \angle HCE.$$

$$\text{Now } \cos \angle C = \frac{HC}{CE} = \frac{r}{r+h},$$

$$\therefore 1 - \cos C = \frac{h}{r+h} \text{ or } \frac{h}{r}, \text{ very nearly; neglecting } h$$

in the denominator, as it is insignificant with respect to r.

$$\therefore 2 \sin^2 \frac{C}{2} = \frac{h}{r},$$

$$\text{or, } 4 \sin^2 \frac{C}{2} = \frac{2h}{r} = \frac{2}{r}h.$$

$$\therefore \text{extracting the square root, } 2 \sin \frac{C}{2} = \sqrt{h} \times \sqrt{\frac{2}{r}}$$

but $2 \sin \frac{C}{2} = \sin C$ very nearly $= C \sin 1'$ very nearly, C being very small, and reckoned in minutes.

$$\therefore \textit{The dip in minutes} = \sqrt{h} \times \frac{1}{\sin 1'} \times \sqrt{\frac{2}{r}}$$

The refraction, however, makes the dip rather less than it would otherwise be, and allowance being made for this, h being reckoned in feet.

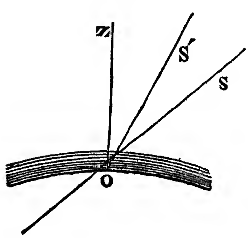
$$\textit{The dip in minutes} = \cdot 9784 \sqrt{h}.$$

From this it will seen that *the square root of the height of the eye in feet is nearly equal to the dip in minutes.*

REFRACTION.

The correction for *refraction* is necessary, on account of the effect of the earth's atmosphere, which bends the rays of light passing through it, into a position more nearly vertical, and thus causes the apparent places of the heavenly bodies to be above the true places.

Fig. 10.



Thus an object situated at S is seen at S', nearer the zenith, or at a greater elevation than it would have, but for the effect of the atmosphere. The atmosphere is represented by the band of parallel lines, and the ray S O is bent downwards to O on its entry and passage through it. An object at Z will not be affected by refraction, no reason existing why the vertical ray Z O should be bent more in one direction than in another. The amount of the correction for refraction varies with the tangent of the zenith distance, so that,

$$\textit{refraction} = 57'' \,.\, \textit{tan zenith distance, nearly};^*$$

$57''$ being the refraction of an object 45° from the zenith, when the thermometer is at 50°, and the barometer $29 \cdot 6$ inches.

* This rule is merely approximate and not applicable for altitudes under 12°.

The amount varies with the changes which take place in the temperature and pressure of the atmosphere, as indicated by the thermometer and barometer. And Table V. gives the correction to be applied to the mean values given in Table III., on account of these changes.

SEMIDIAMETER

The *semidiameter* is applied to the measured altitudes of such objects as have a visible disc, as the sun and moon; the altitude of the upper or lower edge or *limb* being first observed, and then increased or diminished by the angle subtended by half the breadth of the disc; as it would be measured at the place of observation.

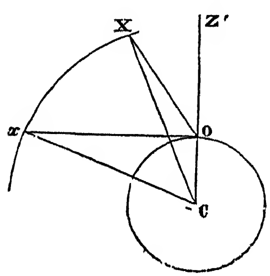
The semidiameter of the sun and moon are given in the "Nautical Almanac;" that of the sun for every day at mean noon in page II. of the month; that of the moon for mean noon and midnight, page III These semidiameters are as they would be observed from the centre of the earth, and therefore require a correction for the situation of the observer, who is on its surface. The sun, however, is at so great a distance, that the *augmentation* due to the nearer position of the observer need not be considered. But the moon's semidiameter is appreciably changed, and must be *augmented* by the quantity given in Table VIII., before it is applied to the observed altitude of the moon's upper or lower limb, to find the altitude of the centre.

The method of computing the augmentation will be presently explained.

PARALLAX

The *parallax* is a correction which is to be added to the apparent altitude, or that which is taken on the earth's surface, to make it what it would have been, if observed at its centre, and estimated from the rational horizon. It is the angle subtended at the object by that radius of the earth which is drawn to the place of observation.

Fig 11.



Thus, if O be the place of the observer, C the centre of the earth,

X any object. Then the angle C X O is the *parallax in alt*. An object at Z′, in the prolongation of the radius C O, has no par. When the object is in the horizon, the parallax is the greatest, a called the *horizontal parallax*, and its value being known, th rallax at any elevation above the horizon can be computed.

Let x represent the same object X in the horizon.

$$\text{Then} \quad Cx = CX;$$

$$\therefore \quad \frac{CO}{Cx} = \frac{CO}{CX};$$

$$\text{or, } \sin \angle x = \frac{\sin \angle X}{\sin \angle XOZ'};$$

$$\therefore \sin x \times \sin XOZ' = \sin X.$$

Or, as x and X are always very small angles, and therefore vary very nearly as their sines—

$$x \times \sin XOZ' = X.$$

But x is the horizontal parallax, X the parallax in altitude, supposing the earth a sphere, X O Z′ is the apparent zenith dist or the complement of the apparent altitude.

Hence the usual formula,

Horizontal parallax × *cos appt. alt.* = *parallax in altitude*

The moon's horizontal parallax, as given in the "Nautical A nac," is, as we are there informed, "the *greatest* angle under w the earth's equatorial semidiameter would appear if seen from centre of the moon."

REDUCTION OF THE MOON'S PARALLAX.

The earth's equatorial radius being the greatest, it follows tha horizontal parallax must be diminished to adapt it to any place distance from the equator, and the reduction for this purpose is in Table VI. The quantity thus found is to be subtracted from horizontal parallax taken from the "Nautical Almanac."

The *reduction* of the horizontal parallax is greatest at the and *nothing* at the equator, and increases in the ratio of the square of the latitude. So that if the reduction be 12″·4 at the in the latitude 30°, it will be—

$$12''\cdot 4 \times \sin^2 30^\circ.$$

log sin 30° . . .	9·698970
	2
	9·397940
12″ 4 . . .	1·093422
3 1 . .	0·491362

Thus in latitude 30° the reduction will be 3″·1

If c represent the compression, or the difference between the polar and equatorial radii, and a the equatorial radius, and C P the radius at any point P whose latitude is l, then by the principles of conic sections—

$$\frac{CP}{a} = 1 - \frac{c}{a} \cdot \sin^2 l \text{ nearly.}$$

$$\text{But } \frac{CP}{a} = \frac{\text{Horizontal parallax for P}}{\text{Horizontal parallax at equator}} = \frac{h}{h'}$$

$$\frac{h}{h'} = 1 - \frac{c}{a} \cdot \sin^2 l.$$

$$\text{or,} \quad h = h' - h' \cdot \frac{c}{a} \cdot \sin^2 l.$$

Therefore $h' \cdot \frac{c}{a} \cdot \sin^2 l$ is the quantity to be subtracted from h' or the equatorial horizontal parallax, in order to find h the horizontal parallax for the place P; the factor $\frac{c}{a}$ depends upon the form of the spheroid, or upon the amount of compression; assuming its value as $\frac{1}{300}$, the computation of the reduction of the horizontal parallax will be as follows, when the horizontal parallax from the "Nautical Almanac" is 60′, and the latitude of the place for which it is required to reduce it 52°,

$$\frac{60'}{300} \cdot \sin^2 52^\circ = 12'' \cdot \sin^2 52^\circ$$

sin 52° . . .	9·896532	Horizontal parallax. . . .	60′ 0″
	2	Reduction	7·45
	9·793064	Reduced horizontal parallax.	59 52·55
12″. . . .	1·079181		
Reduction 7″·45. .	0 872245		

ANGLE OF THE VERTICAL

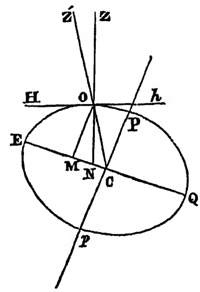
On account of the spheroidal form of the earth, the flattening at the poles, and the bulging at the equator, consequent upon its rapid rotation, the radii are not perpendicular to the surface, except at the equator and the poles, and therefore C Z′ in the last figure is not generally perpendicular to the sensible horizon (O x) of the observer at O, and Z′ therefore is not exactly in the zenith.

The deviation of C Z′ from the vertical is called the "*angle of the vertical*;" the inclination takes place in the plane of the meridian from the pole towards the equator, and varies in amount with the sine of twice the latitude. It is consequently greatest at the parallel

of 45°, where its value is 11′ 27″·3. This being denoted by θ, the formula for computing its value for any other latitude (l) is

$$\theta \cdot \sin 2\, l.$$

Fig. 12.



In the figure, let O be the place of observation, C the centre of the earth, H O h the sensible horizon touching the earth at O, O Z perpendicular to H h meeting the heavens in Z the zenith, whereas the radius C O produced meets the heavens in Z′; then Z O Z′ is the *angle of the vertical.* The angle Z N E or O N E between the vertical Z N and the plane of the equator is called the true latitude of O, and the angle at the centre O C E is the reduced latitude; O C E being less than O N E by the angle of the vertical N O C.

Let O M be drawn perpendicular to E Q, then

$$\frac{\text{O M}}{\text{M N}} = \tan \text{O N M} = \tan l'; \text{ and } \frac{\text{O M}}{\text{M C}} = \tan \text{O C M} = \tan l.$$

Therefore, by division, $\dfrac{\text{M N}}{\text{M C}} = \dfrac{\tan l}{\tan l'}$.

But in an ellipse $\dfrac{\text{M N}}{\text{M C}} = \dfrac{\text{C P}^2}{\text{C E}^2}$.

Or denoting C E by a and C P by b, the equatorial and polar radii of the earth—

$$\frac{\text{M N}}{\text{M C}} = \frac{b^2}{a^2}.$$

Therefore $$\frac{b^2}{a^2} = \frac{\tan l}{\tan l'}, \text{ or } \tan l' = \frac{a^2}{b^2} \tan l.$$

From this, knowing the value of $\dfrac{a^2}{b^2}$, the reduced latitude may be computed from the true latitude.

Assuming the compression at $\frac{1}{300}$ of the greatest radius,

The logarithm of $\dfrac{b^2}{a^2} = \bar{1}.997100$.

Example.—Required the angle of the vertical or *reduction of latitude* for the true latitude 55°.

	tan 55°	10·154773
		1·997100
Reduced lat.	54° 49′ 12″	tan 10·151873
True lat. .	55	
Reduction .	10′ 48″	

When the moon is on the meridian at her upper transit, the zenith distance should be diminished, or the apparent altitude increased, by the whole amount given in Table VII., before the parallax in altitude is computed from the formula

Horizontal parallax × *cos app. alt.* = *parallax in alt.*

When on the meridian below the pole, the correction must be added to the zenith distance or subtracted from the apparent altitude.

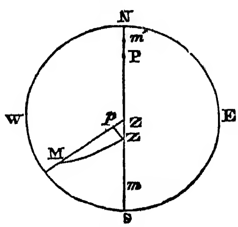
And when she is not on the meridian, the azimuth should be observed or computed, and the correction to be applied to the zenith distance may then be computed by this formula :—

Correction = *angle of vertical* × *cos azimuth.*

The azimuth being reckoned from the north in south latitude, and from the south in north latitude.

This correction will be − while the azimuth is less than 90°, and + when it is greater, and *nothing* when the bearing is east or west.

Fig. 13.



Let N E S W represent the horizon, the eye being supposed to be placed at the zenith looking perpendicularly downward.

Let N S represent the meridian, P the pole, Z the zenith, Z′ the point where the radius produced meets the heavens; then Z Z′ measures the angle of the vertical in the former figure.

Let M represent the position of a heavenly body, M Z its zenith distance, M Z′ its reduced zenith distance.

If Z′ p be drawn perpendicular to M Z, p Z will be very nearly the difference between M Z′ and M Z.

Now $$p\,Z = Z\,Z' \,.\, \cos \angle\, Z.$$

or *Correction* = *angle of vertical* × *cos azimuth.*

THE AUGMENTATION OF THE MOON'S SEMIDIAMETER.

Let X (figure at page 81) represent the moon, O the place of observation, C the centre of the earth It has already been said that the semidiameter as viewed from C is given in the "Nautical Almanac;" let this be denoted by s; then the semidiameter as seen from O will be greater, in the same ratio that the distance C X is greater than X O; let this increased semidiameter be expressed by $s + a$, where a is the augmentation sought.

Let X O Z′, the apparent zenith distance $= z$, the parallax $\angle X = p$, then X C Z′ the true zenith distance $= z - p$.

Then
$$\frac{CX}{XO} = \frac{s + a}{s}.$$

Also by the triangle C X O—
$$\frac{CX}{XO} = \frac{\sin z}{\sin (z - p)}.$$

Therefore
$$\frac{s + a}{s} = \frac{\sin z}{\sin (z - p)}.$$

And resolving this equation—
$$a = \frac{2\, s \,.\, \sin \frac{p}{2} \,.\, \cos \left(z - \frac{p}{2} \right)}{\sin (z - p)},$$

which is the formula employed in computing Table VIII. This augmentation is to be added to the semidiameter of the moon as taken from the "Nautical Almanac," before it is used in correcting an observed altitude.

Given the moon's horizontal parallax 58′ 57″, the apparent altitude 42°, and semidiameter 16′ 4″, to find the augmentation.

Par. in alt = hor par. × cos. app alt.

hor. par 3537″ log	. .	3·548635
app. alt. 42° cos	. . .	9·871073
6,0)262,9	. . .	3,419708
p ... 43 49		
$\frac{p}{2}$. 21 54		
s .. 16′ 4		
2		
$2s$ .. 32·8 = 1928′		

	°	′	″
app. alt.	42	0	0
app. zenith dist. .	48	0	0 . . . z
$\frac{p}{2}$		−21	54
$\left(z - \frac{p}{2}\right)$ . .	47	38	6
z	48	0	0
p		−43	49
$(z - p)$	47	16	11

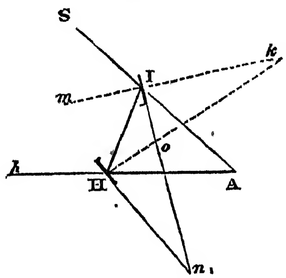
$$a = \frac{2\,s \,.\, \sin\frac{p}{2} \,.\, \cos\left(z - \frac{p}{2}\right)}{\sin\,(z - p)}$$

$2\,s$	1928″	log	3 285107
$\frac{p}{2}$	21′ 54″	sin	7·804173
$\left(z - \frac{p}{2}\right)$	47° 38′ 6″	cos	9·828564
			20·917844
$(z - p)$	47° 16′ 11″	sin	9 866025
a . .	11″·27	.	1·051819

THE SEXTANT.

Let A h be a line directed towards an object h, and across this line suppose a small mirror H to be placed; in the figure the mirror may

Fig. 14



be supposed to stand perpendicularly on the paper. Let H k be perpendicular to the face of this mirror. Draw H I, making the angle I H k = ∠ A H k. At I, suppose another mirror represented by the short black line to be placed, facing the direction I m, or so that I m is perpendicular to it, and draw S I, making ∠ S I m = ∠ H I m.

Now if S represent another object, the ray S I will be reflected by the mirror I, in the direction I H, and again by the mirror H, in the direction H A.

An eye at A, therefore, or at any point in H A, will see the image of the object S in the direction A h.

If then the mirror H be left half unsilvered, so that the object h may also be seen; then both h and the image of S will be seen in the same direction.

Now let S I be produced to meet H A in A; then the angle S A h is the angular distance between the objects S and h.

By making I movable about the centre, the images of objects at various angular distances from h, may be made to coincide with h in the same manner.

It will now be shown that the inclination of the mirrors to each other is equal to half the angle A.

$$\angle \mathrm{I}ok = \angle \mathrm{H}on, \qquad \text{(Euc. I., 15.)}$$
$$\angle k\mathrm{I}o = \angle o\mathrm{H}n, \text{ each a right angle,}$$
$$\therefore \quad \angle n = \angle k.$$

Again $\quad \mathrm{SIH} = \mathrm{IHA} + \mathrm{A}, \qquad$ (Euc. I., 32.)

$$\therefore \tfrac{1}{2}\mathrm{SIH} = \tfrac{1}{2}\mathrm{IHA} + \tfrac{1}{2}\mathrm{A};$$
$$\text{or, } m\mathrm{IH} = \mathrm{IH}k + \tfrac{1}{2}\mathrm{A}.$$
$$\text{but, } m\mathrm{IH} = \mathrm{IH}k + k.$$

Therefore $\quad \mathrm{IH}k + k = \mathrm{IH}k + \tfrac{1}{2}\mathrm{A},$

omitting $\mathrm{IH}k$ from each,

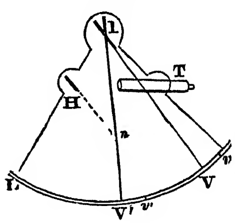
$$k = \tfrac{1}{2}\mathrm{A};$$

and therefore $\quad n = \tfrac{1}{2}\mathrm{A}.$

Or, the inclination of the mirrors to each other $= \frac{1}{2} \angle$ A.

Thus, if any mechanical means can be employed to measure the inclination of the mirrors, the angle A, or the angular distance of

Fig 15



two objects can be ascertained. For this purpose the glasses are attached to a solid frame-work, and to the glass I is attached a movable radius I V, carrying an index V, which moves over the arc or limb V L as I is turned about the centre.

The limb V L is divided into degrees, &c. And these divisions should commence from the point at which the index stands when the mirrors I and H are parallel, for then their inclination is nothing, and the arc moved over by the index from that point will measure the inclination of the glasses. Thus V V′ measures V I V′, or its equal $\angle n$.

As, however, it is not the inclination of the glasses but the angular distance of the two objects which is sought, the half degrees are marked as whole degrees on the divided limb, so that the degrees, &c., read off, are the measure of the angle subtended by the objects observed.

For simplicity, the radius I V′ has been drawn in the same plane with the mirror I: this is seldom the case in the sextant, and does not affect the principle; for if the radius were bent into the position I v,

the divisions would commence at v instead of V, and the index would arrive at v' instead of V' when carried along the limb, and $v\,v' = V\,V'$; or the measured arc would remain the same. If when the mirrors are parallel, the index do not stand at zero, but rather in advance, or to the left, then every arc read off will be too great; and, on the contrary, if it stand to the right, every measured arc will be too small; this constitutes what is called the *index-error*, which must always be ascertained by careful observations in the manner hereafter directed

The index at V is furnished with a scale for subdividing the divisions of the limb; this scale is called, from its inventor, a Vernier, and is thus constructed. Suppose the divisions on the limb to be each 10′, as is usually the case in a sextant; then 59 such divisions will contain 590′, and this length being divided into 60 equal parts, the value of each of the new divisions will be 9′ 50″, or 10″ less than each of the divisions on the limb. And these 60 divisions constitute the Vernier scale, which is attached to the extremity of the movable radius I V′, and slides along the limb I V.

It will be seen that 6 divisions of the Vernier are just 1′ shorter than 6 divisions of the limb. If therefore the stroke which marks the 6th division of the Vernier were to coincide with one of the strokes upon the limb, the index or zero-point of the Vernier would be just 1′ in advance of the corresponding mark or stroke upon the limb. In the same manner, if one of the marks which terminate the 12th, 18th, 24th, &c., divisions of the Vernier were thus to coincide with a mark on the limb, the index of the Vernier would be 2′. 3′, 4′, in advance of the corresponding division of the limb, and thus the 10-minute divisions are divided into single minutes. These points of the Vernier are distinguished by longer strokes, to facilitate the counting.

Moreover, if any mark between these sixths or minute marks coincide with a division of the limb, for each such mark past the preceding minute, the index will be further advanced 10″, and thus the reading may be completed to the nearest 10″ by means of this ingenious contrivance

The general principle may be thus explained:—

Let n represent the value of each division on the principal line or limb;

m, the number of these divisions, taken as the length of the Vernier scale.

Then $m\,n$ = the value of these m divisions.

And this being divided into $m + 1$ equal parts, gives

$\frac{m\,n}{m + 1}$ for the value of each division of the Vernier.

And therefore $n - \frac{m\,n}{m + 1}$ or $\frac{n}{m + 1}$ = the difference between one division of the limb and one of the Vernier.

Now on the sextant $n = 10'$, and if it be required to subdivide to $10''$, we have the equation to solve—

$$\frac{n}{m+1} = 10'' \text{ or } \frac{10'}{m+1} = 10'', \text{ or } \frac{600''}{m+1} = 10'';$$

$$\therefore 600 = 10\,m + 10.$$

Whence it is easy to perceive that $m = 59$.

If $n = 20' =$ two divisions of the limb, m is 119; or 119 divisions of the limb are taken for the length of the Vernier, and this is now a common mode of construction.

The same principle applies to the divisions of Verniers for barometers, &c.

Let $n = \frac{1}{10}$ of an inch, and let it be required to subdivide to 100ths of an inch.

Then $$\frac{\cdot 1}{m+1} = \cdot 01,$$

or, multiplying by 100; $\frac{10}{m+1} = 1 \therefore 10 = m + 1$, or $m = 9$.

Therefore the number of divisions taken from the principal line must be 9, and this length must be divided into 10 equal parts, and then the Vernier scale is constructed.

I is called the *index-glass*, H the *horizon-glass*, L V the *limb*, and a *telescope* T is attached to the instrument, to enable the observer to mark the contact of the objects observed with greater exactness.

In taking altitudes at sea, the instrument is held in a vertical position by means of a handle attached to the back, and then (fig. 14) *h* representing the horizon, and S any celestial object, the arc V V′ will give the altitude of the object above the sea horizon.

On shore an artificial horizon is used, consisting of a small rectangular trough, filled with liquid, generally mercury, and then the angular distance between the object and its image in the mercury is observed, which is double the angle between the visual ray which comes from the object, and the horizontal surface of the mercury, and therefore double the altitude of the object.

The telescope is furnished with a screw to regulate its distance from the plane of the sextant, and to direct it more or less towards the silvered or unsilvered part of the horizon-glass, and thus to render the object which is seen directly through the clear portion, and the image reflected from the silvered portion, equally distinct.

The adjustments of the instruments are, to make all the mirrors *perpendicular*, and the axis of the telescope *parallel* to the plane of the instrument.

There are various screws for making these adjustments, the method of doing which, as well as of using the instruments, will be best learned by practice, under the direction of a skilful teacher; but the following directions for making the requisite adjustments may be found useful.

The frame which holds the index-glass is fastened to the index by means of two screws behind it; and behind these is an adjusting screw. Having placed the index about the middle of the limb, as V′ I (see fig. 15) is placed, turn the face of the instrument upwards, and look obliquely into the glass; and if the image of V V′ appears on a level with V V′ itself, as seen by the eye, the index-glass is perpendicular to the plane of the instrument; but if the image appears *lower* than V V′ tighten the adjusting screw, if *higher*, slacken the adjusting screw till the limb V V′ and its image, seen by reflection in the glass, appear one continued plane.

To make the horizon-glass perpendicular to the plane of the instrument:—

This adjustment is made in some instruments by means of a screw passing through the frame, and in others by a screw behind, turned by a key, or a small capstan-pin, which is put into a hole in the head of the screw.

When the instrument is furnished with a telescope, the adjustment is generally made thus: screw a dark glass on the end of the telescope, and, looking at the sun, make the two images pass over each other, and if they do not exactly cover each other in passing, turn the screw for the perpendicular adjustment of the horizon-glass till they do so. The adjustment may be made with equal readiness by means of the moon or a star, without the use of the dark glass.

To make the axis of the sextant telescope, when it is an inverting one, parallel to the plane of the instrument, turn the eye-piece of the telescope till two of the parallel wires in its focus appear parallel to the plane of the instrument; and bring the sun and moon, the moon and a bright star, or two bright stars, being 90° or more distant from each other, in apparent contact, on the wire next the instrument; instantly bring them to the other wire, on which, if they still appear in contact no adjustment is required; if they separate, *slacken* the screw *furthest* from the instrument, in the ring which holds the telescope, and *tighten* the other; and *vice versâ* if they overlap. On a few repetitions this adjustment may be made perfect, and it is not very liable to alter.

To find the *index-error*, move the index till the horizon, or any distant object, coincides with its image, and the distance of o on the index from o on the limb is the index-error; subtractive when o on the index is to the left, and additive when it is to the right of o on the limb.

Or, move the index forward till the sun and its image appear to touch at the edges, and the difference between the reading on the limb and the sun's known diameter at the time is the index error; subtractive when the reading is the greater, and additive when it is the less.

Or, in the same manner, make the images touch at the edges, above and below, and when the readings are one right and the other left of zero on the limb, half their difference is the index-error; but

when they are both right or both left of zero, half their sum is the index-error; subtractive when the greater reading is left, and additive when it is right of zero

When the readings are on different sides of zero, one-fourth of their *sum*, but when they are on the same side of zero, one-fourth of their *difference* is the sun's semidiameter.

USE OF THE CHRONOMETER.

1. The chronometer, or time-keeper, is a superior kind of watch, the value of which, as an instrument to be employed by a navigator, depends upon the *uniformity* of its performance. Delicate contrivances have been adopted in its construction to compensate for the effect of changes of temperature, which seriously interfere with the going of ordinary watches. It is of great importance to know what we have to expect from this instrument, what it can, and what it cannot do—to what errors its indications are liable, and how to determine and allow for them.

2. Let us first suppose a chronometer to show correctly mean time at Greenwich, and to continue to do so, the hour hand returning to XII. exactly at mean noon. Wherever this instrument is carried, it will show Greenwich mean time, and at any place on the meridian of Greenwich it will show the mean time at that place. If it be taken to a place *east* of Greenwich, then, as it is noon at that place before it is noon at Greenwich, the chronometer will be behind the mean time of that place, or it will be *slow*, and the difference between the time indicated by the chronometer and the actual mean time at that place will be equal to the longitude of the place reduced to time. If the chronometer be taken to a place *west* of Greenwich, the time which it will indicate, and which is supposed to be the true mean time at Greenwich, will now exceed the mean time of the place arrived at, or will be fast for that place, the difference being, as before, the longitude in time.

3. But it can rarely happen that a chronometer shows exactly the mean time at Greenwich, and, therefore, it is necessary to know how much it differs from that time; and this error being known, we shall be able at any time or place to state the actual Greenwich date, by adding the error to the time indicated by the chronometer, if it be too slow, and subtracting the error if the chronometer be fast on Greenwich mean time.

4. Now, as in general such an error will exist in any given chronometer, so it is also true that this error is perpetually changing in amount, and the best that can be expected of the best instrument is, that the daily change of its error, which is called the *rate*, is steady in amount: so that the known error at a given epoch may itself be corrected for the change in it which day after day the *rate* produces.

5. For example, suppose the *error* of a chronometer for Greenwich mean time on June 12th, at noon, to have been 1 h. 27 m. 25 s. slow, and its *daily rate* 7 5 s. gaining, and that it is required to find its error on June 21st, at noon. Here, as 9 days have elapsed, it will have gained 9 times the daily rate, or 67·5 s.,—that is, 1 m 7·5 s.,—its error, which was slow, will therefore have been diminished, and at noon, on June 21st, it will be 1 h. 26 m. 17·5 s. slow on Greenwich mean time.

In precisely the same manner, if the error for a given place, and at any given epoch, and also the rate be known, the error for time at that place can be found from the chronometer wherever we may happen to be, always provided the rate be uniform.

6. A frequent source of embarrassment in interpreting the indications of a chronometer arises from the division of its face into twelve, instead of twenty-four parts, so that the same positions of the pointers represent two periods of the day twelve hours distant. Thus, at 4 hours past noon, and again at 16 hours past noon, the hands are in the same places; and it is necessary to distinguish whether the time should be read as 4 or 16 hours, 5 or 17, 6 or 18 past noon, and so on. Having applied the corrections for error and rate, the day of the month and the hours must be checked thus:—To the approximate astronomical date, at the place of observation, add the longitude in time if west, or subtract if east, and the result is the Greenwich date nearly, and this in the worst circumstances can hardly be an hour wrong.

For example, suppose that at 7 A.M. nearly on June 16th, in longitude 45° east, the corrected chronometer time is 4 h. 24 m. 36 s., what is the correct Greenwich date?

		d.	h	m.
Astronomical date at place, .	June	15	19	0
Longitude in time (subtract)			3	0
Greenwich date nearly . . .	June	15	16	0

Thus we see that the 4 hours of the chronometer must be reckoned as 16, and that the Greenwich rate is June 15 d. 16 h. 24 m. 36 s.

Again, if the longitude be 34° 16′ E, on June 25th, about 6 h. 40 m. P.M., and a chronometer time, read and corrected, be 4 h. 21 m. 50 s., what is the Greenwich date?

		d.	h	m
Date at place, ...	June	25	6	40
Long in time (subtract) . .	. .		2	17
Greenwich date nearly	June	25	4	23

Here the 4 hours indicated by the chronometer are 4, and not 16, past noon, and the Greenwich date is June 25th, 4 h. 21 m. 50 s.

7. *Intervals of time.*—Having thus shown how a correct Greenwich date is to be found from the time indicated by a chronometer, its error and its rate, it is next to be remarked that there is another distinct and important use for which chronometers serve, that of furnishing correct intervals of time.

If a chronometer shows seven hours when it ought to show four, and nine hours when it ought to show six, as the interval between seven and nine is precisely the same as between four and six, it is evident that the error of the chronometer does not affect the intervals of time.

But on account of the *rate*, a chronometer which is three hours fast at *four* will be more than three hours fast at *six*, if it be gaining, and less than three hours fast, if it be losing, by just so much of its *daily rate* as is accumulated in the interval of two hours, that is, by the twelfth part of the daily rate; and therefore the interval shown by the chronometer must be increased by the proportional part of the daily rate due to the interval when the chronometer is losing, or diminished by that amount when the chronometer is gaining, whenever the exact interval of mean solar time is required.

Example.—If between two observations the chronometer exhibits a change of time, or an interval of 3 h. 25 m. 45·6 s., and the daily rate of the chronometer be 12 s. gaining, required the interval of mean solar time.

Here dividing 12 s. by 24, the hourly rate is found to be ·5 s., and this multiplied by 3·43, the hours, &c., in the interval, gives 1·71 s. to subtract, and the work stands thus :—

	h	m	s
Interval by chronometer	3	25	45·6
Cor. for rate			− 1·7
Interval of mean time	3	25	43·9

8. It has already been shown (p. 74) how to reduce an interval of mean solar time into the corresponding interval of sidereal time, by means of the "Table of Time Equivalents" in the "Nautical Almanac;" and here we have the method of securing the exact interval of mean solar time, to which the rules there given apply.

9. It is sometimes necessary to find the interval of apparent time which has elapsed between two observations of the sun. The interval of mean time must first be found by correcting that which the chronometer gives in the manner shown above;

10. And then the change in the equation of time due to the elapsed time must be calculated by multiplying the "Diff in 1 hour" (from page I. of the month in the "Nautical Almanac") by the interval in hours; this product is the correction of the *mean* solar interval to reduce it to the corresponding interval of *apparent* solar time.

11. The correction is to be *added* to the interval of mean time when the equation of time is *additive to mean time and increasing*, or

when it is *subtractive and decreasing;* and it is to be *subtracted* when the equation of time is *additive and decreasing,* or *subtractive and increasing.*

12. To see whether the equation of time is *additive* or *subtractive* on mean time, you must look at the heading of the "Equation of Time" column, in page II. of the month in the "Nautical Almanac."

Example.—Let it be required to convert 4 h 23 m. 13 s. of mean solar time into apparent solar time, supposing the time to be from about two hours before to two hours after apparent noon at Greenwich, on August 16th, 1855.

4 h. 23 m. 13 s. reduced to hours is 4·387, and by page I. for the month of August in the "Nautical Almanac," the "Diff. for 1 hour" in the equation of time is ·504 s.

$$\text{and } \cdot 504 \text{ s.} \times 4\cdot 387 \doteq 2\ 21 \text{ s.}$$

In page II. it is seen that the equation of time is to be *subtracted* from mean time, and a glance at the values in the column shows it to be *decreasing.* The correction 2·21 s. is, therefore, to be added to the interval of mean time, and the corresponding apparent time is 4 h. 23 m. 15·21 s.

13. Once and again the learner is cautioned against confounding an interval of time with a moment of absolute time or a date.

An interval is a portion of time elapsed between any two events whatever. But a date, or moment of absolute time, is indicated by the interval elapsed from a definite or standard event, such as the transit of the sun over the meridian, or of the mean sun or first point of Aries over the meridian.

For example, it is essential to distinguish between the following questions:—

1. What sidereal time corresponds to 2 hours of mean solar time?

2. What is the sidereal time at 2 P.M. mean solar time?

The first question relates to the difference between the length of the hours of mean solar time and of sidereal time, and the answer is, that 2 h. of mean solar time are equivalent to 2 h. 0 m. 19·71 s. of sidereal time; and this is irrespective of any particular place or date.

But in the second question the words "sidereal time" have a different signification; they mean here the time from sidereal noon or from the transit of the first point of Aries; and the question may be read thus:—How many sidereal hours, &c., has the first point of Aries passed the meridian at two o'clock in the afternoon? Or, what time should a sidereal clock show at two in the afternoon? And even when thus interpreted the question does not admit of such a categorical answer as the preceding one.

The proper answer would be—That will depend on the time of year, on the relative positions of the mean sun and of the first point of Aries; in other words, on the right ascension of the mean sun. It will depend also on the meaning we attach to 2 P.M., whether 2 P.M. at Greenwich or at any other place; and therefore before this question can be answered, the date must be completed, and a place or at least the longitude should be mentioned.

For instance,

At 2 P.M. mean time at the observatory of St. Petersburg on August 10th, 1859, what is the sidereal time?

		h	m	s
Date at St Petersburg . . .	August 10	2	0	0
The longitude of St. Petersburg		2	1	15·8 E
Date at Greenwich	August 9	23	58	44·2

	h	m.	s
Right ascension of the mean sun at noon at Greenwich on August 9, 1859 . .	9	1	55·18
Increase in 23 h. 58 m. 44·2 s.		3	56·35
Right ascension at the given date . . .	9	5	51·53
Mean time at St. Petersburg	2	0	0
Sidereal time required	11	5	51 53

That is, the sidereal clock should show 11 h. 5 m. 51·5 s.

As a second example, let us ask—

1. What is the apparent time corresponding to 2 hours of mean solar time?

2. What is the apparent time at 2 hours P.M. mean solar time?

The 2 hours in the first question are evidently an interval taken anywhere out of the day; but in the second question the letters P.M. and the word *at* both indicate that the 2 hours form a portion of a date, to which are wanting the year, the day, and the month, with the addition of which and a special locality an instant of absolute time will be sufficiently specified.

And for the answers to the questions—

For the first it must be said that as the apparent solar days are unequal in length, and the mean solar days uniform, the question is incomplete without a Greenwich date, to enable us to find the relation between mean and apparent time.

When the apparent solar day is shorter than the mean solar day there will be a correction to *add* to the 2 hours of mean solar time, and when the apparent solar day is longer

than the mean solar day, the correction will, of course, be subtracted in order to find the interval of apparent solar time: for the shorter the divisions of time are, the greater will be the number of them required to measure a given interval, and *vice versâ*.

When the equation of time at page II. of the Almanac is additive and increasing, or subtractive and decreasing, the apparent solar day is shorter than the mean solar day by as much as the change of the equation of time in 24 hours: but if the equation of time at page II. is additive and decreasing, or subtractive and increasing, the apparent solar day is longer than the mean solar day by the change of the equation of time per day.

And to convert a mean solar interval into an apparent solar interval, a proportional part of the daily change of the equation of time must be added to the mean solar interval in the first, and subtracted from it in the second case.

And for the second question, it is not the change, but the total amount of the equation of time at the given date, which must be applied to the mean solar date to obtain the apparent solar *date*.

In like manner to obtain a correct *interval* of mean time from a chronometer which is gaining or losing, the proper correction is a proportional part of the *rate* due to the length of the interval.

But to obtain an accurate *date*, the whole *error* must be applied.

Questions.

1. Reduce 2h. 25m. 35s. of mean solar time to its equivalent in apparent solar time, September 8, 1858. Difference for 1h. = 0·85s.
Ans. 2h. 25m. 37s.

2. Reduce 3h. 40m. 55s. of apparent solar time to its equivalent in mean solar time, June 14, 1858. Difference for 1h. = 0·53s.
Ans 3h. 40m. 57s.

3. The interval between two observations, shown by a chronometer which is gaining 7·5 seconds per day on mean time, is 4h. 15m. 10s; required the correct interval of mean solar time?
Ans. 4h. 15m. 8·67s.

4. A correct sidereal clock shows 2h. 10m. 15s. between the transit of two stars, what is the corresponding interval of mean solar time?
Ans. 2h. 9m. 43·6617s.

5. Show that

$$\frac{\textit{An interval of mean solar time}}{\textit{The same interval in apparent solar time}} = \frac{\textit{An apparent solar day}}{\textit{A mean solar day.}}$$

6. What time should a sidereal clock show at Paris, on April 18, 1858, at 5h. 12m. 27s. P.M. Greenwich mean time? Longitude of Paris 0h. 9m. 20·63s. E.
Ans. 7h. 7m. 53·24s.

PRACTICAL RULES AND EXERCISES IN NAUTICAL ASTRONOMY.

TO FIND THE LATITUDE BY A MERIDIAN ALTITUDE OF THE SUN.

1. Find the Greenwich date (apparent time) of the sun's transit.

2. Take the sun's declination from the "Nautical Almanac" (page I. of the month), and correct it with the Greenwich date; notice also if the declination be N or S.

3. Correct the observed altitude for the index error, dip, refraction, semidiameter, and parallax, and subtract from 90° to find the true zenith distance, and note whether the zenith be N or S of the sun.

4. The sum or difference of the zenith distance and declination is the latitude, according as they are of the same or different denominations: and the latitude is N or S as the greater is.

Example.—On May 23rd, 1854, in longitude 73° E, the observed meridian altitude of the sun's lower limb was 59° 14′ 30″, the zenith north of the sun, index error + 2′ 30″, and the height of the eye above the sea 19 feet; required the latitude?

The observation was made when the sun was on the meridian, that is, at apparent noon; the date, therefore, at the place of observation, is May 23rd, 0 h. 0 m. 0 s.

But the meridian of the place of observation is 73° E of Greenwich, and therefore the sun is 73° E of the meridian of Greenwich, or, in time, 4h. 52m.; for 73° is equivalent to 4h. 52m.

It is therefore 4h. 52m. *before* apparent noon of May 23rd, at Greenwich, and the *Greenwich date* will be found by subtracting 4h. 52m. from the time of apparent noon on May 23rd, thus—

	d.	h	m.	s
Apparent noon, May	23	0	0	0
Longitude in time E		4	52	0
Greenwich date, May	22	19	8	0

With this date, the sun's declination must be taken from the "Nautical Almanac," where it will be found for apparent noon, May 22nd, in page I. for May. And in the adjoining column, headed *Diff. in one hour*, is found the change in one hour past noon.

But the Greenwich date shows that there are 19 h. 8 m. past noon; or, reckoning in hours and decimals, 19·133, and therefore, if we retain only two places of decimals, the diff. in one hour must be multiplied by 19·13, to find the change in that number of hours. And as the declination is increasing in May, the computed change must be added to the value of the declination at apparent noon.

As a general rule, and for the sake of order, which is essential to correct calculation, it is recommended that the quantities to be corrected be written to the left, and the correction computed, immediately to the right of them, thus—

	° ′ ″		
☉'s declination at apparent noon, May 22nd . . .	20 22 44·8 N	Diff in 1 hour . . .	29·23+
Correction . . .	+ 9 19·2	Hours past noon . .	19·13
	20 32 4·0		8769
			2923
			26307
			2923
Semidiameter for May 23rd	15′ 49·4″		559·1699
			=9′ 19·17″

The observed altitude must now be corrected.

	° ′ ″	
Observed alt. . . . . ☉	59 14 30	
Index error	+ 2 30	
	59 17 0	
Dip for 19 feet, Table I.	− 4 17	
	59 12 43	
Refraction, Table III.	- 34	
	59 12 9	
Semidiameter	+ 15 49 4	
	59 27 58·4	
Parallax, Table IV.	+ 4	
True alt.	59 28 2·4	
90°—True alt = zenith distance . . .	30 31 57·6 N	(see question.)
Declination	20 32 4·0 N	
Latitude	51 4 1·6 N	= sum (see rule)

The work should be compactly tabulated, as in the following example:—

Example 2.—If on September 2nd, 1855, in longitude 55° E, the observed altitude of the ☉ be 85° 13′ 20″ (zenith north), index error −2′ 10″, and the height of the eye 18 feet; required the latitude?

For Greenwich Date.		Longitude in Time
	d. h. m.	°
Sept	2 0 0	55 E
Longitude E .	3 40	22,0
Sept	1 20 20	3 h. 40 m.

Declination, Page I		Correction	
	° ′ ″		′ ″
On Sept 1 .	8 24 38 0 N	Diff in 1 hour	−54·52
Correction .	−18 28·4	Hours past noon	20 33
Declination .	8 6 9·6 N	Product . .	=18 28·4

To correct the Altitude and find the Latitude	
	° ′ ″
Observed alt.	85 13 20
Index error	− 2 10
	85 11 10
Dip for 18 feet	− 4 11
	85 6 59
Refraction	− 5
	85 6 54
Semidiameter	+ 15 53
	85 22 47
Parallax	+ 1
True alt.	85 22 48
Zenith distance	4 37 12 N
Declination	8 6 9·6 N
Latitude	12 43 21·6 N

To find the Latitude by the Meridian Altitude of a Fixed Star.

Rule.—The rule is the same as for the sun; but such precision is not required in the Greenwich date, as the declinations of the stars change very little, and are given for every tenth day in the "Nautical Almanac," under the heading "Apparent places of the principal fixed stars for the upper transit at Greenwich."

Example.—The observed meridian altitude of α Pegasi was 33° 20′ 50″ (zenith south of the star), on March 31st, 1854, index error of sextant 1′ 20″ +, and height of the eye above the sea 20 feet; required the latitude?

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Declination, April 1	Correction to March 31
On April 1 . 14° 25′ 2″ N	Difference for 10 days . ·1 Therefore in one day ·01 The correction being very small, may be neglected in this case

Next to correct the observed altitude.

	° ′ ″
Observed altitude α Pegasi . . .	33 20 50
Index error.	+ 1 20
	33 22 10
Dip (20 feet)	− 4 24
	33 17 46
Refraction	− 1 26
True altitude	33 16 20
Zenith distance	56 43 40 S
Declination	14 25 2 N
Latitude	42 18 38 S

EXAMPLES FOR EXERCISE.

1. On May 23rd, 1855, in long. 61° 55′ E, the observed meridian altitude of the sun's lower limb was 59° 14′ 22″ (zenith north of the sun), index error − 1′ 54″, height of the eye 16 feet; required the latitude?

Declination of the sun at apparent noon at Greenwich on May 22nd, 1855, 20° 19′ 58·8″ N, "Diff. in 1 hour" + 29·43″, and semidiameter 15′ 50″. Ans. 51° 5′ 52″ N.

2. On May 1st, 1855, in long. 47° 5′ W, the observed meridian altitude of the sun's lower limb was 69° 30′ 16″ (zenith north of the sun), index error + 3′ 33″, height of the eye 14 feet; required the latitude?

Declination of the sun at apparent noon at Greenwich on May 1st, 1855, 14° 59′ 27·0″ N, "Diff. in 1 hour" + 45·23″, and semidiameter 15′ 54″. Ans. 35° 16′ 5″ N.

3. On March 9th, 1855, in long. 4° 10′ E, the observed meridian altitude of the sun's upper limb was 32° 18′ 50″ (zenith south of the sun), index error +55″, height of the eye 16 feet; required the latitude?

Declination of the sun at apparent noon at Greenwich on March 8th, 1855, 4° 59′ 11·2″ S, "Diff. in 1 hour" – 58·50″, and semidiameter 16′ 8″. Ans. 62° 37′ 45″ S.

4. On August 5th, 1855, in long. 44° 58′ W, the observed meridian altitude of the sun's lower limb was 26° 5′ 47″ (zenith south of the sun), index error + 2′ 37″, height of the eye 13 feet; required the latitude?

Declination of the sun at apparent noon at Greenwich on August 5th, 1855, 17° 5′ 0·9″ N, "Diff in 1 hour" – 40·74″, and semidiameter 15′ 48″. Ans. 46° 38′ 9″ S.

5. On July 26th, 1855, in long. 13° 11′ W, the observed meridian altitude of the sun's upper limb was 16° 14′ 34″ (zenith south of the sun), index error + 3′ 37″, height of the eye 18 feet; required the latitude?

Declination of the sun at apparent noon at Greenwich on July 26th, 1855, 19° 31′ 54 8″ N, "Diff. in 1 hour" – 33·27″, and semidiameter 15′ 47″. Ans. 54° 33′ 27″ S.

6. On May 8th, 1855, in long. 105° 17′ W, the observed meridian altitude of the sun's lower limb was 76° 3′ 16″ (zenith south of the sun), index error – 1′ 27″, height of the eye 10 feet; required the latitude?

Declination of the sun at apparent noon at Greenwich on May 8th, 1855, 17° 0′ 38·2″ N, "Diff. in 1 hour" + 40·58″, and semidiameter 15′ 52″. Ans. 3° 19′ 46″ N.

7. On May 23rd, 1855, the observed meridian altitude of Antares was 55° 26′ 17″ (zenith south of the star), index error + 1′ 15″, height of the eye 13 feet; required the latitude?

Declination of Antares on May 23rd, 26° 6′ 31·5″ S.

Ans. 60° 43′ 12·5″ S.

8. On November 12th, 1855, the observed meridian altitude of Marcab was 45° 48′ 40″ (zenith north of the star), index error – 47″, height of the eye 9 feet; required the latitude?

Declination of Marcab on Nov. 12th, 14° 25′ 53·8″ N.

Ans. 58° 41′ 54″ N.

9. On December 11th, 1855, the observed meridian altitude of Capella was 81° 34′ 0″ (zenith north of the star), index error + 2′ 10″, height of the eye 13 feet; required the latitude?

Declination of Capella on Dec. 11th, 45° 50′ 53·8″ N.

Ans. 54° 18′ 26″ N.

10. On November 5th, 1855, the observed meridian altitude of Altair was 57° 2′ 30″ (zenith north of the star), index error – 2′ 9″, height of the eye 18 feet; required the latitude?

Declination of Altair on Nov. 5th, 8° 29′ 27″ N.

Ans. 41° 33′ 54″ N.

PREPARATORY PROBLEMS.

PROB. I.—*To find the mean time Greenwich date, of the moon's meridian passage on a given astronomical day in a given latitude.*

1. Enter the "Nautical Almanac," page IV. of the month, with the given astronomical day, and take out the time opposite to it under the heading "Meridian Passage;" also take the difference between this time and the time on the following day if the longitude be west, or on the preceding day if the longitude be east: this difference is the daily retardation.

2. Find the correction for longitude and daily retardation by this formula:—

$$\frac{\textit{Longitude in degrees} \times \textit{retardation in minutes}}{360} = \textit{correction in minutes.}$$

3. Add the correction to the time of meridian passage at Greenwich when the given longitude is west, and subtract if it be east; the sum or remainder is the mean time at the given meridian, corresponding to the moon's transit.

4. Write the day of the month before the hours, minutes, &c., and add the longitude in time if west, or subtract it if east, and the result is the *mean time, Greenwich date, of the moon's meridian passage.*

Note.—If the civil day be given, and it be evident from the "Nautical Almanac" that the meridian passage is past midnight on that day, the time in the "Nautical Almanac" on the preceding day must be taken.*

Example.—Required the Greenwich date of the moon's meridian passage on May 6th, 1855, *civil date*, in longitude 63° W?

1. Entering the "Nautical Almanac" with May 6th, at page IV. of May, it is at once seen that the moon passes the meridian after midnight on that day. The time must therefore be taken which stands opposite to May 5th: it is 15 h. 31·7 m.

2 The longitude being west, the difference between this time and the time on the *following day* is to be taken: this time is 16 h. 33·6 m., and the difference or retardation is 1 h. 1·9 m. = 61·9 m.

* Many persons add the retardation to the longitude in time, and apply the sum, with the sign of the longitude, to the meridian passage of the moon.

The work is then tabulated thus:—

☽'s Meridian Passage.	Correction for Retardation	Longitude
d. h. m May 5 15 31·7 Correction . . + 10·8 May 5 15 42·5 Longitude . .+4 12 May 5 19 54 5	m. Retardation . 61·9 Longitude . 63 1857 3714 6) 3899·7 6,0) 64,9·9 +10 8 cor.	Longitude . . 63° 4 6,0) 25,2 Long in time 4 h. 12m
∴ Greenwich date (mean time), 1855, May 5th, 19 h 54 5 m		

PROB. 2.—*To find the semidiameter and horizontal parallax of the moon for a given Greenwich date (mean time), from the "Nautical Almanac," page III. of each month.*

1. Take them from the "Nautical Almanac" for the *noon* or *midnight* next preceding the given date, and the change to the following *midnight* or *noon.*

2. Reduce the time past noon or midnight to hours and decimals; multiply the change in 12 hours by the number of hours past noon or midnight, and divide by 12; the quotient is the correction.

3. Add the *correction* to the values of the semidiameter and horizontal parallax which were taken out, when they are increasing, but subtract the correction when they are decreasing, and the result will be their values at the given *Greenwich date.*

Example.—Required the moon's semidiameter and horizontal parallax, for the Greenwich date, 1855, May 5th, 19 h. 54·5 m.

Here, as the hours indicate that the time is past midnight, the semidiameter and horizontal parallax must be taken out for midnight of May 5th, and they must be subtracted from the values on May 6th at noon, and the work stands thus:—

Semidiameter at midnight.	Change in 12 hours.	Change in 12 hours	Hor Parallax at midnight.
16′ 4″ 1 Correction +1·4 16 5·5 Aug. +	+2″·1 7·9 hours past midnight 189 147 12) 16 59 1·38. . . Correction	+7″5 7 9 675 525 12)59 25 . 4 93	58′ 51″·2 Correction +4·9 58,56·1 Red.—
∴ Semidiameter=		Horizontal parallax=	

$+ 2{\cdot}1$ and $+ 7{\cdot}5$ are the changes in the semidiameter and horizontal parallax between midnight, May 5th, and noon, May 6th; and $7{\cdot}9$ the number of hours past midnight of May 5th, according to the Greenwich date.

The blanks marked "Aug." and "Red." are for the augmentation and reduction from Tables VIII. and VII.: the final results should then be recorded in the places left for them below, the parallax being first reduced to seconds.

PROB. 3.—*To take out the moon's declination from the "Nautical Almanac" (pages V. to XII. of each month), with a given Greenwich date.*

Note.—The right ascension and declination of the moon are given for every hour of every day, and also the change of the declination in 10 minutes, under the heading "Diff. Dec. for 10 m:" therefore,

1. Take out the declination for the given day and hour of the Greenwich date, together with the diff. for 10 m. which stands opposite it.

2. Convert the minutes and seconds of the date to minutes and decimals of a minute, and multiply this into the difference for 10 m., and divide by 10: the quotient is the correction.

3. Add the correction to the declination when the declination is increasing, or subtract when it is decreasing, and the result is the declination at the given *Greenwich date.*

Example.—Required the moon's declination for the Greenwich date, 1855, May 5th, 19 h. 54·5 m.?

☽'s Declination.		Correction.	
	° ′ ″		″
At 19 hours	27 32 55·8 S	Diff for 10 m.	− 3 42
Correction	− 18·6	Min. in date	54 5
	27 32 37·2 S		1710
			1368
			1710
			10) 186·390
		Correction	− 18 6
Corrected declination . . . 27° 32′ 37″ S.			

To find the Latitude by the Meridian Altitude of the Moon.

1. Find the Greenwich date (mean time) of the moon's meridian passage. (Prob. 1.)

2. Take from the "Nautical Almanac" the semidiameter and horizontal parallax (page III.), correcting them as in Prob. 2. Add the augmentation to the semidiameter, and subtract the reduction from the horizontal parallax (Tables VIII. and VII.); reduce the horizontal parallax to seconds.

3. Take out also the moon's declination, Prob. 3.

4. Correct the observed altitude of the moon for index error, dip, refraction, and semidiameter, and so get the apparent altitude; and this, subtracted from 90°, leaves the apparent zenith distance.

5. Compute the parallax in altitude from this equation :—

Parallax in altitude = horizontal parallax × sin apparent zenith distance.

And subtract the parallax in altitude from the apparent zenith distance: the remainder is the true zenith distance.

6. The latitude is the sum of the zenith distance and declination when they are both north or both south, or their difference when one is north and the other south; and the latitude is of the same denomination as the greater.

Example.—On March 30th, 1855, in longitude 29° 50′ W, the observed meridian altitude of the moon's lower limb was 46° 23′ 40″ (the zenith north of the moon), index error + 3′ 10″, height of the eye above the sea 20 feet; required the latitude?

☽'s Meridian Passage.	Correction for Retardation.	Longitude.
d. h. m.	m.	° ′
March 30 10 20·1	Retardation . 41·6	Longitude . 29 50 W
Correction . +3·45	Longitude . 29·83	4
March 30 10 23·55	1248	6,0) 11,9 20
Longitude . 1 59·33	3328	1 h. 59 m 20 s
March 30 12 22·88	3744	29° 50′ = 29·83
	832	
	6) 1240·928	
	6,0) 20,6 8	
	3·45 cor.	

(1) ∴ Greenwich date (mean time), 1855, Mar. 30, 12 h. 22·9 m.

Semidiameter at midnight.	Change in 12 hours.	Change in 12 hours	Hor Parallax at midnight.
14′ 54″·2	+2″·7	+9″·8	54′ 35″·0
Correction +·1	·38 Hours past midnight	·38	+·3
14 54 3	216	784	54 35·3
Aug. . +10·3	81	294	Red. . −7
15 4·6	12) 1·026	12) 3·724	54 28·3
	·08 . Correction .	. ·31	

(2) ∴ Semidiameter, 15′ 4·6″ Hor. parallax, 3268·3″

☽'s Declination.		Correction	
At 12 hours . . .	12° 0′ 50″ 3 N	Diff. for 10 m	127″·17
Correction . . .	−4 51·2	Min in date	22·9
	11 55 59·1 N		114453
			25434
			25434
			10) 2912·193
			6,0) 29,1·22
		Correction	−4′ 51·2″

(3) ∴ Corrected declination . . . 11° 55′ 59·1″ N.

To correct the observed Altitude, &c.

Altitude.		Parallax.	
	46° 23′ 40″	Zenith distance, about . .	43° N
Index error . . .	+3 10	Declination	12 N
	46 26 50	Latitude, about	55 N
Dip (20 feet) . . .	−4 24	∴ { Reduction of hor. parallax, Table VI., applied above . . }	−7″
	46 22 26		
Refraction . . .	−54		
	46 21 32	Apparent zenith distance .	43° 23′
Semidiameter . .	+15 4·6	Red. of lat., Table VII. . .	−11
Apparent altitude .	46 36 36·6		43 12
App zenith dist. .	43 23 23·4	43° 12′ . . . . sin	9·835403
Parallax in altitude .	−37 17·3	3268·3″ . . log	3·514322
True zenith dist. .	42 46 6·1 N	6,0) 223,7·3	3·349725
Declination . . .	11 55 59·1 N		
Latitude	54 42 5·2 N	−37′ 17·3″ parallax in altitude.	

(4) ∴ Latitude . . . . . 54° 42′ 5·2″ N.

Questions for Exercise.

1. If on May 25th, 1855, at about 7 h. 10 m. P.M., in longitude 34° 18′ E, the observed meridian altitude of the moon's lower limb e 50° 10′ 20″, the zenith being south of the moon, the index error - 2′ 30″, and the height of the eye 19 feet; required the latitude?
Ans. Lat. 30° 56′ 31″ S.

2. If on July 3rd, 1855, at 3 h. 20 m. A.M. mean time nearly, in ngitude 54° 12′ W, the observed double altitude of the moon's lower mb on the meridian be 45° 2′ 10″, the zenith being north of the oon, and the index error – 1′; required the latitude?
Ans. Lat. 49° 14′ 51″ N.

3. If on September 23rd, 1855, at 10 h. 10 m. P.M. mean time at ace nearly, in longitude 13° 22′ E, the observed meridian altitude the moon's lower limb be 40° 3′ 30″, the zenith being north of the oon, the index error + 2′ 25″, and the height of the eye 14 feet; quired the latitude?
Ans. Lat. 33° 51′ 11″ N.

4. If on December 20th, 1855, at 9 h. 50 m. P.M. mean time at ace nearly, in longitude 63° 51′ W, the observed meridian altitude the moon's upper limb be 55° 2′ 20″, the zenith being south of e moon, the index error – 3′ 10″, and the height of the eye feet; required the latitude?
Ans. Lat. 12° 18′ 41″ S.

Elements taken from the "Nautical Almanac" for computing the above questions.

Meridian Passage at Greenwich.	Retardation	Date	Declination.	Change of Declination in 10 m.
d. h. m.	′	d. h	° ′ ″	″
Iay 25 7 33	41 0	24 22	8 10 16·9 N.	−134·15
uly 2 15 12 7	53 8	2 18	17 17 48·5 S.	−136·75
ept. 23 10 16	56·6	23 9	15 9 41·1 S.	−151·78
Dec. 20 9 43·2	54 5	20 14	22 27 20·6 N.	+ 97·72

Date.	Semidiameter.	Hor Parallax.	Change in 12 hours. S. D.	Change in 12 hours. H. P.
d. h.	′ ″	′ ″	″	″
Iay . . 24 12	14 54·6	54 36·8	+3·1	+11·3
uly . . 2 12	16 30·6	60 28·1	−2·9	−10·3
ept . . 23 00	16 35 7	60 46 9	+2·7	+10
ec. . . 20 12	15 44 9	57 40·7	−3·8	−14

PROB —*To find the mean time at any place, and also the Greenwich mean time of the passage of a star over a given meridian on a given day.*

First, approximately.

RULE 1. Take the star's right ascension from the "Nautical Almanac," for the day nearest to the given date.

2. Take the *sidereal time* from page II. of the month for the given day.

3. *Subtract* the sidereal time (which is the mean sun's right ascension) from the star's right ascension, increasing the latter by 24 hours if necessary, and the remainder is the mean time (nearly) of the star's transit at the given meridian.

4. Write the day of the month before the hours, &c., and add the longitude if west, or subtract it if east, and the sum or remainder is the *Greenwich date* of the meridian passage of the star (nearly).

This will be sufficient, if the object of the calculation be only to prepare for taking the meridian altitude of the star.

Example 1.—About what time will β Tauri pass the meridian on August 10th, 1855 (civil date)?

	h	m	s
Right ascension of β Tauri on Aug. 10th	5	17	8·46
Sidereal time (or right ascension mean sun)	9	13	36·19
	20	3	32·27
or about Aug 10th	8	3	32 A M.

The Astronomical date is Aug 9th, 20 h. 3 m. 32 s.

Observe—the right ascension of the star is taken out of the "Nautical Almanac," 1855, at page 449. "*Apparent places of the principal fixed stars,*" *and not from the table of* "*Mean places of* 100 *principal fixed stars.*"

As the stars are tabulated in the order of their right ascensions, beginning at 0 h. and proceeding to 24 h., the right ascension in the table of "Mean places" may be used as an index to the table of "Apparent places" to facilitate the finding of the stars in the latter table.

Example 2.—About what time will the planet Saturn pass the meridian on February 18th, 1855 (astronomical date)?

	h	m	s
Saturn's right ascension on February 18th	4	30	12·6
Sidereal time or right ascension of mean sun	21	51	32
February 18th	6	38	40·6

∴ Time at place, February 18th, 6 h. 38 m. 40s.

Secondly, to find the time of the star's meridian passage more accurately.

Rule 1. Find the approximate time as shown above.

2. Apply the longitude in time, adding when it is west, subtracting when it is east; and so convert the time at place into a *Greenwich date.*

3. With this Greenwich date take out again the mean sun's right ascension. Prob. 4 (page 78).

4. *Subtract* the mean sun's corrected right ascension from the star's right ascension, increasing the latter by 24 hours if necessary, and the remainder, with the *astronomical* day written before it, is a more correct date of the star's transit.

5. By applying the longitude again a more correct mean Greenwich date is also obtained.

6. If still greater accuracy is required, the operations 3, 4, 5, may be repeated with the Greenwich date found by Rule 5.

Example.—At what time will Markab pass the meridian of a place in longitude 87° 10′ E on June 20th, 1855 (astronomical date)?

	h	m.	s.	
From Markab's right ascension, June 20th	22	57	33·4	
Subtract right ascension of mean sun at mean noon	5	52	31·7	
Astronomical date nearly June 20th	17	5	1·7	
Long. 87° 10′ E in time	5	48	40	
Corresponding Greenwich date, June 20th	11	16	21·7	(1)

	h.	m.	s	
Right ascension of mean ☉ at mean noon	5	52	31·74	
Acceleration for 11 h		1	48·42	
Acceleration for 16 m.			2·63	
Acceleration for 21 s.			·06	
Acceleration for ·7			·00	
Mean sun's corrected right ascension	5	54	22·85	(2)

		h	m	s
Markab's right ascension		22	57	33·40
The mean sun's corrected right ascension		5	54	22·85
Date at given meridian	June 20th	17	3	10·55
Longitude in time		5	48	40
Greenwich date	June 20th	11	14	30·55

Note.—By adding half a sidereal day, or 11 h. 58 m. 2·05 s. of mean time, to the time of the upper transit of a fixed star, the time of transit below pole can be obtained. If 23 h. 56 m. 4·09 s. be added, the sum is the time of the upper transit on the following day, or if it be subtracted, the remainder is the time of transit on the preceding day.

TO FIND THE LATITUDE BY THE ALTITUDE OF THE SUN ON THE MERIDIAN BELOW THE POLE.

RULE 1. The apparent time at the place is *midnight;* apply the longitude in time, adding if W and subtracting if E, and so get the *apparent time Greenwich date.*

2. Take the sun's declination from page I. of the month, in the "Nautical Almanac," and correct it for the time past noon at Greenwich.

3. Subtract the declination from 90°, for the declination.

4. Correct the sun's altitude and add to it the co-declination, and the sum is the latitude, which is N or S as the declination is N or S.

Example.—On June 2nd, 1855, in longitude 79° E, the altitude of the sun's lower limb on the meridian below pole was 10° 22′ 30″, index error − 1′ 10″, height of the eye above the sea 21 feet; required the latitude?

Longitude in Time.		For Greenwich Date.	
	°		h m.
Longitude	79 E	Date at place . . June 2nd	12 0
	4	Long. in time	−5 16
	6,0) 31,6	June 2nd	6 44
Longitude in time. .	5 h. 16 m.		

∴ Greenwich Date . . . . . . 1855, June 2nd, 6 h. 44 m.

Declination, Page I.		Correction.	
	° ′ ″		″
☉'s Dec. June 2nd .	22 9 29·9 N	Difference for 1 h.	+19·27
Correction	+ 2 9·7	Hours past noon	6·73
	22 11 39·6		5781
			13489
90° − Dec. = Co-dec.	67 48 20·4		11562
			6,0) 12,9·6871
Semidiameter	15′ 48·1″	Correction	+2′ 9·7″

∴ Co-declination at time of observation . 67° 48′ 20·4″

To correct the Altitude and find the Latitude	
	° ′ ″
Observed altitude	10 22 30
Index error	− 1 10
	10 21 20
Dip for 21 feet	− 4 31
	10 16 49
Refraction	− 5 7
	10 11 42
Semidiameter	+ 15 48·1
	10 27 30·1
Parallax in altitude	+ 9
True altitude	10 27 39 1
North Polar distance	67 48 20 4
Latitude	78 15 59 5 N

To find the Latitude by the Altitude of a Star on the Meridian below the Pole.

Rule.—As for the sun; but omitting the *Greenwich date*, such precision in the time not being requisite, on account of the small change in the declination of the fixed stars.

Example.—The observed altitude of β Centauri on the meridian below the south pole on May 7th, 1855, was 43° 18′ 20″, index error + 2′ 10″, and height of the eye above the sea 18 feet; required the latitude?

Declination β Centauri.		Correction.	
	° ′ ″		
On May 1st	59 40 26 3 S	10 days' difference	+2″7
	+ 1 6		6
	59 40 27 9		10) 16·2
South Polar Distance	30 19 32·1	6 days' difference	1·62
South Polar Distance . . . 30° 19′ 32·1″.			

To correct the Altitude and find the Latitude

	°	′	″
Observed altitude	43	18	20
Index error	+	2	10
	43	20	30
Dip for 18 feet	−	4	11
	43	16	19
Refraction	−	1	0
True altitude	43	15	19
South Polar distance	30	19	32·1
Latitude	73	34	51·1 S

Questions for Exercise.

1. July 1st, 1855, longitude 59° 13′ E, the observed altitude of the sun's lower limb (artificial horizon) on the meridian below the pole, 18° 33′ 10″, index error + 1′ 50″; required the latitude?

The sun's declination at apparent noon July 1st, 1855, 23° 9′ 8·4″ N, diff. for 1 hour − 10·22″, and semidiameter 15′ 46″.

Ans. Lat. 76° 20′ 00″ N.

2. December 12th, 1855, longitude 101° 20′ E, the observed altitude of the sun's upper limb (artificial horizon) on the meridian below the pole, 20° 10′ 14″, index error − 1′ 10″; required the latitude?

The sun's declination at apparent noon December 12th, 1855, 23° 4′ 45·7″ S, diff. for 1 hour + 10·96″, and semidiameter 16′ 17″.

Ans. Lat. 76° 37′ 28″ S.

3. March 8th, 1855, the observed altitude of α Ursæ Majoris on the meridian below the pole, 45° 18′ 29″, index error + 2′ 10″, height of the eye 15 feet; required the latitude?

Declination of α Ursæ Majoris 62° 31′ 57″ N.

Ans. Lat. 72° 43′ 54″ N.

4. August 23rd, 1855, the observed altitude of η Draconis (artificial horizon) on the meridian below the pole, 48° 30′ 18″, index-error + 2′ 10″; required the latitude?

Declination of η Draconis 61° 50′ 46″ N.

Ans. Lat. 52° 23′ 22″ N.

To find the Latitude by the Altitude of the Moon on the Meridian below the Pole.

Rule 1. Find the Greenwich date.

2. Take the horizontal parallax and semidiameter from the "Nautical Almanac," as directed in the rules for "Meridian Altitudes."

3. Take the declination from the "Nautical Almanac," and subtract it from 90°.

4. Correct the altitude and add to it the co-declination, and the sum is the latitude.

1. *To find the Greenwich date.*

Rule *a.* Take out the time of the preceding meridian passage and the retardation as before directed.

b. Add to this 12 hours, and half the retardation between this passage and the following one.

c. Add the longitude in time when west, or subtract when it is east, and the *Greenwich date* is found.

Having found the Greenwich date; the declination, semidiameter, and horizontal parallax are taken out in the same manner as for the moon's meridian altitude. The altitude is then corrected, and the polar distance is added to the corrected altitude. (Rule 4.)

The elevation of the pole above the horizon is equal to the latitude of the place of observation, and therefore no object can be seen between the pole and horizon, whose polar distance exceeds the latitude.

The polar distance of the sun at the time of the summer solstice, when it is least, is about $66\frac{1}{2}°$, and that of the moon a little less; these bodies therefore can be seen on the meridian below pole only in the Arctic regions.

To find the Latitude by the Altitude of the Pole-Star.

The principle upon which this method depends is, that the altitude of the pole equals the latitude of the place of observation.

The pole-star is situate within about $1\frac{1}{2}°$ of the north pole, and is carried by the daily rotation of the heavens alternately above and below the pole. When the star is on the meridian above the pole, the latitude may be found by subtracting the polar distance of the star from its true altitude; and when the star is on the meridian below the pole, by adding the polar distance to the true altitude.

The polar distance is the greatest difference between the true altitude of the star and the latitude of the place, and the difference or *correction of the star's altitude* varies with the star's distance

from the meridian, between the limits $+p$ and $-p$, if p be allowed to represent the polar distance.

The method of working this problem by means of certain special Tables is shown in the "Explanation" which is appended to the "Nautical Almanac."

There is, however, some trouble in interpolating between the quantities given in the special Tables before so exact a result can be obtained as in the Example usually given in the "Nautical Almanac;" and it may be fairly questioned whether it be not as easy to compute the requisite corrections as to take them from these Tables.

Without the Tables the method is as follows:—

RULE 1. Find the mean time at the place of observation, and also the mean Greenwich date.

2. Take out the right ascension of the mean sun, as in Prob. 4 (p. 78), and also the right ascension of Polaris.

3. To the mean sun's right ascension add the mean time at the place, and from the sum (which is the sidereal time or right ascension of the meridian) *subtract* the right ascension of the star, previously adding mentally 24 hours, if necessary, and the remainder is the westerly meridian distance of the star.

4. Reduce the meridian distance to arc, call it h, and compute the first correction from this formula:—

Polar distance of the star in seconds $\times$ *cos h = correction in seconds*

or $\qquad p \quad \cos h = cor.$

5. If the westerly meridian distance be between 6 hours and 18 hours the star is below the pole, and the first correction will be additive, otherwise it is subtractive.

6. For the second correction, which is always additive, the formula is—

$$\tfrac{1}{2}\sin 1'' \times \tan altitude \times (p \quad \sin h)^2.$$

The logarithm of $\tfrac{1}{2}\sin 1'' = 4{\cdot}384545$

Example.—On March 6th, 1855, at 8 h. 25 m. 32 s. P.M., in longitude 130° W, the observed altitude of Polaris is 37° 30′ 18″; index error $-$ 1′ 13″, height of the eye 18 feet; required the latitude?

Greenwich Date.		Longitude	
	d. h m s		°
Date at place Mar. 6 . .	8 25 32	Long.	130 W
Long in time	+8 40 0		4
Greenwich date Mar. 6 .	17 5 32		6,0) 52,0
		Long. in time . .	8 h. 40 m. 0 s.
Greenwich date		1855, March 6th, 17 h 5 m. 32 s.	

With this, the right ascension of the mean sun must be taken out from the "Nautical Almanac," and also the right ascension and declination of the pole-star.

From "Nautical Almanac."

Right Ascension of Mean Sun, page II.		Right Ascension and Declination of Polaris	
	h m s		h m s
March 6th	22 54 36·84	R. A.	1 5 30·55
Accel. for { 17 h. .	2 47 56		
5 m. .	0 82		° ′ ″
32 s. .	0 09	Dec.	88 32 22·2 N
R. A. of mean sun .	22 57 25·31	N. P. D. . . .	1 27 37 8
			87 37·8
		Polar dist. in seconds	5257″ 8 = p

Right Ascension of Mean Sun 22 h 57 m. 25·31 s., $p = 5257''·8$.

Next, the meridian distance, or h, is to be computed, for which the formula is—

Mean time + mean ☉'s R. A. − star's R. A. = ✱'s westerly meridian distance.

To find the Hour Angle or Westerly Meridian Distance

	h m s
Mean time at place of observation	8 25 32
Right ascension of mean sun	+22 57 25·31
(24 hours rejected) sidereal time, or R. A. of meridian . . .	7 22 57·31
Right ascension of Polaris	1 5 30·55
	6 17 26 76
	60
	4) 377 26·76
Westerly meridian distance	$h = 94° 21'·69$

h is more than 6 hours, therefore the first cor. will be + (*Rule* 5).

We may now proceed to compute the corrections and apply them to the altitude; the formula for the first correction is $p \cdot \cos h$, and it may be noticed that when, as in this case, h exceeds 90°, its cosine

cannot be obtained directly from Table XIV., and whenever such cases occur—

(*a*.) Take the difference between h and 180 or 360 degrees, whichever will leave the remainder less than 90°, and take out the *cosine* of the remainder; or,

(*b*.) Take the difference between h and 90 or 270 degrees, whichever will leave the remainder less than 90°, and take out the *sine* of the remainder.

1st Cor $= p \cos h$			Altitude, &c	
p = 5258	log	3·720821	Obs alt.	37° 30′ 18″
h 94° 22′	cos	8·881607	Index error	−1 13
				37 29 5
6,0) 40,0 4		2·602428	Dip	−4 11
+6′ 40″·4 1st correction.				37 24 54
			Refraction	−1 14
				37 23 40
			1st correction	+6 40·4
			Latitude (nearly)	37 30 20·4

The second correction is to be computed from the formula.

$\frac{1}{2}$ *sin* 1″ . *tan alt* (p . *sin* h)^2, *log* $\frac{1}{2}$ *sin* 1″ *being* 4·384545.

2nd Correction always +			Latitude	
p	log	3·720821	Latitude (nearly)	37° 30′ 20″·4
h	sin	9·998738	2nd correction	+50·9
		3·719559		37 31 11·3
		2	Latitude 37° 31′ 11″·3	
(p . sin h)^2		7 439118		
Constant		4·384545		
Alt. 37° 24′	tan	9·883410		
50·9		1·707073		
2nd Correction + 50·9″				

The two corrections being in this case both additive, their sum 7′ 31″·3 is the total difference between the true altitude of the star and the latitude of the place. The value of the second correction may amount to 3 minutes.

The following modification of the formula for the second correction

will be found convenient; the polar distance and the first correction, which are denoted by p and c, are here reckoned in minutes:—

$$\tfrac{7}{800}\left\{(p+c)\,.\,(p-c)\,.\,\tan\text{alt.}\right\}$$

p in minutes	87′·6		
1 cor. in minutes	6·7		
Sum	94·3	log	1·9745
Diff.	80·9	log	1·9079
Tan alt.			9·8834
5832			3·7658
$\frac{1}{100}$ . . . 58·32			
$-\frac{1}{8}$ . . . 7·29			
$\frac{7}{800}$ = + 51·03 = second correction in seconds.			

Questions for Exercise.

1. January 4th, 1855, at 5 h. 35 m. 42 s. A.M. mean time at place, longitude 144° 20′ E, the observed altitude of the pole-star 43° 12′ 20″, index error − 1′ 53″, height of the eye 27 feet; required the latitude?

The star's declination 88° 32′ 28″·6 N, and right ascension 1 h 6 m. 16·28 s.; and the sidereal time at Greenwich mean noon January 3rd, 18 h. 50 m. 10·36 s. Ans. 44° 30′ 36″ N.

2. June 12th, 1855, at 9 h. 31 m. 40 s. P.M mean time at place, longitude 29° 17′ 30″ W, the observed altitude of Polaris 49° 18′ 50″, index error + 45″, height of the eye 20 feet; required the latitude?

The declination of the star 88° 31′ 58″·8 N, right ascension 1 h. 5 m. 56·62 s.; and the sidereal time at Greenwich mean noon June 12th, 5 h. 20 m. 59 s. Ans. 50° 32′ 55″·4 N.

3. September 19th, 1855, at 4 h. 15 m. 25 s. A.M. mean time at place, longitude 155° 30′ W, the observed altitude of Polaris 56° 28′ 45″, index error − 3′ 17″, height of the eye 16 feet; required the latitude?

The declination of the star 88° 32′ 17″·4 N, right ascension 1 h. 7 m. 5·74 s., and the sidereal time at Greenwich mean noon Sept 19th, 11 h. 51 m. 18·37 s. Ans. 55° 19′ 46″ N.

4. November 5th, 1855, at 8 h. 26 m. 15 s. P.M. mean time at place, longitude 16° 11′ W, the observed altitude of the pole-star 38° 29′ 30″, index error + 2′ 10″, height of the eye 19 feet; required the latitude?

The declination of the star 88° 32′ 35″·5 N, right ascension 1 h. 7 m. 7·27 s., and the sidereal time at Greenwich mean noon Nov. 5th, 14 h. 56 m. 36·42 s. Ans. 37° 7′ 35″·7 N.

5. August 9th, 1855, at 10 h. 44 m. 40 s. P.M. mean time at place, longitude 10° 43′ E, the observed double altitude of Polaris 100° 22′ 15″, index error − 1′ 28″; required the latitude?

The star's declination 88° 32′ 4″·6 N, right ascension 1h. 6m. 43·82s., and the sidereal time at Greenwich mean noon August 9th, 9 h. 9 m. 39·63 s. Ans. 49° 52′ 6″ N.

Method of finding the Latitude by Altitudes of the Sun when it is near the Meridian.

The altitudes are to be taken when the sun is within about 20 minutes of its meridian passage, either before or after.

The altitudes will generally change slowly, and therefore may be taken deliberately and carefully.

In observing the sun, the upper and lower limbs may be observed alternately, so that an equal number of altitudes of both limbs be taken. The time must be taken by a chronometer, whose error is known.

The error of the chronometer for *apparent time* at the place of observation, should be determined from observations taken on the same morning; and from this error the time which the chronometer will show at noon, may be directly inferred.

And then the method is as follows:—

1. Take the mean of the times, and thence deduce the corresponding Greenwich date. Take also the mean of the observed altitudes.

2. Find the sun's declination from the "Nautical Almanac" for the Greenwich date.

3. Find the time the *chronometer* shows when the sun is on the meridian.

4. Take the differences between this time and each of the times shown by the chronometer when the observations were taken, and these differences are the *hour angles* or meridian distances of the sun.

5. With these hour angles, enter Table XI., and take out the numbers which correspond to them; and take the mean of those numbers; call the quotient N.

6. Compute the "Reduction" from this formula:—

$$Red^n. = cos.\ lat. \times cos.\ declin. \times cosec.\ (mer^n.\ zen.\ dist) \times N.$$

7. Correct the mean of the altitudes, find the zenith distance, and subtract the "Reduction" from it, when the sun is observed at his upper transit, but add it when he is observed below the pole.

8. The sum of this corrected zenith distance and the declination found as directed in (2) is the latitude, when the declination and zenith

The difference between this time and each of the times shown by the chronometer when the altitudes were observed, must now be taken, and with the remaining hour angles,

the corresponding values of $\dfrac{2\sin^2\dfrac{h}{2}}{\sin 1''}$ are taken from Table XI.

Chronometer At noon 8 h 56 m. 49 s	Hour Angles	Nos Table XI	☉'s Meridian Zenith Distance nearly
h. m. s	m s		
9 1 42	4 53	47	{ Dec + lat = Z D.
4 25	7 36	113	{ lat. S and the dec N
4 50	8 1	126	Dec. noon 22° 6′ N
5 11	8 22	137	Lat. (acct) 12 0 S
5 33	8 44	150	
6 0	9 11	166	Mer Z D. 34 6
6 23	9 34	180	
6 49	10 0	196	
7 18	10 29	216	
		9) 1331	
	Mean . .	147·9=N.	

Next, to compute the reduction, correct the altitude, and find the latitude:—

Reduction . . . R = Cos L Cos D Cosec Z D × N.				Altitude and Latitude.	
	° ′				° ′ ″
Lat. . .	12 0	Cos	9·990404	Obs alt ☉	111 17 52·2
Dec. . .	22 6	Cos	9 966859	Index error . . .	−55
M. Z. D. .	34 6	Cosec	·251317		2) 111 16 57·2
N. . .	147 9	Log	2·169968		55 38 28·6
	239·1		2·378548	Refraction . .	−39
Red. . .	3′ 59″·1				55 37 49·6
				Semidiameter. . .	+ 15 46·2
					55 53 35·8
				Parallax	+5·0
					55 53 40·8
				Reduction . . .	+3 59·1
				Meridian altitude .	55 57 39·9
				Meridian zenith distance	34 2 20·1 S
				Declination . . .	22 5 39·9 N
				Latitude . . .	11 56 40·2 S

The following example will illustrate the method when stars are observed:—

Example 2.—On September 29th, 1854, at Greenwich, the following observations were taken of Jupiter when he was near the meridian; required the latitude?

The index error of the sextant being $-9''\cdot5$, and the error of the chronometer for Greenwich mean time 2 m. 23·0 s. fast—

	Chronometer		Sextant.
	h m s		° ′ ″
	6 59 2		31 37 45
	7 0 17		38 15
	7 1 40·5		36 30
	7 2 29		34 10
	7 3 5		34 25
	7 4 20		33 50
	7 5 4·5		32 15
	7) 15 58·0		7) 37 10
	7 2 16·85		31 35 18·6
Error	− 2 23	Index error	−9·5
Sept. 29th . . .	6 59 53·85		2) 31 35 9 1
			15 47 34·5

Greenwich date, corresponding to mean of observations, 1854, Sept. 29th, 6 h. 59 m. 53·85 s. (mean time)

To find the time by chronometer when Jupiter was on the meridian:—

$*$*'s right ascension − mean sun's right ascension = mean time.*

The time of Jupiter's transit might be found by the time of his meridian passage, given in the "Nautical Almanac" to the nearest tenth of a minute. But the method below applies to all stars:—

Mean Time of Transit (nearly)

	h. m. s.	
$*$'s right ascension	19 18 37 01	Houlr diff. ·54
Corr. = hourly diff. × 7 hours	3·78	7
$*$'s right ascension at transit	19 18 40 79	3·78
Mean ☉'s right ascension at noon	12 31 40 95	
Mean time at place (nearly)	6 46 59·84	

Or Sept. 29th, 6 h. 46 m. 59·84 s.

As the observation was made at Greenwich, this is a Greenwich date, and with it the right ascension of the mean sun may be found more correctly.

Right Ascension of Mean Sun			
			h m. s
At noon, Sept 29th			12 31 40·95
Acceleration for	6 h.		59·14
	46 m.		7·56
	59 s		·16
	·84 s.		·00
Right ascension of mean sun			12 32 47 81

If the right ascension of the planet were changing rapidly, its right ascension should also be again corrected; as this is not the case we may subtract the right ascension of the mean sun just deduced from the right ascension of the star found before; and the remainder will be the time of transit more correctly, and then the error of the chronometer being added, the result is what the chronometer showed at the time of the star's transit.

*'s R A – mean ☉'s R A, = Mean Time of Transit	
	h. m s
Jupiter's R. A.	19 18 40·79
Mean ☉'s R. A.	12 32 47·81
Greenwich time of transit	6 45 52·98
Error of chronometer (fast)	+ 2 23
Chronometer shows at transit	6 48 15·98

Next the declination of the star.

Declination		Correction	
	° ′ ″		
	22 42 56·0 S	Hourly difference	·9
Correction	– 6·3		7
	22 42 49 7	Correction	6·3

We must now find the hour angles or meridian distances; and if the object were a fixed star these would be correctly expressed by the intervals shown by a sidereal clock between the time of each observa-

tion and the star's transit over the meridian. These sidereal intervals may, however, be found from the mean solar intervals obtained from the chronometer : the time shown by the chronometer at the time of transit being found as above, and the reductions to sidereal time being made by the "Tables of Time Equivalents," in the "Nautical Almanac."

In the present example, the object being a planet, a further small correction on the sidereal intervals should be made for the change of the planet's right ascension, by subtracting a proportional part of the hourly change from the westerly meridian distances when the right ascension is increasing, and adding when it is decreasing.

Jupiter's right ascension is changing only about ·65 seconds per hour, and the largest hour angle is about 17 m.; and therefore, in this instance, this correction may be omitted.

Chronometer	Differences M Time	Hour Angles, Sid Time	Nos Table XI	Meridian zenith distance (nearly)
h m s Transit . 6 48 16				
First Obs. 6 59 2	m s 10 46	m s 10 48	229	Dec. . . 22° 43′ S
7 0 17	12 1	12 3	285	Lat. . . 51 29 N
1 40·5	13 24·5	13 27	355	
2 29	14 13	14 15	399	M. Z D.. 74 12
3 5	14 49	14 51	433	
4 20	16 4	16 6	509	
5 4·5	16 48·5	16 51	557	
			7)2767	
		Mean .	. 395 3	

To compute the reduction, correct the altitude, and find the latitude.

R = cos L cos D cosec M Z D × N			Altitude, &c.	
Lat. . .	51° 29′ cos .	9·794308	Altitude	15° 47′ 34″·5
Dec. . .	22 43 cos .	9·964931	Refraction	− 3 19
				15 44 15·5
M. Z D. .	74 12 cosec .	·016727	Parallax	+ 1
N. .	395·3 log .	2·596927		15 44 16 5
R. .	236·0. . .	2·372893	Reduction	+ 3 56·0
Reduction 3′ 56″·0			Mer. alt.	15 48 12 5
			Z D.	74 11 47 5 S
			Dec.	22 42 49·7 N
			Latitude	51 28 57·8 N

Examples for Practice.

1. If on July 12th, 1855, in lat. by account 12° S, long. 150° E, the following observations were taken near noon on shore, with an artificial horizon, the index error of the sextant being − 55″, and the error of chronometer for apparent time at the place of observation 2 h. 23 m. 18 s. fast; required the latitude?

Chronometer.		Sextant.
h. m. s.		° ′ ″
2 28 11	☉	111 22 50
30 54		20 40
31 20		19 0
31 42		18 40
32 4		18 0

Declination of the sun at apparent noon July 11th, 22° 10′ 22″·9 N, diff. in 1 hour − 20″·05; sun's semidiameter 15′ 46″·2.

Ans. Lat. 11° 56′ 34″ S.

2. If on September 3rd, 1855, in lat. by account 46° 40′ N, long. 25° W, the following observations were taken near noon, with an artificial horizon, the index error of the sextant being + 42″ 5, and the error of chronometer for apparent time at place of observation 3 h. 12 m. 40 s. slow; required the latitude?

Chronometer		Sextant
h m s		° ′ ″
8 49 38	$\overline{☉}$	102 25 20
49 50	$\underline{☉}$	101 21 0
50 0	$\overline{☉}$	102 24 40
50 16	$\underline{☉}$	101 21 0
50 35	$\overline{☉}$	102 24 20
50 46	$\underline{☉}$	101 20 45

Declination of the sun at apparent noon, Sept. 3rd, 7° 40′ 53″ N, diff. in 1 hour − 55″·17; sun's semidiameter 15′ 54″.

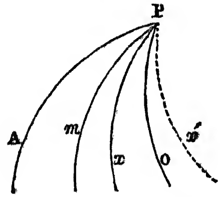
Ans. Lat. 46° 42′ 56″ N.

Hour Angles—Meridian Distances—Right Ascensions—Computations of Time—Error and Rate of Chronometer.

Preparatory Rules.

Let P A, P m, P x, and P O represent the meridians of the first point of Aries, the mean sun, any other object x, and of the observer respectively.

Fig. 16.



1. Then since A P m + m P O = A P O, therefore

Mean time + mean sun's right ascension = right ascension of the meridian.

Hence to find the right ascension of the meridian add the mean sun's right ascension to the mean time at place.

2. Again, since x P O = A P O − A P x, therefore

The westerly meridian distance of the object x = R.A. of meridian − R A. of object.

Hence the hour angle, or meridian distance of any celestial object is found by subtracting the object's right ascension from the right ascension of the meridian.

If the object be east of meridian, as at x', its right ascension A P x' exceeds the right ascension of the meridian, and

$$\text{O P } x' = \text{A P } x' - \text{A P O},$$

or the *easterly* meridian distance is found by subtracting the right ascension of the meridian from the right ascension of the star.

The westerly meridian distance in this case is

$$24\text{ h.} - \text{O P } x' = 24\text{ h.} + \text{A P O} - \text{A P } x',$$

and therefore may be found by subtracting the object's right ascension from the right ascension of the meridian increased by 24 hours.

Hence the following general rule for finding the *westerly* meridian distance or hour angle :—

Subtract the right ascension of the object from the right ascension of the meridian, increasing the latter, if necessary, by 24 hours, and the remainder is the westerly meridian distance.

3. Again, since $x\,\mathrm{PO} + \mathrm{AP}\,x = \mathrm{APO}$, and

$$m\,\mathrm{PO} + \mathrm{AP}\,m = \mathrm{APO}.$$

Therefore to find the *sidereal time* or right ascension of the meridian add the right ascension of any object to its westerly meridian distance, and the sum is the right ascension of the meridian or sidereal time; and the mean time at any place added to the right ascension of the mean sun gives the sidereal time at that place.

4. And because $\mathrm{APO} - \mathrm{AP}\,m = m\,\mathrm{PO}$.

Therefore, subtracting the mean sun's right ascension from the right ascension of the meridian (increased if necessary by 24 hours), gives the mean sun's westerly meridian distance, or the *mean time*.

The above rules serve for determining the meridian distance or hour angle of any object, the time being given, or the time when the meridian distance is known.

The westerly meridian distance of the sun is the measure of apparent time.

The westerly meridian distance of the mean sun is the measure of mean time.

The westerly meridian distance of the first point of Aries is the measure of sidereal time.

To Compute the mean or apparent Time at any Place from the observed Altitude of a Heavenly Body.

A set of altitudes, not less than five, should be taken, and the corresponding times marked by a chronometer, the mean of the altitudes and the mean of the times must be found, and the calculation then proceeds as follows:—

1. Find the *Greenwich date* expressed astronomically; this may be deduced from the chronometer by applying its error for Greenwich time. Or it may be found from the time at the place and the longitude.

2. If the object be the sun, take out from the "Nautical Almanac" its declination and the equation of time; and find the polar distance.

3. If the object be a star, take out the right ascension and declination of the star, and also the mean sun's right ascension. Find also the star's polar distance.

4. The meridian distance or hour angle must then be computed from this formula:—

$$Sin\ \tfrac{1}{2}\ hour\ \angle = \sqrt{\left(sec\ lat.\ cosec\ pol.\ dist.\ cos\ S.\ sin\ (S - alt.)\right)}$$

$$Where\ S = \tfrac{1}{2}\ (altitude + latitude + polar\ distance).$$

5. Convert the hour angle into time, and if the object be east of the meridian, subtract this hour angle from 24 hours, to find the westerly meridian distance.

6. If the object be the sun, the westerly meridian distance is the apparent time at the place of observation, and may be converted into mean time by applying the equation of time.

7. If the object be a star or planet, add its right ascension to the westerly meridian distance, and from the sum, which is the right ascension of the meridian, subtract the mean sun's right ascension, first adding 24 hours, if necessary, and the remainder is the mean time at the place.

Example 1.—On December 8th, 1855, at 9 h. 0 m. A.M. mean time at the place of observation nearly, in latitude 32° 40′ 18″ S, longitude 152° E, when the chronometer showed 10 h 46 m. 44·0 s., double the altitude of the sun's lower limb, taken with an artificial horizon, is 97° 49′ 55″, index error − 0′ 25″; required the error of the chronometer for mean time at the place of observation?

Greenwich Date.					Longitude in Time
		d	h	m	°
Date at place . .	December	7	21	0	152 E
Longitude E . .	. . . .	−	10	8	4
	December	7	10	52	6,0) 60,8
					10 h. 8 m.

Greenwich Date nearly, 1855, December 7th, 10 h 52 m.

Sun's Declination, Page II.				Correction, Page I	
	°	′	″		″
December 7th . . .	22	36	3·6 S	Difference for 1 hour . .	+ 16·63
Correction	+ 3		0·8	Hours past noon . . .	10·87
	22	39	4·4		11641
					13304
South polar distance	67	20	55 6		1663
					180·7681
				Correction	+ 3′ 0″·8

South Polar Distance, 67° 20′ 56″.

Equation of Time, page II		Correction, page I.	
	m s		s
December 7th	8 27·77	Difference for 1 hour . . .	−1·094
Correction	−11·92	Hours past noon	10·9
	8 15·85		9846
			10940
		Correction	−11·9246

Equation of Time, 8 m. 15·85 s. — for Apparent Time.

Correction of Observed Altitude		Sin ½ hour ∠ = $\sqrt{\frac{\text{Cos S Sin (S}-a)}{\text{Cos } l \text{ Sin } p}}$			
	° ′ ″		° ′ ″		
⊙̲	97 49 55	Altitude . . .	49 10 19		
Index error	−25	Latitude . . .	32 40 18	Sec . . .	·074802
	2) 97 49 30	Polar distance	67 20 56	Cosec . .	·034861
	48 54 45		2) 149 11 33		
Refraction	−49	S	74 35 46	Cos . . .	9·424264
	48 53 56	S − alt. . .	25 25 27	Sin . . .	9·632778
Semidiameter	+ 16 17				2) 19·166705
	49 10 13	½ hour ∠ . .	22 31 41	Sin . . .	9·583352
Parallax	+ 6		2		
True altitude	49 10 19		45 3 22	hour angle.	
			4		
			6,0) 18,0 13 28		

	h m s.
Hour angle east, because the time is A.M.	3 0 13·5
Subtract from	24
Hour ∠ west = apparent time	20 59 46·5
Equation of time	− 8 15·8
Mean time at place	20 51 30·7

	h m s
Therefore the chronometer, if correct, would show . .	8 51 30·7
But the chronometer showed	10 46 44·0
Therefore the chronometer was fast on mean time at the place	1 55 13·3

Example 2.—Altitude of α Leonis or Regulus.

If on February 10th, 1855, at 9 h. 5 m. P.M. mean time nearly, in latitude 28° 20′ N, and longitude 31° 2′ W, a chronometer shows 11 h. 16 m. 25 s. when the altitude of α Leonis is 41° 55′ 10″, the star east of the meridian, index error + 1′ 20″, and the height of the eye above the sea 25 feet; required the error of the chronometer for mean time at the place of observation?

Greenwich Date			Longitude in Time	
	h	m		° ′
Date at place, Feb. 10th .	9	5	Longitude	31 2 W
Long. W.	+2	4		4
February 10th . . .	11	9		6,0)12,4 8
				2h. 4 m. 8s.

Greenwich Date, 1855, Feb. 10th, 11 h 9 m.

R. A α Leonis				Declination α Leonis	
	h	m	s.		° ′ ″
February 10th . .	10	0	39·74		12 40 24·5 N
				North polar distance	77 19 35 5

Right Ascension of the Mean Sun.

		h.	m.	s.
Sidereal time at mean noon		21	19	59·54
Acceleration for	11 h.		1	48·42
	9 m.			1·48
	0 s			..
		21	21	49·44

Correction of Observed Altitude.

	°	′	″
Alt. obs. . .	41	55	10
Index error .	+	1	20
	41	56	30
Dip	−	4	55
	41	51	35
Refraction .	−	1	3
True alt . .	41	50	32

$$\text{Sin } \tfrac{1}{2} \text{ hour } \angle = \sqrt{\frac{\text{Cos } S \text{ Sin } (S-a)}{\text{Cos } l \text{ Sin } p}}$$

	°	′	″		
Altitude . . .	41	50	32		
Latitude . . .	28	20	0	Sec	·055418
Polar distance .	77	19	36	Cosec	010712
2)	147	30	8		
	73	45	4	Cos	9·446864
	31	54	32	Sin	9 723102
					2) 19·236096
	24	31	9·5	Sin	9·618048
			2		
	49	2	19·0		
			4		
6,0)	19,6	9	16·0		
	3	16	9·3		

		h.	m.	s.
Hour angle east of the meridian		3	16	9·3
Subtract from		24		
Hour angle west of the meridian		20	43	50·7
Right ascension of *a* Leonis	*add*	10	0	39·7
Right ascension of the meridian		30	44	30·4
Right ascension of the mean sun	*sub*	21	21	49·4
Mean time at the place of observation		9	22	41·0

	h.	m.	s.
Therefore the correct mean time at the place.	9	22	41
Time shown by the chronometer	11	16	25
Chronometer fast on mean time at the place of observation . . .	1	53	44

Examples for Exercise.

1. Given the true altitude of the sun in the morning 38° 29′ 54″, the sun's polar distance 73° 40′ 26″, the latitude of the place of observation 22° 40′ 0″ S, and the equation of time 14 m. 12·7 s. additive to apparent time, required the error of the chronometer both for mean and apparent time? The time by the chronometer being 10 h. 27 m. 57 s. Ans. Fast for mean time, 1 h. 51 m. 45·8 s.
Fast for apparent time, 2 h. 5 m. 58·5 s.

2. Given the true altitude of the sun in the morning 30° 19′ 40″, the sun's polar distance 73° 58′ 0″, the equation of time 14 m. 17·9 s. additive to apparent time, and the latitude of the place of observation 22° 11′ S, required the error of the chronometer on mean and apparent time? The time by the chronometer being 9 h. 53 m. 0 s.

Ans. Fast for mean time, 1 h. 51 m. 40·5 s.
Fast for apparent time, 2 h. 5 m. 58·4 s.

3. Given the true altitude of the sun in the afternoon 19° 57′ 8″, the sun's polar distance 71° 59′ 38″, the equation of time 6 m. 1·7 s. additive to apparent time, and the latitude of the place of observation 74° 36′, required the error of the chronometer for mean and apparent time at the place of observation? The chronometer showing at the observation 7 h. 15 m. 45 s.

Ans. Fast for mean time, 1 h. 49 m. 8 s.
Fast for apparent time, 1 h. 55 m. 9·7 s.

4. Given the true altitude of the sun's centre in the afternoon 19° 14′ 25″, the polar distance 71° 59′ 45″, the equation of time 6 m. 1·6 s. additive to apparent time, and the latitude of the place of observation 74° 36′, required the error of the chronometer on mean and apparent time at the place? The time by the chronometer 7 h. 26 m. 26·4 s.

Ans. Fast on mean time, 1 h. 49 m. 6·8 s.
Fast on apparent time, 1 h. 55 m. 8·3 s.

To find the Error in the Hour Angle from a given Error in the Altitude.

The formula for computing the error of the hour angle arising from an error in the observed altitude is—

$$Error\ of\ hour\ angle = \frac{error\ of\ alt.}{15 \times \cos latitude\ .\ \sin azimuth};$$

therefore if the error of altitude be 15″ = 1 second of time—

$$Error\ of\ hour\ angle = \frac{1}{\cos lat.\ \sin az.} = \sec lat.\ \operatorname{cosec} azimuth;$$

and this will give the error of the hour angle in seconds of time. Thus, if the latitude be 50° N, and the azimuth of the object N 36° E.

	°	
Sec	50	10·191933
Cosec	36	10·230781
2·647 s.		0·422714

Therefore an error of 15″ in the altitude would produce an error of 2·6 s. in the time computed from it under the given circumstances.

When the azimuth = 90°, the cosecant of the azimuth = 1, and then supposing the altitude 15″ in error.

Error of the hour angle in seconds of time = 1 second × sec latitude.

The effect of a given error of altitude is then least, and it is therefore best to observe celestial objects on the prime vertical, when the intention is to obtain the time at the place of observation. In the case above given, an error of 15″ in the altitude would cause an error of 1·56 s. in the time deduced from it, or about 5-10ths of a second for every second of error in the observation.

Another form of the expression for the error of the hour angle, in seconds of time, is,

$$Error\ of\ hour\ angle = \frac{cos\ alt.}{sin\ p\ .\ cos\ l\ .\ sin\ h} \times \frac{Error\ of\ alt.}{15}$$

Where p = polar dist., l = latitude, and h = hour angle. In the first example worked at page 130.

South polar distance	67° 21′ sin	9·965143
Hour angle	45 3 sin	9·849864
Latitude	32 40 cos	9·925222
Log. Denominator		29·740229
Altitude 49° 10′ cos		9 815485
Subtract		9·740229
·119		·075256

Therefore 15″ of error in the altitude would have produced an error of little more than 1-10th of a second in the computation of the time.

To find the Rate of a Chronometer.

The rate of a chronometer, or its daily gain or loss, is determined by comparing its errors for mean time, as found by observation at a given place on different days. Thus, if by observation a chronometer is found 20 s. fast, and at the end of ten days 30 s. slow for mean time at the same place, it has evidently lost 50 s. in ten days; whence its daily rate is 5 s. *losing*. If on a given day a chronometer be 12 s. fast, and at the end of thirteen days 57 s. fast for mean time at any place, it must have gained 45 s. in thirteen days, or its rate is about 3·5 s. per day *gaining*. Hence the method of finding the rate of a chronometer is evident.

When the longitude of a place at which observations are taken for the error of a chronometer is accurately known, it will be found convenient in all cases to find its error for Greenwich time, which may be

done by finding the time at Greenwich corresponding to the time at the given place as computed from observation, and comparing that time with the time shown by the chronometer. When the errors are all referred to Greenwich time, the rate can easily be found from observations taken at different places; and it is often desirable to do this: for a seaman cannot always stop long enough at one place to obtain a rate that is entitled to confidence; as the unavoidable errors of any observation render it advisable that a rate should not be deduced from observations separated from each other by a very short interval of time. If more than two sets of observations can be obtained, it will be seen from the results whether the rate of the chronometer is uniform or not; and for this purpose the mariner should lose no opportunity to multiply observations, that any change in the rate may be detected and allowed for.

Questions.

Example 1.—If on July the 29th the chronometer be slow 20 m. 57 s. and on August 18th, slow 20 m. 38 s. at the same place, what is the rate of the chronometer?

The chronometer has gained 19 s. between July 29th and August 18th, that is in 20 days, and therefore its daily rate $= \frac{19}{20} = \cdot 95$ seconds per day, gaining.

2. A chronometer is fast for mean time 1 h. 28 m. 29·5 s. at a place in longitude 33° 45′ W, what is its error on Greenwich mean time?

	h	m	s
Suppose the chronometer to show	12	0	0
Subtracting the error	1	28	29·5
Therefore the chronometer ought to show	10	31	30·5
Adding the longitude in time W	2	15	0
The corresponding Greenwich mean time is	12	46	30·5
But the chronometer showed	12	0	0
Therefore the chronometer is slow on Greenwich mean time	0	46	30·5

3. A chronometer is slow 3 h. 36 m. 18 s. for mean time at a place in longitude 151° 25′ 31″ E; required its error on Greenwich mean time?

	h	m	s
Suppose the chronometer to show	12	0	0
Adding the error	3	36	18
Corresponding time at the place	15	36	18
Subtracting the longitude in time	10	5	42·07
The corresponding Greenwich mean time is	5	30	35·93
But the chronometer shows 12 h. or	0	0	0
Therefore the chronometer is slow on Greenwich mean time	5	30	35 93

4. A chronometer is fast 1 h. 25 m. 30·7 s. at a place A, in longitude 37° 18′ 50″ E; what is its error for a place B, in longitude 10° 15′ 15″ W?

	h	m	s.
Suppose the chronometer to show	12	0	0
Subtracting what it is fast for A	1	25	30·7
The time at A	10	34	29 3
Subtracting the longitude of A, east	2	29	15·3
The time at Greenwich	8	5	14 0
Subtracting the longitude of B, west	0	41	1
Time at B	7	24	13 0
But chronometer showed	12	0	0
Therefore chronometer fast for B	4	35	47·0

5. A chronometer losing 3·8 s. per day, is fast for mean time at a place A, 1 h. 29 m. 13·7 s. on May 5th, and fast for mean time at a place B on May 12th, 1 h. 25 m. 17·5 s.; required the difference of longitude between A and B?

	h	m	s
Chronometer fast for A on May 5th	1	29	13·7
Lost between May 5th and May 12th			26 6
Therefore the chronometer is fast for A on May 12th	1	28	47·1
But the chronometer is fast for B on May 12th	1	25	17·5
Difference of longitude in time	0	3	29·6

The time at B being the most advanced, B is the eastward place, and the difference of longitude is 52′ 24″.

6. A chronometer on July 1st was 18 m. 45·5 s. fast for mean time at a place A, and on sailing to B it is found to be 19 m. 37 s. fast on July 20th, and again on August 1st, at A, it is 18 m. 10 s. fast for that place; required the difference of longitude between A and B?

	m.	s.
Chronometer fast at A on July 1st	18	45·5
,, ,, Aug. 1st	18	10
Therefore chronometer lost in 31 days		35·5

Hence daily rate of the chronometer $\frac{35 \cdot 5}{31} = 1 \cdot 14$ s. losing.

Between July 1st and July 20th, 19 days, the chronometer lost $19 \times 1 \cdot 14 = 21 \cdot 66$ s.

	m	s.
Therefore, since the chronometer was fast for A on July 1st	18	45·5
And lost between July 1st and July 20th		21 7
The chronometer was fast for A on July 20th	18	23 8
But the chronometer was fast for B July 20th	19	37
Difference of longitude between A and B	1	13·2

and B is west of A, because the time at B will be less than the time at A (see page 75). The difference of longitude is 0° 18′ 18″.

7. A chronometer is slow on mean time at Liverpool on June 21st, 28 m. 27·5 s., and on July 10th, 4 h. 17 m 58·6 s. fast for mean time at New York; what has been the rate of the chronometer?

Longitude of Liverpool, 2° 58′ 55″ W, longitude of New York, 73° 59′ 0″ W.

	h	m	s
1. Suppose the chronometer to show	12	0	0
The chronometer was slow at Liverpool on June 21st		28	27·5
Therefore the mean time at Liverpool is	12	28	27·5
Adding the longitude of Liverpool	0	11	55 7
The Greenwich mean time is	12	40	23 2
But the chronometer showed	12	0	0
Therefore the chron. slow on Greenwich mean time, June 21st		40	23·2

	h	m	s
2. Suppose chronometer to show	12	0	0
Chronometer fast at New York, July 10th	4	17	58·6
Mean time at New York	7	42	1·4
Longitude of New York	4	55	56
Greenwich mean time	12	37	57·4
But the chronometer showed	12	0	0
∴ The chronometer slow on Greenwich mean time, July 10th		37	57 4

	m.	s.
The chronometer slow on Greenwich mean time, June 21st	40	23·2
" " " July 10th	37	57·4
	2	25·8
The chronometer has gained in 19 days		145·8 s.

∴ The daily rate of the chronometer has been 7·67 s. gaining.

TO FIND THE LONGITUDE BY A CHRONOMETER.

This problem consists in finding the correct mean time at Greenwich, when an altitude of any celestial object is taken, by means of the known error and rate of the chronometer; and secondly, in finding the correct mean time at the place whose longitude is required by the method given at page 128. And then, the difference between the Greenwich time and the time at the place of observation, is the longitude of the place in time; east when the Greenwich time is the least, and west if the Greenwich time is greater than the time at place.

In keeping a record of observations such as this for future working or for reference, it is quite necessary to retain all the following particulars:—

1. The date at the place, with the civil or astronomical time.

2. The latitude and the longitude, as nearly as they may be known.

3. The time shown by the chronometer, with their mean, and the error of the chronometer for Greenwich mean time at a given date, together with its daily rate. It will give very little trouble, and is desirable at the time of recording the observations, to find an astronomical Greenwich date.

4. The observed altitudes, with their mean—together with the index error of the sextant, or, better still, the list of observations made for ascertaining the index error, properly reduced.

5. The height of the barometer and thermometer.

Many valuable observations have been altogether untranslatable, for the want of some one or other of these elements.

The date at the place serves the purpose of checking the Greenwich date deduced from the chronometer, by verifying, at least, the day and hour at Greenwich.

Example 1.—An altitude of the sun. On March 12th, 1855, at 4 h. 10 m. P.M. mean time nearly, in latitude 50° 48′ N, longitude by account 65° E, when the chronometer showed 11 h. 50 m. 20 s., the observed altitude of the sun's lower limb was 14° 50′ 10″, index error − 2′ 20″, and height of the eye above the sea 18 feet; required the longitude?

On February 25th, at noon, the chronometer was slow on Greenwich mean time 1 m. 40·2 s., and its daily rate 3·5 s. gaining.

Greenwich Date nearly.				Longitude in Time
	d.	h.	m.	°
Date at place . .	March 12	4	10	65 E
Longitude E		4	20	4 h. 20 m.
	March 11	23	50	

Greenwich Date from the Chronometer				Error from Rate	
	h.	m.	s.		
Chronometer	11	50	20	Daily rate	3·5 s.
Slow on Feb. 25th .		+1	40·2	Feb. 25th to March 12th	15 days
	11	52	0·2		175
Gain since		−	52·5		35
March 11th	23	51	7·7	Gain . . .	52·5

Greenwich date, 1855, March 11th, 23 h. 51 m. 7·7 s.

Sun's Declination, page II				Correction, page I	
	°	′	″	Diff. in 1 hour	59″·0
March 12th	3	25	25·1 S	1 min.	·98
Correction		+	8 8	9 min.	8·82
	3	25	33·9		
North polar distance	93	25	33·9		

Equation of Time, page II			Correction, page I.	
	m.	s.		
March 12th	10	2·30	Diff in 1 hour	·671
Correction		+ ·10	6 min.	·0671
			3 min.	·033
Equation of time . . .	10	2·40	9 min.	·100

Correction of the Observed Altitude		$\text{Sin } \tfrac{1}{2} \text{ hour } \angle = \sqrt{\dfrac{\text{Cos S Sin (S}-a)}{\text{Cos } l \text{ Sin } p}}$			
	° ′ ″		° ′ ″		
Obs alt. ☉ .	14 50 10	Altitude . .	14 56 20		
Index error .	−2 20	Latitude . .	50 48 0	Sec	·199263
	14 47 50	Polar distance	93 25 34	Cosec	·000777
Dip . . .	−4 11		2) 159 9 54		
	14 43 39		79 34 57	Cos	9·257245
Refraction .	−3 34		64 38 37	Sin	9·956006
	14 40 5				2) 19 415291
Semidiam. .	+ 16 7	½ hour ∠ . .	30 35 29	Sin	9 706645
	14 56 12		2		
Parallax . .	+ 8		61 10 58		
True alt. .	14 56 20		4		
			6,0) 24,4 43 52		
			4 4 43·9		

	h m s
Hour angle west = apparent time	4 4 43·9
Equation of time	+ 10 2·4
Mean time	4 14 46·3

	d h m s
∴ Date at the place of observation March	12 4 14 46·3
Greenwich date . . . . . March	11 23 51 7·7
Difference = longitude in time	4 23 38·6
	4) 263 38·6
Longitude	65° 54·6 E

Example 2.—On March 27th, 1855, at 6h. 10m. P.M. mean time nearly, in latitude 22° 0′ 0″ N, and longitude by account 12° 47′ W, when a chronometer showed 7 h. 2 m. 20 s., the observed altitude of Regulus (east of meridian) was 40° 20′ 0″, index error of the sextant + 1′ 10″, and the height of the eye above the sea 20 feet; required the longitude?

On March 13th, the chronometer was fast on Greenwich mean time 10 m. 20·5 s., and its daily rate 2·5 s. gaining.

Greenwich Date nearly

	h	m
Date at place, March 27th .	6	10
Longitude W.		51
March 27th	7	1

Longitude in Time

12° 47′ W

In time . . . . . 0 h. 51 m. 8s.

Greenwich Date from the Chronometer

	h.	m	s
Chronometer	7	2	20
Error on March 27th .	−	10	20·5
	6	51	59·5
Gain since		−	35
March 27th	6	51	24·5

Error from Rate

Daily rate	2·5 s.
March 13th-27th . . .	14 days.
	100
	25
Gained	35·0

∴ Greenwich Date, 1855, March 27th, 6 h. 51 m. 24·5 s.

Right Ascension of Regulus

10 h. 0 m. 39·80 s.

Declination of Regulus N

	°	′	″
	12	40	24·7
North polar distance	77	19	35·3

Right Ascension of the Mean Sun

		h	m	s
Sidereal time at mean noon, March 27th		0	17	24·47
Acceleration for	6 h.			59·14
	51 m.			8·38
	24·5 s.			·07
Right ascension of the mean sun		0	18	32·06

Correction of the Observed Altitude

	° ′ ″
Obs. alt. .	40 20 0
Index error .	+ 1 10
	40 21 10
Dip . . .	−4 24
	40 16 46
Refraction .	−1 8
True alt .	40 15 38

$$\text{Sin } \tfrac{1}{2} \text{ hour } \angle = \sqrt{\frac{\text{Cos S} \quad \text{Sin (S}-a)}{\text{Cos } l \quad \text{Sin } p}}$$

	° ′ ″		
Altitude . . .	40 15 38		
Latitude . . .	22 0 0	Sec	·032834
Polar distance .	77 19 35	Cosec	·010713
2)	139 35 13		
	69 47 36	Cos	9·538332
	29 31 58	Sin	9·692778
		2)	19·274657
½ hour ∠ . . .	25 42 41	Sin	9 637328
	2		
	51 25 22		
	4		
6,0)	20,5 41 28		
	3 25 41·47		

		h. m s
Easterly hour angle		3 25 41·47
Subtract from		24
Westerly hour angle		20 34 18 53
R. A. of Regulus	*add*	10 0 39 80
R. A. of meridian		30 34 58 33
R A. of mean sun	*sub*	0 18 32·06
(24 hours rejected)		6 16 26·27
		d h m s
∴ Correct date at place		March 27 6 16 26·3
Greenwich date		March 27 6 51 24·5
Difference = longitude in time		34 58·2
Longitude . .		8° 45′ W

Examples for Exercise.

1. On May 24th, 1855, at 2 h. 20 m. P.M. mean time at place nearly, in latitude 40° 17′ 26″ N, longitude by account 88° 57′ W, when a chronometer showed 8 h. 22 m. 7 s., the observed altitude of the sun's lower limb was 53° 48′ 34″, index error − 1′ 15″, height of the eye 17 feet; required the longitude?

April 10th, 1855, at noon, the chronometer was fast on Greenwich mean time 3 m. 27·5 s., gaining daily 4·5 s.

Declination at noon, May 24th, N 20° 43′ 11·9″, difference for 1 hour 27·66″ +, and semidiameter 15′ 49·4″.

Equation of time at noon, May 24th, − on apparent time 3 m. 29·56 s., difference for 1 hour − ·221 s. Ans. 89° 1′ 39″ W.

2. On October 20th, 1855, at 10 h. 15 m. A.M. mean time at place nearly, in latitude 37° 20′ N, longitude by account 37° 20′ W; the observed altitude of the sun's lower limb was 37° 21′ 0″, index error − 4′ 15″, height of the eye 15 feet, time by chronometer 0 h. 58 m. 52·5 s; required the longitude?

October 1st, 1855, at noon, the chronometer was fast on Greenwich mean time 17 m. 27·5 s, losing daily 3·5 s.

Declination at mean noon, October 20th, S 10° 14′ 52″·7, difference for 1 hour + 53·82″, and semidiameter 16′ 6″·6.

Equation of time at mean noon, October 20th, − on apparent time 15 m 3·0 s., difference for 1 hour + ·417 s. Ans. 38° 7′ 18″ W.

3. On December 13th, 1855, at 10 h. 27 m. A.M. mean time at place nearly, in latitude 41° 13′ S, longitude by account 84° 10′ E, when a chronometer showed 5 h. 5 m. 27·5 s., the observed altitude of the sun's lower limb was 64° 10′ 39″, index error − 2′ 5″, height of the eye 19 feet; required the longitude?

November 10th, at noon, the chronometer was fast on Greenwich mean time 10 m. 27 s., gaining daily 8·5 s.

Declination at mean noon, December 12th, S 23° 4′ 46″·9, difference for 1 hour + 10″·96, and semidiameter 16′ 17″·2.

Equation of time at mean noon, December 12th, − on apparent time 6 m. 12·04 s., difference for 1 hour − 1·180 s. Ans. 84° 12′ E.

4. On February 20th, 1855, at 6 h. 21 m. P.M. mean time at place nearly, in latitude 50° 51′ 39″ S, longitude by account 34° 17′ E, time by chronometer 4 h. 22 m. 45·7 s., the observed altitude of α Eridani was 64° 41′ 24″, index error − 1′ 10″, height of the eye 19 feet; required the longitude?

January 3rd, 1855, at noon, the chronometer was fast on Greenwich mean time 16 m. 10 s., gaining daily 3 s.

Declination February 20th, S 57° 58′ 43″·8, difference for 10 days − 2·3″.

Right ascension February 20th, 1 h. 32 m. 17·52 s., difference for 10 days − ·24 s.

Sidereal time at mean noon, 21 h. 59 m. 25·09 s.

Ans. 34° 46′ 42″ E.

The formula for computing the change in the hour angle for a given small difference in the *latitude* employed in computing it is—

$$d \cot p \,.\, \operatorname{cosec} h - d \,.\, \tan l \,.\, \cot h$$

d = given increment or decrement of latitude, p = polar distance, h = hour angle, l = latitude.

Thus, in the example at page 138, let it be required to find how

much the hour angle would be changed if computed with a latitude 10′ or 600″ greater than the one there employed. Here $d = 600''$.

+	600″	log	2·778151	−	600″	log	2·778151
− p 93° 26′		cot	8·778114	+ l 50° 48′		tan	·088533
+ h 61° 11′		cosec	·057413	+ h 61° 11′		cot	9·740468
−41·08			1·613678	− 404·7			2 607152
				− 41·0			
				− 445 7 . . . 7′ 25″·7			

Hence, as the negative sign shows, the hour angle would have been about 7′ 26″ less, if the latitude 50° 58′ had been used instead of 50° 48′.

Let us see:

	° ′ ″		
Altitude . .	14 56 20		
Latitude .	50 58 0	Sec	·200816
Polar distance	93 25 34	Cosec	·000777
	2) 159 19 54		
	79 39 57	Cos	9 253795
	64 43 37	Sin	9 956305
			2) 19·411693
	30 31 45·7	Sin	9·705846
	2		
Hour angle .	61 3 31·4		
And by the former calculation	61 10 58		
Difference .	−7 26·6		

The error in longitude, therefore, for 10 miles error of latitude would *in this case* be about $7\frac{1}{2}$ miles

The longitude found with the last computed hour angle is 65° 47′ 12″ E; and, tabulating these results, we see that—

with lat	50° 48′ N, the longitude is	65° 54′ 39″ E	. . . A
,,	50 58 N ,,	65 47 12 E	. . . B

Difference of latitude 10′ N, Difference of longitude 7′ 27″ W = 7·45 W
Middle latitude 50° 53′ N.

Thus a doubt in the correctness of the latitude to the extent of 10 miles north will make the longitude also doubtful to the extent of 7·45 miles westward. So also 10 miles to the southward in the latitude would make a difference of 7·45 miles eastward in the longitude, as deduced from this particular observation.

Let it be required to find how the points determined by these latitudes and longitudes lie with respect to each other.

$$\textit{Tan course from A to B} = \frac{\textit{Diff long} \times \textit{cos mid lat}}{\textit{Diff lat.}}$$

Difference of longitude	7·45	. . .		0·872156
Middle latitude	50° 53′	.	cos	9·799962
				10 672118
Difference of latitude	10	. . . . . .		1·000000
Course	N 25° 10′ W,		tan	9·672118

Therefore, if these places were found upon the chart, and a line drawn through them, this line would cut the meridians at an angle of N 25° 10′ W, and is called *a line of position* of the ship, because the ship's actual place lies upon it, and it can be found even when the latitude is not exactly known.

The line of position is also perpendicular to the sun's bearing at the time of observation; therefore, subtracting 25° 10′ from 90°,

	°	′	
	90	0	
	25	10	
Sun's bearings nearly S	64	50	W

This may be verified by the actual computation of the azimuth of the sun from this formula:—

$$\textit{Sin azimuth} = \frac{\textit{sin hour angle} \times \textit{sin polar distance}}{\textit{cos altitude.}}$$

	° ′		
Hour angle	61 11	sin	9·942587
Polar distance	93 26	sin	9·999220
			19·941807
Altitude	14 56	cos	9 985079
Azimuth	S 64 51 W	sin	9·956728

Agreeing within a minute with the former result. The principles here practically illustrated are the foundation of the method of finding the latitude and longitude from two observations, which is given at page 158.

Azimuths, Amplitudes, Variation of the Compass, Determination of True Bearings, &c.

The principle of all methods of finding the variation of the compass is the same; the bearing of any celestial object is to be taken with the compass, the time being also noted; the true bearing is then computed, and if the true and observed bearings agree, there is no variation; and if they disagree, the difference is the variation.

If both the true and observed bearing be supposed to be marked on the compass card, then if the true bearing is to the right of the observed bearing, the north end of the needle is drawn to the right, or the variation is east; and if the true is to the left of the observed bearing, the variation is west.

The methods of computing the variation given in the following pages are distinguished by the data used in the computation, viz.—

1. The altitude, date, and observed bearing.
2. The hour angle and observed bearing.
3. The amplitude, at rising or setting.

I. *When the altitude is observed at the same instant that the bearing is taken.*

Rule 1. Find the Greenwich date, and with it, enter the "Nautical Almanac," and take out the declination of the object observed and find its polar distance.

2. Compute the true azimuth from the following formula—

$$\mathit{Sin}\ \tfrac{1}{2}\ \mathit{azimuth} = \sqrt{\frac{\cos S \ . \ \cos (S - \mathit{polar\ dist.})}{\cos \mathit{alt.} \ . \ \cos \mathit{lat.}}}$$

where S denotes half the sum of the altitude, latitude, and polar distance.

3. The azimuth is reckoned from the north in south latitude, and from the south in north latitude, and towards the east or west according as the altitude is increasing or decreasing at the time of observation.

4. If the true and observed azimuths are both east or both west the difference is the variation, otherwise their sum is the variation, easterly when the true azimuth is to the right, and westerly when the true azimuth is to the left of the observed.

Example 1.—October 17th, 1855, at 9 h. 28 m. A.M. mean time at the place nearly, in latitude 66° 12′ S, longitude 114° E, the observed altitude of ☉ 28° 2′ 00″, index error + 4′ 40″, height of the eye 25 feet, also the bearing of the sun by compass N 13° 47′ E; required the variation?

Greenwich Date

		d	h	m.
Date at place . .	Oct.	16	21	28
Longitude E.			−7	36
	1855, Oct.	16	13	52

Long in Time

114° E
456
7 h 36 m.

☉'s Declination, page II

	°	′	″
Oct 16th	8	47	17·7 S
Cor	+12	46·3	
	9	00	4·0
South polar distance .	80	59	56

Correction

	″
Diff in 1 hour	+55·25
Hours past noon	13 87
Product = cor . . .	+12 46 3

Correction of Observed Alt

	°	′	″
Obs alt ☉ .	28	2	0
Index error .	+	4	40
	28	6	40
Dip . . .		−4	55
	28	1	45
Refraction .		−1	47
	27	59	58
Semidiameter		+16	6
	28	16	4
Parallax .			+8
True alt. .	28	16	12

$$\text{Sin } \tfrac{1}{2} \text{ az} = \sqrt{\frac{\text{Cos S} \quad \text{Cos (S} - \text{pol dist)}}{\text{Cos alt} \quad \text{Cos lat}}}$$

	° ′		
(*a*) Altitude . . .	28 16	sec	·055146
(*l*) Latitude . . .	66 12	sec	·394108
(*p*) Polar distance .	81 0		
	2) 175 28		
S	87 44	cos	8·597152
S − *p* . . .	6 44	cos	9 996994
			2) 19·043400
½ azimuth . .	19 25	sin	9·521700
azimuth . . .	38 50		

		° ′	
True azimuth . .	N	38 50	E
Observed azimuth .	N	13 47	E
Variation . . .		25 3	E

The variation is east because the true azimuth is to the right of the observed azimuth.

II. *When the altitude is not observed, but the bearing and time of observation only.*

1. Find the Greenwich date.
2. Take out the declination and find the polar distance.
3. Find the meridian distance of the object.

(*a*). If the object be the sun, the apparent time at place is the meridian distance.

(*b*). For any other object, the

westerly meridian distance = mean time at place + mean ☉'s R. A. − object's R. A.;

and therefore the right ascension of the object and the right ascension of the mean sun must also be found.

4. Compute S and D from these formulæ—

$$\tan S = \sec. \tfrac{1}{2}(\text{polar dist.} + \text{colat.}) \cos \tfrac{1}{2}(\text{polar dist.} - \text{colat.}) \times \cot. \tfrac{1}{2} \text{hour} \angle$$
$$\tan D = \operatorname{cosec} \tfrac{1}{2}(\text{polar dist.} + \text{colat.}) \sin. \tfrac{1}{2}(\text{polar dist.} - \text{colat.}) \times \cot. \tfrac{1}{2} \text{hour} \angle$$

Observing particularly that when ½ (polar dist. + colat.) is greater than 90°, S is also greater than 90°, and when less, less.

5. 180 − (S + D) is the azimuth when the polar distance is greater than the colatitude, otherwise 180° − (S − D) is the azimuth; estimated from the north in south, and south in north latitude.

Example 1.—On Sept. 28th, 1855, at 4 h. 18 m. 20 s. P.M. apparent time at place, latitude 64° 10′ S, longitude 32° 15′ W, the sun's bearing by compass S 84° 1′ W; required the variation?

Greenwich Date (App Time)		Long in Time.
	h. m s.	° ′
1855, Sept. 28th	4 18 20	32 15 W
Long. W.	+2 9 00	129 0
1855, Sept. 28th	6 27 20	2 h. 9 m.

Declination (Page I)		Correction	
	° ′ ″		″
Sept. 28th	1 53 59·5 S	Diff in 1 hour	+ 58·46
Cor.	+6 17	Hours past noon	× 6·45
	2 00 16·5	Product = Cor.	+ 6′ 17″
S. P. D.	87 59 43·5		

Hour ∠ = Apparent Time at place	Colatitude
h m s 4 18 20 4) 258 20 Hour ∠ . . . 64° 35′ ½ Hour ∠ . . 32 17·5 = ½ h	° ′ 90 0 Latitude 64 10 Colatitude 25 50

Polar distance	88° 00′	
Colatitude	25 50	
Polar distance + colatitude	113 50	= A
Polar distance − colatitude	62 10	= B
½ A (*less than* 90°)	56 55	
½ B	31 5	

Tan S = sec ½ A cos ½ B cot ½ *h* | Tan D = cosec ½ A sin ½ B cot ½ *h*

½ A	56° 55′	sec	·262920	. . . . . .	cosec	·076819
½ B	31 5	cos	9·932685	. . . . . .	sin	9·712889
½ *h*	32 18	cot	·199164	. . . . . .	cot	199164
S	68 3	tan	10 394769	D 44° 16′	tan	9·988872

And because the polar distance is greater than the colatitude,

$$\text{Azimuth} = 180^\circ - (S + D)$$

S =	68° 3′	
D =	44 16	
	112 19	
180° − (S + D) =	67 41	= azimuth.

The azimuth is to be reckoned from the north because the latitude is south, and westward because the observation was taken in the afternoon.

True azimuth	N 67° 41′ W
Observed azimuth. . . .	S 84 1 W
Sum.	151 42
180°−sum	28 18

As the true azimuth is estimated from the north, and the observed azimuth from the south, and both towards the west, the angle between them, or the difference between them, is found by subtracting their sum from 180°.

The remainder is the variation, east because the true bearing is to the right of the observed bearing. Therefore the variation is 28° 18′ E.

Examples for Exercise.

1. If on May 17th, 1855, at 2 h. 0 m. P.M. mean time, in latitude 48° 30′ S, and longitude 90° E, the altitude of the sun's upper limb corrected for index error be 17° 16′ 0″, height of the eye 10 feet, and at the same time the sun's bearing by compass be due N; required the variation of the compass?

Sun's declination at mean noon on May 16th, 19° 2′ 12″·9 N, diff. in 1 hour + 34″·5, and semidiameter 15′ 50″·6.

Variation 30° 21′ W.

2. If on April 25th, 1855, at 3 h. 30 m. P.M. mean time, in latitude 63° 14′ N, and longitude 10° W, the observed altitude of the sun's lower limb be 29° 46′ 37″, index error −32″, height of the eye 17 feet, and at the same time the sun's bearing by compass S 78° 13′ W; required the variation?

Sun's declination at mean noon on April 25th, 13° 5′ 55″·6 N, diff. in 1 hour + 48″·74, and semidiameter 15′ 55″·5.

Variation 22° 11′ W.

3. On March 5th, 1855, at 10 h. 0 m. P.M. mean time, in latitude 21° 3′ N, and longitude 17° E, when the altitude of Spica is 22° 2′ 30″ increasing, index error − 1′ 30″, height of the eye 15 feet, the bearing of Spica is S 33° 45′ E; required the variation of the compass?

Declination of Spica 10° 24′ 18″·2 S. Variation 34° 57′ W.

In the following questions the altitude is not given.

4. On May 25th, 1855, in latitude 40° 50′ N, and longitude 67° 30′ W, at 9 h. 27 m. 30 s. A.M. mean time, the sun bore by compass S 80° 25′ E; required the variation of the compass?

The sun's declination on May 25th at mean noon 20° 54′ 15″·6 N, diff. in 1 hour + 26″·76, and semidiameter 15′ 49″·2.

The equation of time to be added to mean time 3m 24·27 s., diff. in 1 hour − ·241 s. Variation 11° 17′ E.

5. On September 24th, 1855, at 4 h. 18 m. 20 s. P.M. apparent time, in latitude 64° 10′ S, longitude 32° 15′ W, the sun bore by compass S 94° 28′ W, and the deviation of the compass at the time 10° 17′ W; required the variation of the compass?

Sun's declination at apparent noon on September 24th, 0° 20′ 22″·5 S, diff. in 1 hour + 58·52, and semidiameter 15′ 59″·4.

Variation 28° 48′ E.

To find the True Bearing of a point of Land, or of any other Object in the Horizon, by means of its observed angular distance from the Sun.

1. Let the time be noted when the distance of either limb of the sun from the object is measured, or, better still, let several distances and times be noted, and their mean taken.

If the further limb be observed, *subtract* the semidiameter from the observed distance, but if the nearer limb, *add*. Write down also whether the object is *east* or *west* of the sun.

2. The sun's altitude should also be taken (or it may be computed for the mean of the times).

3. Find the difference between the bearing of the sun and the bearing of the object from this formula.

$$\cos\ (\textit{difference of bearings}) = \frac{\cos\ (\textit{apparent distance})}{\cos\ (\textit{apparent alt. of}\ \odot)}.$$

4. Compute the sun's true bearing by either of the preceding methods.

5. If the sun be *east* of the meridian and the object more *east*, or the sun be *west* and the object more *west*, *add* the difference of the bearings to the sun's true bearing. In any other case take the difference between the sun's true bearing and the difference of bearings, and the result is the true bearing of the object.

In like manner may any other heavenly body be used to find the true bearing of a point in the horizon.

Example from Sir Edward Belcher's Treatise on Surveying.

On May 1st, 1834, at eight, observed the land about one point on the starboard bow; about 9 h. 30 m. (smooth water and very clear day) determined on proving the reckoning. Put the patent log. over, and steered S 41° 30′ W, so as to bring the objects well on the starboard bow, and prevent the action of leeway.

The following observations were then taken:—

Date at Place.	Sextant	☉ to Outer Island	Index error.
May 1st, 1834, 9 h. 35 m. 52 s.	☉ 52° 25′ 30″	111° 34′ 0″	0

Height of the eye 18 feet, latitude 33° 8′ N, longitude 16° 10′ W.

Greenwich Date		Longitude in Time.
	h. m. s.	
May 1st	9 35 52	16° 10′ W.
Long.	1 4 40 W.	1 h. 4 m. 40 s.
May 1st	10 40 32	

Declination.		Correction
	° ′ ″	
Dec. .	15 0 48 N.	− 1′ 0″
Cor. .	− 1 0	
Cor. Dec.	14 59 48	
N. P. D.	75 0 12	

Altitude Corrected		Sin ½ az = $\sqrt{\dfrac{\text{Cos S} \quad \text{Cos (S}-\text{p)}}{\text{Cos lat. Cos alt}}}$		
	° ′ ″		° ′ ″	
☉ . . .	25 25 30	Alt.	52 36 33 sec	·216633
Dip . .	− 4 11	Lat	33 8 0 sec	·077067
	52 21 19	N.P.D.	75 0 12	
Ref. . .	− 44		2)160 44 45	
	52 20 35		80 22 22 cos	9·223333
Semidiam..	+ 15 53		5 22 10 cos	9·998090
	* 52 36 28			2) 19·515123
Par.. . .	+ 5		34 54 18 sin	9·757561
	52 36 33		2	
		☉'s true bearing S	69 48 36 E	

The refraction added again to the altitude marked with the * gives 52° 37′ 12″ for the apparent altitude of the sun.

$$\frac{\text{Cos App Dist}}{\text{Cos App Alt}} = \text{Cos A}$$

	° ′ ″		
Apparent Dist ☊ — Island	111 34 0		
○'s Semidiameter	+ 15 53		
Dist from ☉'s centre	111 49 53	cos	9 570399
Apparent Alt ☉	52 37 12	cos	9 783259
	52 13 35	cos	9 787140
From 180°, because the distance greater than 90°	127 46 25		

To find the true bearing of the Island

	° ′ ″
True bearing of Sun	S 69 48 36 E
Island W of Sun	127 46 25 W
Bearing of Island	S 57 57 49 W

To find the Variation of the Compass from the observed amplitude of the Sun or a Star.

From the effect of refraction, celestial objects appear on the horizon when they are 33′ below it, and therefore the altitude of the sun's centre, or the altitude of a star, should be about 33′ + *the dip of the horizon*, when the amplitude is observed to find the variation; or the altitude of the sun's lower limb should be about 17′ + *the dip*.

To compute the true amplitude and variation of the Compass.

1. Find the Greenwich date.
2. Take out the declination of the object.
3. Compute the amplitude,

$$\textit{sin amplitude} = \textit{sin declination} \times \textit{sec latitude}.$$

4. The true amplitude is estimated from the *east* or *west*, according as the object is *rising* or *setting*. It is also to be reckoned toward the *north* or *south* according as the declination is *north* or *south*.

5. When the true amplitude and the observed amplitude are both north or both south, their difference is the variation; but when one is north and the other south, their sum is the variation.

6. The variation is *east* or *west* according as the true amplitude is to the *right* or *left* of the observed amplitude.

Example 1.—If on October 27th, 1855, at 4 h. 34 m. P.M. mean time at place nearly, in latitude 52° 36′ N, and longitude 5° 16′ W, the observed amplitude of the sun be W 42° 15′ S; required the variation of the compass?

Greenwich Date.				Longitude in Time	
		d	h m		
Date at place,	Oct.	27	4 34	Longitude . . .	5° 16′ W
Longitude in time W			21		0 h. 21 m 4 s.
	Oct.	27	4 55		

Declination, page II		Correction	
	° ′ ″		
October 27th	12 42 1 4 S	Difference in 1 hour . .	+50″ 72
Correction	+ 4 9 0	Hours past noon	4 91
	12 46 10 4 S		5072
			45648
			20288
		4′ 9″=	249 0352

Sin amplitude = sin declination × sec latitude

	° ′		
Declination	12 46 S	sin . . .	9·344355
Latitude	52 36 N.	sec . . .	·216542
Amplitude . . .	W 21 20 S	sin	9 560897
Observed amplitude .	W 42 15 S.		
Variation	20 55 E.		

Example 2—On January 24th, 1855, at 6 h 45 m. A.M., in latitude 21° 14′ N, longitude 31° W, the sun's rising amplitude by compass was E 35° 20′ S; required the variation?

Greenwich Date				Longitude in Time	
		d	h m		
Date at place . . .	Jan.	23	18 45	Longitude	31° W
Longitude in time. .	W		+2 4	In time	2 h. 4 m
	Jan.	23	20 49		

☉'s Declination, page II		Correction, page I	
	° ′ ″		″
January 23rd . . .	19 30 29·9 S	Difference in 1 hour . .	−35·54
	−12 19 2	Hours past noon	20·8
	19 18 10 7	Product = correction . .	12′ 19″ 2

Sin amplitude = sin declination × sec latitude

Declination	19° 18′	sin	9 519190
Latitude	21 14	sec	·030531
True amplitude	20° 46′	sin	9·549721

The amplitude is to be reckoned from the east towards the south, therefore,

	° ′
True amplitude . .	E 20 46 S.
Observed amplitude . .	E 35 20 S.
Difference = Variation . .	14 34 W.

Examples for Exercise.

1. On April 29th, 1855, in latitude 36° 22′ N, and longitude 12° W, at 5 h. 32 m. A.M. mean time, the sun's amplitude is observed to be E 3° 20′ N; required the variation?

Sun's declination at mean noon on April 28th, 14° 3′ 45″ N, diff. in 1 hour + 47″·04, and semidiameter 15′ 54″ 8.

Variation 14° 32′ W.

2. If on June 6th, 1855, at 7 h. 40 m. P.M. mean time, in latitude 48° 27′ N, and longitude 33° W, the sun's amplitude observed with a compass at his setting be W 14° 23′ N; required the variation?

Sun's declination at mean noon on June 6th, 22° 37′ 59″·9 N, diff. in 1 hour + 15″·35, and semidiameter 15′ 47″·5.

Variation 21° 9′ E.

3 On August 27th, 1855, at 5 h. 30 m. P.M. mean time, in latitude 21° 4′ S, and longitude 6° E, the sun's setting amplitude is observed by compass to be W 10° 0′ N; required the variation?

Sun's declination at mean noon on Aug. 27th, 10° 11′ 32″·8 N, diff. in 1 hour − 52″·69, and semidiameter 15′ 52″·4.

Variation 0° 51′ E.

4. On January 25th, 1855, in latitude 16° 21′ S, longitude 15° W, the amplitude of Spica when rising is E 16° 3′ N by compass; required the variation?

Declination of Spica at this time 10° 24′ 11″·5 S.

Variation 26° 53′·5 E.

5. On January 3rd, 1855, in latitude 38° 17′ S, longitude 25° E, the amplitude of Sirius when rising is observed to be E 16° 20′ S; required the variation?

Declination of Sirius 16° 31′ 11″ S. Variation 4° 54′ E.

At the poles, the diurnal paths of the stars are parallel to the horizon, and they consequently do not meet the horizon at all; and in high latitudes generally, the variable refractions, as well as the very acute angle at which the objects cross the horizon, render observations of amplitudes useless, when not impossible. A very small error in estimating the *apparent* altitude which the object should have, will make enormous differences in the observed bearings; but at the equator the circles of declination meet the horizon at right angles, and any probable error in the time of the observation will hardly affect the observed amplitude; for the bearing will be nearly constant for some time before the object sets or after it rises. The rising or setting sun may, under these circumstances, be advantageously used in determining the true bearings of headlands, &c., for the difference of bearing between the sun and any other object may then be measured with a sextant, and this difference applied to the true bearing of the sun will give the true bearing of the object, by a simple operation of addition or subtraction. It may be noted, that at the equator, the latitude being 0, the *secant* of the latitude = 1, and the formula for the computation of the amplitude becomes *sin amplitude* = *sin declination;* therefore the true *amplitude* is equal to the *declination.*

To reduce the Altitude of any Celestial Object observed at one Place, to what it would have been if observed at the same instant at another Place.

1. The bearing of the object must be taken at the same time that the altitude is taken, or it must be computed for that time. The course and distance run since the observation was taken must also be noted. And the correction of the altitude for this change of place may then be computed.

2. Let A = the angle between the object's bearing and the ship's course, the distance being estimated in minutes.

$$Correction = distance \times \cos A$$

3. The correction is to be added when A is less, and subtracted when A is greater than 90°.

Example 1.—An altitude of the sun was taken at 10 h. 30 m. A.M.; the sun at the time bearing N 37° E.

Required the correction for the place arrived at, after sailing N 58° W, 8 6 miles per hour, until 12 h. 39 m.?

1		2			
	h m 12 39 10 30	2·15 hours 8·60 rate	☉'s bearing . . .		N 37° E
			Ship's course . . .		N 58° W.
Interval	2 9	1290 1720	A . . .		95°
Interval	2 15 hours	18 490 run	A = 95° . . .	cos	8 940296
			18·49	log	1 266937
			−·161		0·207233
			60		
			−9·660″		

The correction is 9″·7, to be subtracted from the observed altitude after correcting it for dip, refraction, semidiameter, and parallax.

Example 2.—What is the correction to be applied to an observed altitude, to reduce it to the second place, supposing the bearing at the time of observation S 35° E, and the course and run to the second place, S 59° E, 20 6 miles.

Object's bearing	S 85° E		
Ship's course .	S 59 E		
	26° . . .	cos +	9·953660
	20·6 . . .	log .	1·313867
	18·51		1 267527
	60		
correction +	18′ 30″ 60		

This problem is used when an observation is taken on board ship, and after some time another observation is taken, and it is required to combine the results of the two observations to find the position of the ship at the time of taking the second observation.

To find the Latitude and Longitude by Sumner's Method.

(1.) *Results deduced from the mean of a single set of Observations.* (2.) *From the means of two sets.* (3.) *Sumner's method of finding the Latitude and Longitude.*

(1.) *One Observation.*

1 In the discussion of the method of finding the longitude by chronometer it was shown how an error in the latitude with which the time has been computed affects the longitude deduced from an observation. If several latitudes not differing very greatly from the truth be successively employed in calculating the longitude, each latitude will give a different longitude; and if with each latitude and the longitude deduced from it, a point is found upon the chart, it will be seen that these points lie very nearly in a straight line, whose course is the complement of the bearing of the object; in other words, the *line of position* is perpendicular to the direction of the object at the time of observation.

2. How does this happen? At the time of observation the object must be vertically over some point of the earth's surface, having there an altitude of 90°; and if a circle be supposed to be described about this point as a pole at the distance which the observer is from it, then to all persons situate upon the circumference of this circle the object would have the same zenith distance, and the same altitude. A small portion of this circle in the neighbourhood of the observer's true position may be considered as a straight line, and this line is evidently perpendicular to the direction of the observed object; and it is this line which has been called the *line of position.*

3. *Two* assumed latitudes with the two longitudes calculated from them will be sufficient to determine the *line of position,* provided the latitudes do not differ greatly from the truth Let the places be found upon the chart, and a line be drawn through them: somewhere in this line, or very near it, the observer's place is to be found. This much can be ascertained from one observation

4. After the observation the ship changes her place; but if from any point in the line of position the ship's run be laid off in proper direction according to the ship's course, and through the point thus found a line be drawn parallel to the line of position, the ship's actual place is still somewhere on *this line.* And as a ship, near a known coast, when her latitude has been found is sometimes run along the parallel of latitude until the land is seen, and thus her position verified, so in this case, by running upon the *line of position*, or any line parallel to it, the land may be recognised, and all the advantages of parallel sailing be made available, even without precise knowledge of either the latitude or the longitude.

5. If the observation be not immediately worked, allowance may be made upon the observed altitude for the run of the ship, and the line found from the corrected altitude when drawn upon the chart will pass through the present position of the ship.

(2.) *Two Observations.*

6 If a second observation of the same or any other celestial object be taken and treated in the same manner, another *line of position* may be drawn, which will also pass through the place of observation; and therefore the intersection of the two lines will give the ship's place; the latitude and longitude of which will then both be ascertained.

7. As the place is ascertained by the intersection of two lines, it will be best defined when they are at right angles to each other.

And, as the lines are at right angles to the bearings of the objects, their inclination to each other is always equal to the difference of the bearings of the objects observed, which should therefore be such as will give a well-defined point of meeting, say from 45° to 135°.

8. The method of *computing* the latitude and longitude is now to be explained. The first altitude must be reduced to what it would have been if it had been taken at the same place as the second observation (see the problem at page 156).

1. Assume two latitudes differing about a degree, and near the supposed latitude of the ship. Compute the longitude with each of them, first from one observation and then from the other; thus four longitudes will have been found.

2. The form of Tabulation will be best illustrated by an example. Suppose that with the assumed latitudes 51° N and 52° N four longitudes have been computed, they must then be arranged thus :—

D Lat = 60 = D.	With Lat 51° N.		With Lat 52° N	
1st Obs^{n}. Longitude	a	33° 34′ 40″ W	a'	33° 30′ 17″ W
2nd Obs^{n}. Longitude	b	33 29 51 W	b'	33 31 9 W

Differences of Long. A . . . . 4′ 49″ E . . . . . 0′ 52″ W B
B . . . 0 52 W

A + B . . . . . 5 41 = 341″

3. The longitudes are distinguished by the letters a, a', b and b', to facilitate references to them. The differences of longitude are marked E or W, according as the longitudes in the second line, b and b', are east and west of those above them.

The greater difference is to be denoted by A, and the less one by B. The difference between the assumed latitudes is distinguished by the letter D.

4. When A and B are both E or both W, their difference must be taken; but when one is E and the other W, as in this example, their sum is to be taken.

5. Either latitude may be corrected, but it will be found more convenient to correct that which stands over the larger difference A, because the rules for applying the correction can be expressed and remembered more easily.

6. The formula for computing the correction of the latitude is then $\frac{A \times D}{A - B}$, when A and B are both E or both W, and $\frac{A \times D}{A + B}$, when one of them is E and the other W.

In the present instance $\frac{A \times D}{A + B} = \frac{289'' \times 60'}{341''} = 50'{\cdot}85 = 50'\ 51.''$

7. The correction to be applied to the latitude 51° N is therefore 50′ 51″, and it is to be considered as a north difference of latitude if the other assumed latitude is to the north of the one we are correcting, and *vice versâ*. Therefore in the example before us the correction is 50′ 51″ N.

	°	′	″
Assumed latitude	51	0	0 N
Correction		50	51 N
Corrected latitude	51	50	51 N

8. The longitude is now to be found. For this purpose take the longitudes a and a' from the table; find the difference of longitude, and call the difference C.

9. And the correction of that longitude which stands over A in the table can be computed from the formula $\frac{A \times C}{A - B}$ or $\frac{A \times C}{A + B}$, according as A and B are of the same or different names.

	°	′	″
Longitude a (to be corrected)	33	34	40 W
Longitude a'	33	30	17 W
Difference of longitude	C =	4	23 E = 263″

$$\text{Correction} = \frac{A \times C}{A + B} = \frac{289 \times 263''}{341} = 223'' = 3'\ 43''$$

10. And this correction is to be considered as an east or west difference of longitude, according as the longitude *not* to be corrected (or that which stands over B) is east or west of the one to be corrected. In the present case the correction is $3' \; 43''$ E.

	°	′	″	
Longitude *a*	33	34	40	W
Correction		3	43	E
Corrected longitude	30	30	57	W

11. Or the longitudes *b* and *b′* may be employed thus :—

Longitude *b* . . .	$33° \; 29' \; 51''$ W	(to be corrected)
Longitude *b′* . . .	$33 \;\; 31 \;\; 9$ W	
	$C = 1 \;\; 18 = 78$ W	

$$\text{Correction of } b = \frac{A \times C}{A + B} = 66'' = 1' \; 6'' \text{ W}$$

Longitude *b*	=	$33° \; 29' \; 51''$ W
Correction	=	$1 \;\; 6$ W
Corrected longitude		$33 \;\; 30 \;\; 57$ W

Agreeing with the former calculation.

12. The corrections for the latitudes and longitudes which give the smaller difference B are $\frac{B \times D}{A + B}$ and $\frac{B \times C}{A + B}$ respectively.

13. A little sketch will be found useful in verifying the mode of applying the corrections to the latitude and longitude.

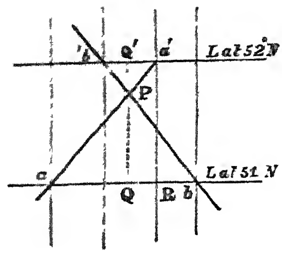
First distinguish the two longitudes deduced from one observation by the letters *a* and *a′*, and the others by *b* and *b′* as in the table in article 2.

Draw two horizontal lines to represent the parallels of the two assumed latitudes, and four vertical lines *lightly* to represent the meridians corresponding to the four calculated longitudes. Consider the lower of the two horizontal lines to represent the more southern parallel, and the vertical line most to the left as the most westward meridian.

Select the most westerly longitude (in this case it is $33° \; 34' \; 40''$, and stands under the latitude $51°$); make a dot at *a* to represent the place in this latitude and longitude, for this point is on the parallel of $51°$, and also on the most westerly meridian. Proceeding thus in order, the next most westerly longitude (*b′*) is

seen to be upon the other parallel, so also is the next (a'), and the most easterly (b) is on the parallel 51° N.

Fig. 17.



The four points being thus arranged and lettered, join $a\ a'$ and $b\ b'$, and the point P where these lines intersect represents the actual place of the ship.

14. The point P, then, is to the north of the latitude 51° N, and to the east of the longitude of a, 33° 34′ 40″ W. P Q is the correction of the latitude 51° N, and a Q the correction of the longitude a, 33° 34′ 30″ W. P Q′ represents the correction of the latitude 52° N, and a' Q′ the correction of the longitude a'.

15. $a\ b$ (fig. above) is the quantity denoted by A in the calculation, $a'\ b'$ is the difference which is there called B, Q Q′ the difference of the assumed latitudes = D, and a R = C the difference of longitude between the meridians of a and a'. Now by the similar triangles a P b, a' P b',

$$\frac{a'\,b'}{a\,b} = \frac{PQ'}{PQ}$$

$$\therefore \quad \frac{a'\,b'}{a\,b} + 1 = \frac{PQ'}{PQ} + 1$$

$$\text{or} \quad \frac{a'\,b' + a\,b}{a\,b} = \frac{PQ' + PQ}{PQ} = \frac{QQ'}{PQ}$$

$$\text{or} \quad \frac{A + B}{A} = \frac{D}{PQ}$$

Whence P Q, or the correction of the latitude of the parallel $a\,b = \dfrac{A \times D}{A + B}$.

Again, by the similar triangles a P Q and a R a'

$$\frac{a\,Q}{PQ} = \frac{a\,R}{a'\,R}; \therefore a\,Q = \frac{a\,R}{a'\,R} \times PQ.$$

But a R = C, R a' = D, and P Q has been shown to be $= \frac{A\ D}{A+B}$;

$$\therefore a\,Q = \frac{C}{D} \times \frac{A\,.\,D}{A+B} = \frac{A\,.\,C}{A+B},$$

which is the formula for computing the correction of the longitude a.

When the *lines of position* have to be produced to meet, the appropriate formulæ will be found by subtracting each side of the equation (1) from 1, which is equivalent to making B negative, and therefore the corrections of latitude and longitude become

$$\text{Cor. of lat.} = \frac{A\,.\,D}{A-B} \text{ and cor. of long.} = \frac{A\,.\,C}{A-B}.$$

Example 2.—Suppose that with the latitudes 15° 0′ S and 15° 30′ S, the longitudes have been computed from two observations and tabulated as under; required the latitude and longitude?

D. Lat = 30′ = D		With Lat. 15° 0′ S		With Lat 15° 30′ S	
1st Obs^n. 2nd Obs^n.	Longitudes Longitudes	a b	100° 7′ 56″ E 100 28 23 E	a' b'	100° 14′ 2″ E 100 19 51 E
			A . . 20 27 E B . . 5 49 E		B . . 5 49 E
	D = 30′	A − B . . 14 38 = 878″		A = 1227″	

1. Here the difference between A and B is taken, because the longitudes b and b' are both eastward of a and a' respectively.

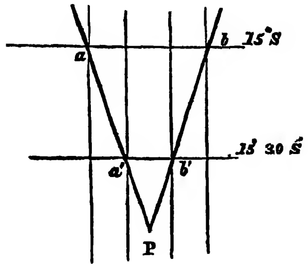
2. The latitude 15° 0′ S will be corrected because it stands over the greater difference A.

3. The correction when found will be applied southwards because the other latitude 15° 30′ S is to the south of the one we propose to correct.

$$\text{Cor. of lat} = \frac{A\,.\,D}{A-B} = \frac{1227 \times 30'}{878} = 4'.19\ \text{S} = 41'\ 54''\ \text{S}.$$

Assumed latitude	15° 0′ 0″ S
Correction	41 54 S
Corrected latitude	15 41 54 S

Fig 18.



Longitude a	100° 7′ 56″ E	
Longitude a'	100 14 2 E	
Diff. long = C	6 6	= 366″ E

$$\text{Cor. of long} = \frac{A\ .\ C}{A - B} = \frac{1227 \times 366''}{878} = 511'' = 8'\ 31''\ \text{E}$$

Longitude a	100° 7′ 56″ E
Correction	8 31 E
Corrected longitude	100 16 27 E

Example 3.—From actual observation at Greenwich.

D Lat. 20′ = D		With Lat. 51° 20′ N		With Lat 51° 40′ N	
1st Obs^n. 2nd Obs^n.	Longitudes. Longitudes	a b	h m s 0 0 8 W 0 0 20 E	a' b'	h. m s. 0 0 11 E 0 0 25 W
			B . . 28 E		A . . 36 W B . . 28 E
	D = 20′ A = 36″ C = 19″ W				A + B . . 64

$$\text{Cor. of lat.} = \frac{A\ .\ D}{A + B} = 11'\ 15''\ \text{S}$$

$$\text{Cor. of long.} = \frac{A\ .\ C}{A + B} = 10{\cdot}7\text{s. W.}$$

Assumed lat.	51° 40′ 0″ N
Correction . . .	11 15 S
Latitude	51 28 45 N

	h m s
Long a'	0 0 11 E
Correction . . .	10·7 W
Long in time . .	0 0 0·3 E

Example 4, *at length.*—If on September 30th, 1855, in latitude by account 15° 50′ S, longitude by account 100° 10′ E, the following observations were taken; determine the latitude and longitude?

M time nearly	Chronometer	Obs alt ☉		
h m	h m s	° ′ ″		
8 48 A M.	1 14 53	42 57 40	Index error	−4′ 15″
2 19 P M.	6 45 40	51 12 14	Height of the eye	15 ft.

On August 27th, at noon, the chronometer was slow on Greenwich mean time 48 m. 11 s., losing daily 5·5 s.

First Observation.

Greenwich Date nearly		Longitude in Time.
	h. m	
1855, September 29th .	20 48	100° 10′
Longitude	6 41 E	6 h. 40 m. 40 s.
September 29th	14 7	

Greenwich Date.		Error from Rate	
	h m. s.		s.
Chronometer . . .	1 14 53	Daily rate —	5·5
Slow, Aug. 27th . . .	48 11	Days ×	33·5
	2 3 4		
Lost since	3 4·3	Loss 3 m. 4·3 s.	
September 29th	14 6 8·3		

Declination, page II		Correction.	Equation of Time, page II.	Correction.
	° ′ ″	″	m. s.	s.
Declination	2 17 31·8 S	+ 58·41	9 33·76	+ ·819
	+ 13 43·6	× 14·1	+ 11·55	× 14·1
	2 31 15·4	13′ 43·6″	9 45·31	11·55
Polar dist.	87 28 44·6		Sub. from apparent time.	

Correction of Observed Altitude

	°	′	″
Obs alt . .	42	57	40
Index cor . .		−4	15
	42	53	25
Dip		−3	49
	42	49	36
Refraction . .		−1	1
	42	48	35
Semidiam. . .		+16	1
	43	4	36
Parallax . .			+6
True alt. . .	43	4	42

Computation of Time and Longitude

1st.—With Latitude 15°.

	°	′	″		
a	43	4	42		
l	15	0	0	sec	·015056
p	87	28	45	cosec	000421
	145	33	27		
S	72	46	43	cos	9·471387
S − a	29	42	1	sin	9·695011
					19·181875
	22	56	50	sin	9·590937

East hour angle in degrees, &c. .	45°	53′	40″	
	h	m	s	
East hour angle in time	3	3	34·7	
Equation of time — September 29th	20	56	25·3	
. . . . . .		−9	45·3	
Greenwich date — September 29th	20	46	40·0	. . .
September 29th	14	6	8·3	
Longitude in time . . . . . a	6	40	31·7	= 100° 7′ 56″ E

2nd.—With Latitude 15° 30′

	°	′	″		
l	15	30	0	sec	·016089
p	87	28	45	cosec	·000421
S	73	1	43	cos	9·465225
S − a	29	57	1	sin	9·698317
					19·180052
	22	53	47	sin	9·590026

East hour angle in degrees, &c. . .	45°	47′	34	
	h	m	s	
East hour angle in time	3	3	10·3	
Equation of time — September 29th	20	56	49·7	
. . . .		− 9	45·3	
Greenwich date — September 29th	20	47	4·4	
September 29th	14	6	8·3	
Longitude in time . . . . . . a'	6	40	56·1	= 100° 14′ 1″ E

Second Observation.

Greenwich Date nearly			Longitude in Time
		h m	
1855 . .	September 30th	2 19	6 h. 40 m 40 s E
Longitude . . .	. .	6 41	
	September 29th	19 38	

Greenwich Date			Error from Rate
		h m s	s
Chronometer . .		6 45 40	Daily rate 5 5
Slow on	August 27th	48 11	Days . × 33·8
		7 33 51·	
Lost since		3 5 9	Loss 3 m 5·9 s
	September 29th	19 36 56·9	

Declination, page II		Correction	Equation of Time, page II	Correction
	° ′ ″	″	m s	s
Declination	2 17 31·8 S	+ 58 41	9 33·8	+ ·819
	+ 19 4·8	19 6	+ 16·0	19 6
	2 36 36·6	19′ 4·8″	9 49·8	16″·05
Polar dist.	87 23 23·4		Sub from app. time.	

Correction of Observed Altitude	
	° ′ ″
Obs alt . .	·51 12 14
Index cor .	− 4 15
	51 7 59
Dip	− 3 49
	·51 4 10
Refraction .	− 46
	51 3 24
Semidiameter	+ 16 1
	51 19 25
Parallax . .	+ 5
True altitude	51 19 30

Computation of Time and Longitude

1st—With Latitude 15°

	° ′ ″		
a .	51 19 30		
l	15 0 0	sec	·015056
p	87 23 23	cosec	·000451
	153 42 53		
S	76 51 26	cos	9·356750
S − *a*	25 31 56	sin	9·634496
			19·006753
	18 35 2	sin	9·503376
	37 10 4		

			h m s
West hour angle			2 28 40·3
Equation of time			− 9 49·8
Date at place	Sept. 30th		2 18 50·5
Greenwich date . . .	Sept. 29th		19 36 56·9
Longitude in time		*b*	6 41 53·6 = 100° 28′ 24″ E

2nd.—With Latitude 15° 30′

	° ′ ″		
l	15 30 0	sec	·016089
p	87 23 23	cosec	·000451
S	77 6 26	cos	9·348553
S − a	25 46 56	sin	9·638441
			19·003534
	18 30 46	sin	9·501767
	37 1 32		

		h. m. s.
West hour angle		2 28 6·1
Equation of time		− 9 49·8
Date at place	Sept. 30th	2 18 16·3
Greenwich date . . .	Sept. 29th	19 36 56·9
Longitude in time	b'	6 41 19·4 = 100° 19′ 51″ E

D. Lat. 30′ = D.	With Lat. 15° 0′ S.		With Lat. 15° 30′ S	
		h. m. s.		h. m. s.
Obs. 1 } Longitude in time.	a	6 40 31·7 E	a'	6 40 56·1 E
Obs. 2 }	b	6 41 53·6 E		6 41 19·4 E
	A	1 21·9 E	B	23·3 E
	B	23·3 E		
D = 30′	A − B	58·6	A = 81·9 s.	C = 24·4 s

$$\frac{A \times D}{A - B} = \frac{81 \cdot 9 \times 30'}{58 \cdot 6} = 41' \cdot 9 \text{ S} \quad . \; . \quad \frac{A \times C}{A - B} = \frac{81 \cdot 9 \times 24 \cdot 4}{58 \cdot 6} = 34 \cdot 1 \text{ s E.}$$

	° ′ ″			h. m. s.
Assumed latitude . .	15 0 0 S		Longitude a . .	6 40 31·7 E
Correction	41 54 S		Correction . . .	34·1 E
Corrected latitude . .	15 41 54 S			6 41 5·8 E

Longitude 100° 16′ 27″ E

To find the Bearing of the Object at the time of either Observation.

1. With the assumed latitudes, and the longitudes deduced with them from either observation, compute the course of the *line of position* by middle latitude or Mercator's sailing.

2. Subtract this *course* from 90°, and the remainder is a bearing *from* the N if the course is N, or *from* the S if the course is S; and towards the E if the course is W, and towards the W if the course is E.

3. This bearing or the one directly opposite to it is the bearing of the object; and no difficulty can arise in actual practice in distinguishing which should be taken.

Example.—Required the bearing of the object at the time of the second observation in Example 4, the sun being west of the meridian at the time of observation?

Latitudes	15° 0′ S 15 30 S	Longitudes	100° 28′ 24″ E 100 19 51 E
Diff. latitude	30 S	Diff longitude	8 33 = 8′·55 W

Middle lat = 15° 15′

$$\text{Tan course} = \frac{\text{Diff long} \times \cos \text{mid. latitude}}{\text{Diff. lat.}} = \frac{\text{Dep.}}{\text{D lat.}}$$

Diff. long 8′·55 . . .	0·931966	90° 0′
Mid. lat. 15° 15′ cos. . .	9·984432	S 15 22 W
	10·916398	S 74 38 E
Diff. lat 30′	1·477121	or N 74 38 W
Course S 15° 22′ W tan	9·439277	

Therefore the bearing is N 74° 38′ W; for the sun is evidently to the north of the place of observation, for the declination is only about 2° 36′ S and the latitude 15° 42′ S.

To find the Time at the Place, corresponding to either Observation after the Latitude has been found.

1. Take the difference between the times at place deduced from one observation, and call the difference C.

2. The correction of the time deduced with the latitude which gives the greater difference of longitude A is $\frac{A \times C}{A + B}$ or $\frac{A \times C}{A - B}$.

3. And the correction is additive if the time to be corrected is less, and subtractive if it is greater than the other.

Example.—Let it next be required to find the time of the second observation and the error of the chronometer, in Example 4.

		h m s
Date computed with lat 15° . . (mean time) . . .	Sept 30th	2 18 50 5
Date computed with lat 15° 30′ (mean time) . . .	Sept 30th	2 18 16 3
	C′ . . .	34·2

Correction of the time found with lat $15° = \frac{A \cdot C'}{A - B}$

$= \frac{81\ 9 \times 34 \cdot 2}{58\ 6} = 47 \cdot 8$ s nearly.

This correction 47·8 s. is to be subtracted from the first time, for as the time found with lat. 15° 30′ is less than that with 15°, it will be still less for the correct lat. 15° 41′ 54″. (See Rule 3.)

		h m s
Mean time at place computed with lat. 15°	Sept 30th	2 18 50·5
Correction		− 47 8
Corrected date	Sept 30th	2 18 2·7
Chronometer showed at 2nd observation		6 45 40·0
Chronometer fast for mean time at place		4 27 37 3

Questions for Exercise.

1. On June 3rd, 1855, in lat. by account 37° 35′ N, long. by account 19° 26′ W, the following observations were taken; required the latitude and longitude by Sumner's method?

Mean Time nearly	Chron. Time	Obs Alt ☉
h m	h m s	° ′ ″
10 22 A M.	11 44 58	64 25 4
2 45 P M	4 6 6	50 48 1·5

Index error + 1′ 21″, height of the eye 20 feet, sun's bearing at the first observation S 59° 10′ E, ship's run in the interval S S W 6 miles per hour.

On May 20th, at noon, the chronometer was fast on Greenwich mean time 1 m. 48·5 s., gaining daily 4 25 s

Note. The answers are found with assumed latitudes 37° 20′ and 37° 50′.

Lat. . 37° 39′ 51″ N
Long. 19 29 59·5 W

2. On July 1st, 1855, in lat. by account 14° 37′ N, long. by account 70° 40′ W, the following observations were taken; required the latitude and longitude by Sumner's method?

Mean Time nearly.	Chron Time	Mean of Obs Alt ☉'s L L
h m	h m s	° ′ ″
8 45 A M.	1 51 33	41 14 21 5
10 20 A M.	3 24 51	63 19 43

Index error + 4′ 25″, height of the eye 23 feet, sun's bearing at the first observation N 71° 45′ E, ship's run in the interval W b S ½ S 5 5 miles per hour.

On June 18th, at noon, the chronometer was fast on Greenwich mean time 25 m. 42 s., losing daily 7 s.

Note. The answers are found with assumed latitudes 14° 50′ N, and 14° 20′ N.

Lat. . 14° 29′ 58″ N
Long. 70 44 54 W

Data from the "Nautical Almanac" for the above questions:—

No.	Date.	Declination at Mean Noon	Diff in 1 hour.	Equation of Time Mean Noon	Diff in 1 hour	☉'s S D
		° ′ ″	″	m. s	s	′ ″
1	June 2nd	22 9 30·7 N	+ 19 27	2 25·25 − a. t	−·388	
	June 3rd	22 17 13·0 N	+ 18·30	2 15 95 − a. t.	−·404	15 48
2	July 1st	23 9 9 N	− 10 22	3 23·31 + a. t.	+·479	15 46

3. If, on January 2nd, 1855, the following quantities be deduced from observations made at the same place, viz.:

Greenwich Dates.	d.	h.	m.	s	True Altitudes.	°	′	″
January . .	2	6	38	44	Altair	34	46	16
January . .	2	7	33	44	Vega	42	21	6

required the latitude and longitude by Sumner's method, using the

data tabulated hereunder, which are taken from the "Nautical Almanac" for 1855?

Corrected Right Ascensions.					Corrected Declinations			
	h.	m.	s.			°	′	″
Altair	19	43	40·38		Altair	8	29	12 N
Vega	18	31	59·43		Vega	38	38	57 N
Right Ascension of the mean sun at mean noon						h 18	m 46	s 13·81

Ans. Latitude 55° 16′ 25″ N, longitude 48° 26′ W.

4. Nov. 12th, 1858, in latitude by account 34° 25′ N, longitude by account 45° 15′ W, the following observations were taken to determine the latitude and longitude by Sumner's method.

Mean Time nearly.	Chronometer.	Observed Altitudes ☉
h. m.	h. m. s.	° ′ ″
9 25 A.M.	1 43 10	28 8 30
2 20 P.M.	6 37 25	25 42 35

Index error + 1′ 50″, height of the eye 19 ft.

On November 2nd, at noon, the chronometer was fast on G. M. T. 1 h. 16 m. 15 s., losing daily 3 s.

Corrected declination for the first observation 17° 43′ 15″·6 S, and the equation of time 15 m. 41·7 s. minus on apparent time.

Corrected declination for the second observation 17° 46′ 35″ S, and the equation of time 15 m. 40·0 s. minus on apparent time.

Sun's semidiameter 16′ 12″·3.

Ans. Lat. 34° 28′ 0″ N.
Long. 45° 17′ 22″ W.

IVORY'S METHOD OF DOUBLE ALTITUDES,

WITH RIDDLE'S MODIFICATIONS.

This Problem applies only to two altitudes of the same Object and the elapsed time between the Observations.

1. The first thing to be done is to prepare the altitudes and the corresponding times as shown by the chronometer for the application of the mathematical rules to which they are to be subjected. This preparatory work consists in correcting the observed altitudes for the index error, dip, refraction, semidiameter, and parallax, and if the ship have changed her place between the observations, a correction on this account must be applied to the first observation as directed at page 156. Take half the sum (S), and half the difference (D) of the corrected altitudes.

2. And next, the elapsed time, found by taking the difference between the times shown by the chronometer, should be reduced to an interval of mean solar time by allowing for the rate and divided by 2.

3. Thirdly, the Greenwich date corresponding to the middle time must be found, and with this date the declination and equation of time must be taken from the "Nautical Almanac." Find the polar distance (P).

4. Also multiply the "diff. in 1 hour" both for the declination and the equation of time by the half-elapsed mean time *in hours*, denoting the products by the letters c and e.

5 Look at the column of equation of time at page II. for the month in the "Nautical Almanac," and if the equation of time be *additive to mean time and increasing*, or *subtractive and decreasing*, add the correction e to the half-elapsed mean time found by Rule 2. But if the equation of time be *additive* and *decreasing*, or *subtractive* and *increasing*, subtract e, and the result is half the elapsed apparent time; reduce this to arc and call it H.*

6. The rules above are preparatory to the logarithmic calculation which now commences. Compute **A**, **E**, **I**, **O**, from these formulæ :—

$$\sin \mathbf{A} = \sin \text{P}.\ \sin \text{H} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\cos \mathbf{E} = \cos \text{P}.\ \sec \mathbf{A} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$\sin \mathbf{I} = \operatorname{cosec} \mathbf{A}.\ \cos \text{S}\ \sin \text{D} \quad . \quad . \quad . \quad (3)$$

$$\cos \mathbf{O} = \sec \mathbf{A}.\ \sin \text{S}.\ \cos \text{D}.\ \sec \mathbf{I} \quad . \quad . \quad (4)$$

$$\text{and generally } \sin \textit{latitude} = \cos \mathbf{I}.\ \cos (\mathbf{E} - \mathbf{O}) \quad . \quad . \quad . \quad (5)$$

* The directions at (4) and (5) relate to observations of the sun When a star is observed, the half-elapsed *mean* time found as directed in (2) should be reduced to *sidereal* time by the tables given in the "Nautical Almanac" for that purpose.

Note. In tropical latitudes it may happen that

$$\textit{sin lat.} = \textit{cos}\ \mathbf{I}.\ \textit{cos}\ (\mathbf{E} + \mathbf{O}) \quad . \quad . \quad . \ (6)$$

and therefore when the computed latitude differs greatly from the latitude by account, it should be computed by both formulæ (5) and (6), and that which is nearest to the latitude by account will be the result to be adopted.

E is greater or less than 90° according as P is greater or less than 90°; **A**, **I**, and **O** are less than 90°.

7. The hour angle at the middle time may now be computed.

$$\sin H' \ (= \sin \text{hour angle}) = \sin \mathbf{I}.\ \sec \text{lat}.$$

This hour angle is *east* when the second altitude is *greater*, and *west* when the second altitude is *less* than the first.

Convert it into time, and if *east* subtract it from 24 hours, and the result is the *middle time at the place nearly*, and the difference between this and the *middle Greenwich date* will be the longitude in time nearly.

8. But, if the sun be observed, both the *latitude* and *computed middle time* will be slightly erroneous, and should be corrected for the change of the sun's polar distance between the times of observation, for hitherto the polar distance at the middle time only has been used.

To compute the correction of the latitude.

Write under each other c (see Rule 4), the hour angle H′ (Rule 7) and H (Rule 5), and compute the correction of latitude thus:—

$$\mathbf{C} = \text{correction} = c\ .\ \textit{sin}\ H'\ .\ \textit{cosec}\ H.$$

The correction is additive when the second altitude is the greater and the days increasing in length, or when the second altitude is the less and the days decreasing in length, otherwise it is subtractive.

To compute the correction of the middle time.

Compute the corrections a and b from these formulæ:—

$$a = \mathbf{C}\ .\ \textit{tan lat.}\ .\ \textit{cot}\ H'.$$
$$b = \ c\ \ \textit{tan dec.}\ .\ \textit{cot}\ H.$$

$a - b$, or $a + b$ is to be taken according as the latitude and declination are of the same or of different denominations, and the result is to be *added* to the computed middle time when the days are *increasing* in length, and *subtracted* when they are *decreasing*.

In other words, to be added or subtracted according as the polar distance is *decreasing* or *increasing*.

Noting always that the word subtraction is used in its *algebraic* sense, and that when b is greater than a, $a - b$ is a negative quantity, and then subtraction means addition, and addition subtraction.

With the *middle time* thus corrected, and the Greenwich date at the middle time, the longitude will be correctly found.

Example 1.

It is required to compute the latitude and longitude, from the following observations, taken in the southern hemisphere.

Greenwich Date, mean time	Observed Altitudes
h m s 1855. March 9th 20 45 10 March 10th 0 30 16	° ′ ″ $\underline{\odot}$ 21 30 0 $\overline{\odot}$ 39 1 0

Index error + 2′ 30″, and height of the eye 14 feet, the bearing of the sun at the time of the first observation NNE, and in the interval between the observations the ship ran NW b W 6 miles per hour.

Greenwich Date	Half-elapsed Mean Time
d h. m s March 9th 20 45 10 March 10th 0 30 16 2) 19 21 15 26 March 9th 22 37 43	h m s March 9th 20 45 10 March 10th 0 30 16 2) 3 45 6 1 52 33
Declination, page II	**Correction**
° ′ ″ March 9th . . . . 4 35 57·8 S —22 7·2 4 13 50·6 South polar distance 85 46 9·4	Diff in 1 hour . −58·65″ Hours past noon 22·63 Product = 22′ 7″ 2

Equation of Time, page II		Correction	
	m s		s
March 9th	10 49·70	Diff. in 1 hour —	·645
	−14·58	Hours past noon	22·6
			s.
+ to App. T.	10 35·12	Product. . .	14·58

c		*e*.	
	"		s.
Diff in 1 hour for declination	−58 65	Diff in 1 hour for equation of time	·645
Half-elapsed time in hours ·	1·9	Half-elapsed time, hours .	1·9
Product = *c* =	111 435	Product = *e* =	1·2255

The quantities *c* and *e* are the changes in the declination and equation of time respectively in half the elapsed time, the quantity *e* is to be *added* to the half-elapsed mean time 1 h. 52 m. 33 s. to convert it into apparent time, because the equation is *subtractive* on mean time and *decreasing*, as may be seen by referring to page II. for the month of March in the "Nautical Almanac."

	h m s.
Half-elapsed time	1 52 33
Correction *e*	+ 1·2
Apparent time	= 1 52 34 2
∴ In arc the half-polar angle, or H, =	28° 8′ 33″

The correction to be applied to the first altitude on account of the run of the ship between the observations may next be found. The angle between the sun's bearing and the course of the ship is 7 points; = A; and the distance run in 3·75 hours is 22·50 miles, and

$$^{*}\text{ Correction} = 22{\cdot}50 \times \cos 7 \text{ points} = 4' \, 24''$$

to be added to the first altitude, because the angle A is less than 8 points; the altitude having been previously corrected, however, in the usual manner.

* The correction found in this manner is only approximate See Appendix (Note 3).

The altitudes being corrected, we have—

			° ′ ″
First true altitude . .		=	21 47 5 2
Second true altitude .		=	38 42 37 8
½ Sum of altitudes .	S	=	30 14 51 5
½ Difference . . .	D	=	8 27 46·3
The polar distance .	P	=	85 46 9·4
Polar angle . . .	H	=	28 8 33

With the quantities H, P, S, and D, the logarithmic calculation proceeds as under :—

sin	9 673635	H	28° 8′ 33″						
sin	9 998815	P	85 46 9	*cos*	8·867909				
sin	9 672450	A	28 3 33	*sec*	·054304				
		E	85 12 16	*cos*	8·922213				
		A		*sec*	054304	. . . . . .		*cosec*	·327550
				sin	9 702203	S	30° 14′ 51″	*cos*	9·936442
				cos	9 995245	D	8 27 46	*sin*	9·167810
				sec	·016471	I	15 40 50	*sin*	9·431802
		O	54° 5′ 43″	*cos*	9 768223				
		E—O	31 6 33	*cos*	9 932567				
		I	15 40 50	*cos*	9·983529				
Latitude nearly			55 31 11	*sin*	9 916096	. . . . . .		*sec*	·247090
H′, Hour angle at the middle time nearly . .							28° 30′ 59″	*sin*	9·678892

The hour angle is east of the meridian because the second altitude is the greater, therefore, after converting it into time, it must be subtracted from 24 hours, the result is 22 h. 5 m. 56·07 s., and is the *apparent time nearly* at the place of the second observation, corresponding to the middle instant between the observations. The equation of time is then to be applied, to convert it into *mean time nearly*, and this will be found to be 22 h 16 m. 31·19 s.

The little corrections should then be computed from the three formulæ.

$$\mathbf{C} = c \times \sin H' \times \operatorname{cosec} H \quad . \quad . \quad . \quad 1$$
$$a = \mathbf{C} \times \tan \textit{latitude} \times \cot H' \quad . \quad . \quad . \quad 2$$
$$b = c \times \tan \textit{declination} \times \cot H \quad . \quad . \quad . \quad 3$$

1		2		3	
c 111″·4	2·0469	**C** 112″·8	2·0523	c 111″·4	2·0469
H′ 28° 31′ sin	9·6789	Lat 55° 31′ tan	·1631	Dec. 4° 14′ tan	8·8693
H 28 8 cosec	3265	H′ 28 31 cot	·2649	H′ 28 8 cot	·2719
112·8	2·0523	a 302·2	2·4803	b 15·43	1·1881
C 1° 52″·8		b 15·4			
		a—b 286 8	Diff. because Lat. and Dec. both S.		

The quantity $a - b$ must be divided by 15 to convert it into time, or, which is the same, multiplied by 2 and divided by 30, the result is 19·12 s.

In March the sun is advancing northwards, and, therefore, the days are *decreasing* in length in south latitude. And the second altitude is *greater* than the first. Therefore **C** is to be *subtracted* from the computed latitude.

And $(a - b)$ is to be *subtracted* from the computed time, because the days are *decreasing* in length.

	° ′ ″		h. m. s.
Approximate latitude .	55 31 11	Mean time (nearly) .	22 16 31·19
C	— 1 52·8	a—b . . .	— 19·12
Latitude . . .	55 29 18·2	Corrected time . . .	22 16 12·07

The longitude may now be found.

		d. h. m. s.	
Corrected mean date at place . . .	March	9 22 16 12·07	
Greenwich date at middle time		9 22 37 43	
Difference = longitude in time		21 30·93	W
Therefore the longitude	=	5° 22′ 28″·9	W

The omission of the corrections for the change of declination in this instance would make an error of very nearly 2 miles in the latitude, and nearly 5 miles in the longitude.

EXAMPLE 2.

The following are the results of observations made at the magnetic observatory of Hobart Town, Van Diemen's Land, January 27th, 1852, P.M.:—

Chronometer		Altitudes
h m s		° ′ ″
5 22 45·1	☉	103 16 38
7 3 23 8	☉	68 7 42

Each altitude being the mean of five, index error + 45″, and the chronometer 39 m. 44·1 s. fast on Greenwich mean time.

From these data it is found that the Greenwich date at the middle time was January 26th, 17 h. 33 m. 20·3 s.

The corrected altitudes are,

	° ′ ″	
	51 54 17 1	
	34 19 12·1	
And half their sum .	43 6 44 6 . . .	S
And half their difference	8 47 32 5 .	D

Half the elapsed apparent time reduced to arc is	12° 34′ 42″·6	H
The sun's south polar distance	71 20 18	P
The equation of time for the Greenwich date above	12 m. 54 8 s. + to app. time.	
The change of the declination in the half-elapsed time . . .	31″·78 = c	
The change of the equation of time in the half-elapsed time . .	·4375 s = e	

With these quantities the logarithmic calculation proceeds as follows:—

sin	9·3380158	H	12° 34′ 43″						
sin	9 9765446	P	71 20 18	*cos*	9·5051217				
sin	9 3145604	A	11 54 26·5	*sec*	·0094470				
		E	70 54 44	*cos*	9·5145687				
		A		*sec*	·0094470	. . . . . .		*cosec*	·6854396
				sin	9·8346960	S	43° 6′ 44″·6	*cos*	9·8633307
				cos	9·9948661	D	8 47 32·5	*sin*	9·1842838
				sec	·0751338	I	32 44 23	*sin*	9·7330541
		O	34 51 11	*cos*	9·9141429				
		E—O	36 3 33	*cos*	9·9076315				
		I	32 44 23	*cos*	9·9248662				
Latitude . . .			42 50 32	*sin*	9·8324977	. . . . . .		*sec*	·1347604
H′, Hour angle at the middle time							47° 31′ 35″	*sin*	9·8678145

Next for the corrections.

c	31″·78		1·5022	C	108″		2·0320	c	31″·78			1·5022
H′	47° 32′	sin	9·8678	lat	42° 51′	tan	9·9677	dec	18° 40′	tan		9·5287
H	12 35	cosec	·6620	H′	47 32	cot	9·9616	H	12 35	cot		·6513
C	108″		2·0320	a	91·5		1·9613	b	48·1			1 6822
	1′ 48″			b	48·1							

$$\frac{a-b}{15}=\frac{43\cdot4}{15}=2\cdot8\text{s.}$$

	° ′ ″
Latitude nearly . . .	42 50 32
Correction	+ 1 48
Latitude	42 52 20

The hour angle at the middle time (nearly) H′ = 47° 31′ 35″, and in time 3 h. 10 m. 6·3 s.; and this is west, for both observations were made P.M.

		h m. s.	
Therefore apparent time nearly		3 10 6·3	
Correction		— 2·8	
Apparent time at place at the middle time between the observations		3 10 3 5	
Equation of time		+ 12 54 8	
Mean time	Jan. 27th	3 22 58·3	
Middle Greenwich date . . .	Jan. 26th	17 33 20·3	
Longitude in time east		9 49 38	= 147° 24′ 30″ E

Examples for Exercise.

1. If on April 13th, 1855, in lat. by account 41° N, long. by account 30° W, the following observations were taken; required the latitude and longitude?

Mean Time nearly. h. m.	Chronometer. h. m. s.	Obs. Alt ☉'s lower limb ° ′ ″
10 40 A.M.	1 25 52	53 23 7
2 30 P.M.	5 11 46	44 39 7·4

Index error − 4′ 17″, height of the eye 31 feet, sun's bearing at the first observation S 34° 46′ E, ship's run in the interval NE by E 7 miles per hour.

On April 2nd, at noon, the chronometer was fast on Greenwich mean time 43 m. 12 s, gaining 6 s. daily.

{Latitude . 40° 50′ 0″ N
{Longitude 30 19 26 W

2. If on May 19th, 1855, in lat. by account 54° N, long. by account 141° E, the following observations were taken, required the latitude and longitude?

Mean Time nearly. h m	Chronometer h m s	Obs Alt ☉'s lower limb. ° ′ ″		′ ″
2 10 P.M.	2 56 15	46 38 52	Index error −	3 44
4 0 P.M.	4 47 31	32 2 7	Index error −	2 58

Height of the eye 34 feet, sun's bearing at the first observation S 50° 20′ E, ship's run in the interval NNE ¾ E 8 miles per hour.

On May 12th, at noon, the chronometer was slow on Greenwich mean time 1 h. 53 m. 24 s., and gaining daily 7 s.

{Latitude . 54° 20′ 26″ N
{Longitude 140 56 0 E

Elements from the "Nautical Almanac" for the questions above.

No.	Date.	Dec at Mean Noon	Diff in 1 hour.	Equa T Mean Noon	Diff in 1 hour	Semi-diameter.
1	April 13th	° ′ ″ 8 56 38 9 N	″ +54·3	m s 0 37·36 + a t.	s − ·644	′ ″ 15 58·6
2	May 18th	° ′ ″ 19 29 29 N	″ +32·85	m s 3 50·64 − a. t.	s. − ·091	′ ″ 15 50·4

The interval of time between the observations as shown by the chronometer is 5 m. 30 s., and this should be converted in the first place into *mean solar time* by subtracting a proportional part of the gaining rate of the chronometer. The hourly rate is evidently $\frac{6}{24}$ seconds or ·25 seconds, and therefore in 1 minute ·004, and therefore in 5 m. 30 s. it will be ·022 seconds; a very small quantity, for the interval is very small; and subtracting it from 5 m. 30 s. the result is 5 m. 29·98 s. This must now be converted into *sidereal time*.

Mean Time. m. s.		Sidereal Time. m. s.
5	. . . .	5 0·82
29	. . . . . .	29·08
·98	. . . .	·98
5 29·98		5 30 88

The sidereal interval is therefore 5 m. 30·88 s., and this is to be added to the right ascension of Markab, which is the star first observed. The right ascension must therefore be now taken from the "Nautical Almanac."

	h m. s.		h m. s.
R. A. of Markab .	22 57 31·64	*R. A. of Altair* .	19 43 41·72
Sidereal interval .	5 30·88		
Corrected R. A. .	23 3 2·52		

From the corrected right ascension of Markab *subtract* the right ascension of Altair (*rule c*), the remainder is the polar angle in time, half this reduced to arc is P, and will be found to be

$$P = 24° \; 55' \; 6''.$$

The work then proceeds thus:—

(1.) Polar Distances.	(2.) Zenith Distances.	(3)
° ′ ″	° ′ ″	° ′ ″
p 75 34 35·1	z 44 48 54	p 75 34 35·1
p' 81 30 43·2	z' 32 45 6	z 44 48 54·0
2)157 5 18·3	2)77 34 0	2)120 23 29·1
A 78 32 39·1	Z 38 47 0	C 60 11 44·5

Then by the formulæ in article (4),

		° ′ ″		
Half polar angle	P	24 55 6	cos	9·957564
				2
				19·915128
Polar distance of Markab .	p	75 34 35	sin	9 986091
Polar distance of Altair . .	p'	81 30 43	sin	9 995217
Half sum of polar distances .	A	78 32 39		2) 19 896436
Auxiliary angle	B	62 34 24	sin	9 948218
	A + B	141 7 3	sin	9·797769
	A − B	15 58 15	sin	9·439566
				2) 19 237335
Half dist betw. the two stars ½ D		24 33 23·5	sin	9·618667
Dist. between the two stars .	D	49 6 47		

The processes described in rule (5) now commence.

		° ′ ″		
Half dist. betw. the two stars ½ D		24 33 23·5		
Half sum of polar distances .	A	78 32 39·1		
	A′	103 6 2 6	sin	9 988547
Polar dist. of Altair, sub. . .	p'	81 30 43 2		
	A′ − p'	21 35 19·4	sin	9·565779
Polar distance of Markab . .	p	75 34 35·1	cosec	10 013909
Distance between the stars. .	D	49 6 47	cosec	10·121476
				2) 19·689711
	M	45 36 16	cos	9·844855

		° ′ ″		
Half dist. betw. the two stars, ½ D		24 33 23 5		
Half sum of Zenith distances	Z	38 47 0 0		
	Z′	63 20 23 5	sin	9·951184
Zenith distance of Altair . .	z'	32 45 6		
	Z′ − z'	30 35 17·5	sin	9·706601
Zenith distance of Markab .	z	44 48 54	cosec	10 151922
Distance between the stars .	D	49 6 47	cosec	10 121476
				2) 19·931183
	N	22 30 30	cos	9 965591

The difference between **M** and **N** is 23° 5′ 46″ = **Q** (see rule 6), and next, the latitude is to be computed.

	Q	23° 5′ 46″	cos	9·963716
				2
				19·927432
Polar distance of Markab	p	75 34 35	sin	9·986091
Zenith distance of Markab	z	44 48 54	sin	9·848078
Half sum of polar dist. and zenith dist. .	C	60 11 44 5		2) 19·761601
Auxiliary angle	D′	49 27 43	sin	9·880800
	C + D′	109 39 27·5	sin	9·973921
	C − D′	10 44 1 5	sin	9·270086
				2) 19·244007
Half colatitude		24 45 31·2	sin	9·622003
		2		
Colatitude		49 31 2·4		
		90 0 0		
Latitude		40 28 57·6		

To find the Position of the Stars at the Time of Observation.

Find the right ascension of the meridian by adding the mean time at the place to the mean sun's right ascension; and if the right ascension of either star be greater than the right ascension of the meridian, that star is *east* of the meridian. Moreover, that star is furthest from the meridian whose right ascension differs most from the right ascension of the meridian.

In the example above, the time at the place of observation was about 10 h. 30 m., or half-past 10 at night.

		h. m.
Therefore time at place	August 17th	10 30
Longitude in time		5 5 W
Greenwich date nearly	August 17th	15 35
		h. m.
Mean sun's right ascension on August 17th, about		9 44*
Mean time at the place nearly		10 30
Right ascension of meridian nearly		20 14
		h. m.
Therefore we have the right ascension of Markab		22 57
,, ,, Meridian		20 14
,, ,, Altair		19 43

* See sidereal time at mean noon in the "Nautical Almanac."

From this it is seen that Markab was *east* and Altair *west* of the meridian. And Markab was furthest from the meridian.

To find the Longitude at the Time of taking the Second Observation.

We must first find the Greenwich date for either of the observations, that of Markab for instance:—

	h	m.	s
The chronometer showed	4	34	26
Chron. fast on Greenwich mean time on July 22nd	1	2	13
	3	32	13
The chronometer has gained since		2	36
	3	29	37

From this it is seen that the Greenwich date is—

August 17th, 15 h. 29 m. 37 s.

And the reason that it is 15 and not 3 hours is, that the time at place was about 10 h. 25 m. P.M. and the longitude in time 5 h. 5 m. W; and therefore the astronomical date at Greenwich about 15 h. 30 m. With this Greenwich date, the right ascension of the mean sun is found:—

	h.	m.	s
R. A. of mean ⊙ at noon August 17th	9	42	9·13
Acceleration for 15 h.		2	27·85
Acceleration for 29 m.			4·76
Acceleration for 37 s.			·10
R. A. of mean ⊙ at time of observation	9	44	41·84

All the other elements for the computation of the longitude by chronometer are already found, viz.—the *true altitude* and *polar distance* of Markab, and the *latitude* of the place of observation.

Altitude a	45° 11′ 6″		
Latitude l	40 28 58	sec	·118843
Polar distance p	75 34 35	cosec	·013909
	2) 161 14 39		
S	80 37 20	cos	9·212036
S−a	35 26 14	sin	9·763286
			2) 19·108074
	20° 59′ 7″	sin	9 554037
	2		
East hour angle in degrees, &c.	41 58 14		
,, in time	h. m. s. 2 47 52·93		
West hour angle	21 12 7·07	} sum − 24 hours.	
Right ascension of Markab	22 57 31·64		
Right ascension of meridian	20 9 38·71	} diff.	
Right ascension of the mean sun	9 44 41·84		
Date at place . . . . August 17th	10 24 56·87	} diff.	
Date at Greenwich . . . August 17th	15 29 37·		
Longitude in time, W.	5 4 40 13 = 76° 10′ 1″·9 W		

EXAMPLE 2.

At Greenwich, on October 19th, 1852, the following observations were taken to determine the latitude:—

Greenwich Dates.	h. m. s.	Altitudes corrected for index error.	° ′ ″
October 19th,	7 46 57·3	Saturn	17 52 41
October 19th,	7 59 52	Altair	39 25 52

Greenwich Dates.		Sidereal Interval.	
	h. m. s.		m s
October 19th . .	7 46 57·3	12 m.	12 1·97
October 19th . .	7 59 52	54 s	54·15
Interval, mean time . .	12 54·7	·7	·70
			12 56·82

R A of Saturn	Correction	Dec of Saturn	Correction.
h m s 2 56 44·8 — 5·7 2 56 39 1	s — 7 3 7·8 584 511 5·694	° ′ ″ 14 7 47 N — 25·7 14 7 21 3 N.P.D 75 52 38·7 p	″ — 3 3 7·8 264 231 25 74

R A of Altair	Correction	Dec of Altair	Correction
h m s 19 43 34·9	s — ·034	° ′ ″ 8 29 6 1 N N P D. 81 30 54 p'	″ — ·08

Altitude of Saturn		Altitude of Altair.	
Ref. and Par. . .	° ′ ″ 17 52 41 — 2 55 17 49 46	Refraction . . .	° ′ ″ 39 25 52 — 1 9 39 24 43
Zenith distance .	72 10 14 z	Zenith distance . .	50 35 17 z'

To find the Polar Angle

		h m s
R. A. of Saturn		2 56 39·1
Sidereal interval	*add*	12 56 8
		3 9 35·9
R. A. of Altair	*sub*	19 43 34 9
		7 26 1 0
		60
	4)	446 1 0
Polar angle		111° 30′ 15″
Half polar angle		55 45 7 5 = P

A

	°	′	″
p . .	75	52	39
p' . .	81	30	54
	157	23	33
A . .	78	41	46

Z

	°	′	″
z . .	72	10	14
z' . .	50	35	17
	122	45	31
Z . .	61	22	46

C

	°	′	″
p . .	75	52	39
z . .	72	10	14
	148	2	53
C . .	74	1	26

Computation of D

	°	′	″		
P	55	45	8	cos	9·750333
					2
					19·500666
p	75	52	39	sin	9·986672
p'	81	30	54	sin	9·995220
A	78	41	46	2)	19·482558
B	33	26	48	sin	9 741279
A+B	112	8	34	sin	9·966727
A−B	45	14	58	sin	9·851367
				2)	19 818094
½D	54	11	55	sin	9·909047
			2		
D	108	23	50		

Computation of M

	°	′	″		
½D	54	11	55		
Add A	78	41	46		
A′	132	53	41	sin	9·864870
Sub p'	81	30	54		
A′ − p'	51	22	47	sin	9·892818
p	75	52	39	cosec	·013328
D	108	23	50	cosec	·022784
				2)	19·793800
M	37	56	16	cos	9 896900

Computation of N

	°	′	″		
½D	54	11	55		
Add Z	61	22	46		
Z′	115	34	41	sin	9·955206
Sub. z'	50	35	17		
Z′ − z	64	59	24	sin	9·957240
z	72	10	14	cosec	·021376
D	108	23	50	cosec	·022784
				2)	19 956606
N	17	57	38	cos	9 978303
M	37	56	16		
Q	19	58	38		

Computation of L

	°	′	″		
Q	19	58	38	cos	9·973049
					2
					19·946098
p	75	52	39	sin	9·986672
z	72	10	14	sin	9·978624
C	74	1	26	2)	19·911394
D′	64	33	28	sin	9 955697
C+D′	138	34	54	sin	9·820564
C−D′	9	27	58	sin	9·216071
				2)	19·036635
	19	15	35	sin	9·518317
			2		
Colat.	38	31	10		

Lat 51° 28′ 50″ N

To find the Positions of the Observed Stars with respect to the Meridian.

Mer. dist. of star = mean time at place + R. A. of mean sun − ✱'s R. A.

For Saturn

	h. m. s
R. A. of mean sun at noon October 19th	13 52 26·39
Acceleration for 7 h.	1 9·00
Acceleration for 46 m.	7·56
Acceleration for 57 s.	·16
R. A. of mean ☉ at 7 h. 46 m 57 s.	13 53 43·11
Mean time at place of observation	7 46 57·30
R. A of meridian	21 40 40 41
R. A. of Saturn	2 56 39 10
Mer. dist. of Saturn, W of meridian	18 44 1·31
or E "	5 15 58 69

For Altair

	h. m. s
R. A. of mean sun at noon October 19th	13 52 26 39
Acceleration for 7 h.	1 9·00
Acceleration for 59 m.	9·69
Acceleration for 52 s.	·14
R. A. of mean ☉ at 7 h 59 m. 52 s.	13 53 45·22
Mean time at place of observation	7 59 52
R. A. of meridian	21 53 37 22
R. A. of Altair	19 43 34·90
Mer. dist. of Altair, W of Meridian	2 10 2·32

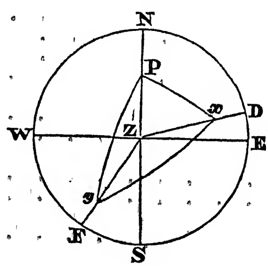
Thus it is seen that Saturn was about 5 h. east, and Altair about 2 h. west of the meridian.

To Compute the Bearings of the Stars.

To compute the bearing of Altair				To compute the bearing of Saturn.			
	° ′ ″				° ′ ″		
alt.	39 24 43	sec	·112044	*alt*	17 49 46	sec	·021376
lat	51 28 50	sec	·205665	*lat*	51 28 50	sec	·205665
p. d.	81 30 54			*p. d.*	75 52 39		
2)	172 24 27			2)	145 11 15		
S	86 12 13	cos	8·820930	S	72 35 37	cos	9·475884
S−*p*	4 41 19	cos	9·998545	S−*p*	−3 17 2	cos	9·999286
		2)	19·137184			2)	19·702211
	° ′ 21 44		9·568592		° ′ 45 13		9·851105
	2				2		
	S 43 28 W				S 90 26 E		

Now let the circle N E S W represent the horizon; P, the north pole. Then x may be taken to represent Saturn, rather more than 90° from the S towards the E. S D, or ∠ S Z D, is its azimuth. The other object, Altair, whose bearing is $43\frac{1}{2}$° W, may be repre-

Fig. 19.



sented by the point y, and its azimuth is S F, or ∠ S Z F. P y and P x are the polar distances of Altair and Saturn, denoted in the calculation by p and p', and Z y and Z x their zenith distances, denoted by z and z' in the calculation. The angle y P x is that which is denoted by P.

In the triangle y P x; P x and P y, and the angle y P x are known, and from these $x y$ is computed; $x y$ is the distance of the stars = D.

In each of the triangles y P x and y Z x all three sides are now known, and thence the angles P $x y$ and Z $x y$ can be computed; and the halves of the angles are denoted in the calculation by the letters

M and N; and the difference of M and N is half the angle P x Z, denoted by Q.

Lastly, in the triangle P Z x; P x, Z x, and ∠ P x Z, being known, the colatitude P Z is computed with them.

Examples for Exercise.

1. On November 24th, 1855, in lat. by account 40° N, long. by account 20° 20′ W, at 8 h. 10 m. P.M. mean time at place nearly, when a chronometer showed 10 h. 18 m. 44 s., the observed altitude of α Arietis was 63° 56′ 37″, index error + 4′ 10″, and at the same time the observed altitude of β Tauri was 30° 28′ 31″, index error − 2′ 55″, height of the eye 21 feet; required the latitude and longitude?

On November 17th, at noon, the chronometer fast on Greenwich mean time 45 m. 16 s., and losing daily 4·5 s.

Ans. Latitude 39° 51′ 20″ N
Longitude 20 18 0 W

2. On April 15th, 1855, in lat by account 49° S, long. by account 35° 20′ W, the following altitudes of stars were observed; required the latitude and longitude?

Mean T nearly h m.	Chronometer h. m s		Observed Altitude ° ′ ″
9 0 P. M.	10 5 53	Antares . .	22 5 15
9 20 P. M.	10 23 44	β Centauri .	62 31 56

Index error − 1′ 25″, height of the eye 17 feet.

Chronometer slow on Greenwich mean time on April 8th, at noon, 1 h. 18 m. 47 s., gaining daily 6 s.

Ans. Latitude 48° 55′ 20″ S
Longitude 35 17 30 W

Quantities from the "Nautical Almanac" for these Questions.

No	Date	R. A m. ⊙ at Noon	Date.	Star's Name	R A	Diff for 10 Days	Dec.	Diff. for 10 Days
		h. m. s.			h m s	s.	° ′ ″	″
1	Nov 24th	16 11 31·0	Nov 17th	α Arietis	1 59 3 80	+0·01	22 46 51·4 N	+ 0·6
				β Tauri	5 17 11 62	+0·22	28 28 58 4 N	+ 0 4
2	Apr. 15th	1 32 18 99	Apr 11th	Antares	16 20 32 41	+0 25	26 6 28 9 S	+ 0 7
				β Centauri	13 53 39 81	+0 15	59 40 20 4 S	+ 3 0

To find the Error of the Chronometer by equal Altitudes of the Sun.

Definition.—The equation of equal altitudes is a correction for the change of the sun's declination, to be applied to the mean of the times shown by a chronometer when the sun has equal altitudes A.M. and P.M. on a given day, to find the time shown by the chronometer at the instant that the sun's centre is on the meridian. It is in fact half the difference between the sun's meridian distances, east and west, at the instants of the two observations.

1. Find the mean between the times shown by the chronometer when the sun had equal altitudes.
2. Find half the interval, and reduce it from time into arc.
3. Find the Greenwich date of the sun's transit as for a meridian altitude.
4. Take out the declination and equation of time for this date. (Page I. for the month in the "Nautical Almanac.")
5. Multiply the "difference in one hour" of the declination by the number of hours, &c., in the half interval, call this *c*.
6. Compute A and B from these formulæ:—

$$A = c \times \textit{tan latitude} \times \textit{cosec half-elapsed time.}$$
$$B = c \times \textit{tan declination} \times \textit{cot half-elapsed time.}$$

7. A + B, when the latitude and declination are of contrary names, and A − B when they are of the same name, is the "equation of equal altitudes."
8. "The equation of equal altitudes" is to be *added* to the mean of the times shown by a chronometer when the sun had equal altitudes east and west of the meridian on the same day; provided the sun's polar distance be *increasing;* but it must be subtracted from the mean of those times if the sun's polar distance be decreasing. The result is the time which the same chronometer showed at *apparent noon.*

9. (*a.*) The difference between the time so found and 12 hours is the error of the chronometer for apparent time at the place of observation.

   (*b.*) Apply the equation of time to 12 hours, which will give the *mean time at apparent noon.* The difference between the result and the corrected middle time is the error of the chronometer for *mean time* at the place of observation.

   (*c.*) Apply the longitude in time to the *mean time at apparent noon*, and so find the *Greenwich time.* The difference between this and the corrected middle time is the error of the chronometer for Greenwich mean time.

EXAMPLE I.

If on April 6th, 1855, in latitude 32° 40′ S, longitude 153° 2′ E, the sun had equal altitudes at the following times by chronometer; required the error of the chronometer at apparent noon?

CHRONOMETER

A. M.	P. M.
h m s	h m s
9 28 20	14 33 47·5
29 59	32 7·5
31 40·5	30 27
3) 89 59 5	3) 96 22
9 29 59·8	14 32 7 3
	9 29 59·8
	2) 24 2 7·1

	h m s
Middle time by Chronometer	12 1 3·55 = m
P. M.	14 32 7·3
A. M.	9 29 59·8
Difference	5 2 7·5
Half-elapsed time	2 31 3·75
" " " in arc	37° 45′ 56″ = h

Greenwich Date at App Noon		Longitude in Time.
	h m s	
1855, April 6th . .	0 0 0	152° 2′ E
Long. in time .	10 8 8 E	10h. 8m. 8s.
April 5th . . .	13 51 52	

☉'s Declination, Page I.		Correction.	
Dec April 5th . .	5° 58′ 6″ N	Diff. in 1 hour . .	+ 56″·83
Cor.	+ 13 7·7	Hours past noon . .	13·86
	6 11 13 7 N	∴ Cor. . . .	+ 13′ 7″·7
South Polar distance	96 11 13·7		

Equation of Time		Correction	
	m s		s
April 5th	2 51·58	Diff. in 1 hour . .	− ·735
	− 10·19	Hours past noon . .	13·86
	2 41·39	∴ Correction . .	10·19s
Add to apparent time.			

c = Diff. in 1 hour × hours in half-elapsed time

$56{\cdot}83 \times 2{\cdot}5 = 142{\cdot}1$ nearly $= c$.

A = c tan l cosec h				B = c. tan d. cot h.			
c	142″·1	. . .	2·152594	c	142″·1	. . .	2·152594
lat	32° 40′	tan . .	9 806971	dec	6° 11′	. tan	9·034791
h	37° 46′	cosec .	·212931	h	37° 46′	. cot	·110840
	148″·76	. .	2·172496		19·87″	. . .	1·298225
	19·87						
	168″·63 sum, because lat. S and dec. N						

$\frac{168{\cdot}63''}{15} = 11{\cdot}24$ sec. = equation of equal altitudes

+ to the middle time by chron. (= *m*) because in April the south polar distance of the sun is increasing.

	h	m	s.
Middle time by chronometer	12	1	3·55 m.
Equation of equal altitudes			11·24
Time by chronometer at app. noon	12	1	14·79

	h.	m	s.
1. If the chronometer were right for app time, the chronometer would show at noon	12	0	0
But it shows	12	1	14·79
Therefore it is *fast* for app. time at place		1	14·79

	h.	m.	s.
2. Applying the equation of time to	12	0	0
Equation of time	+	2	41·39
If right for mean time at place, the chronometer would show at noon	12	2	41·39 A
But it shows	12	1	14·79
Therefore the chronometer is *slow* on mean time at the place of observation		1	26·60

	h	m.	s.
3. Applying the longitude to (A.)	12	2	41·39
Longitude in time	10	8	8
The corresponding Greenwich mean time	1	54	33·39
But the chronometer shows	0	1	14·79
Chronometer *slow* on G. M. T. at app. noon	1	53	18·60

Therefore the error on apparent time at the place is 1 m. 14·79 s. *fast;* on mean time at the place 1 m. 26·6 s. *slow;* and on Greenwich mean time 1 h. 53 m. 18·6 s. slow.

SECOND METHOD.

1. Let the altitudes be taken in the usual manner, setting the index of the sextant so that the successive altitudes may differ by 10 minutes; and let the observations extend at least over one degree.

2. By taking the mean of the differences of the times, the time in which the altitude changes 10 minutes may be found with great accuracy, but if the altitudes are well taken it will be quite sufficient to take the difference between the times at the beginning and end of one degree of altitude.

3. And as 4 minutes of time correspond to 1°, divide this interval by 4, and the quotient is the ratio of the change of time to the change of altitude. Let this be denoted by A. Or if the change of altitude should be half a degree, the interval must be divided by 2.

4. Find the change of the declination in the half-elapsed time, as before directed; reduce it to time; multiply it by A; call the product B.

5. Then the equation of equal altitudes is computed as follows—

$$A \times \textit{cos declination} = \textit{cosec } S.$$
$$B \times \textit{cos } S = \textit{equation of equal altitudes.}$$

S = the angle between the sun's zenith distance and his polar distance.

And the equation of equal altitudes is in seconds of time.

6. The equation of equal latitudes is (generally) additive when the polar distance is increasing, and subtractive when the polar distance is decreasing; and is applied to the middle-time as determined by the chronometer.

Notes on both Methods.

1. In the tropics, when the sun's declination exceeds the latitude of the observer, and is of the same denomination, the sun's azimuth at his rising first decreases, and, after attaining the minimum value, increases; and again, after he has passed the meridian, his azimuth first decreases, and again reaching a minimum value, it increases until he sets.

2. At the minimum points the azimuth neither increases no decreases; the angle S = 90°, and the equation of equal altitude vanishes.

3. If the observations are taken in the morning before the sun attains his minimum azimuth, and in the afternoon, after he has passed it, the equation of equal altitudes retains its ordinary sign; and is *plus* when the polar distance is increasing, and *minus* when the polar distance is decreasing.

4. But if the observations are taken between the points of minimum azimuth, the ordinary rule is reversed, and the equation of equal altitudes is *plus* when the polar distance is decreasing, and *minus* when it is increasing.

5. The first method of computing the equation of equal altitudes distinguishes the different cases by the relative values of the two parts A and B.

A is always greater than B in the temperate zones, and the common rule holds good.

In the tropics when A equals B it indicates that the observations were taken at the times when the sun was at his minimum azimuth; and when B is greater than A, the observations are taken between the points of minimum azimuth, and the rule is reversed.

6. If the second method of computing the equation of equal altitudes is used, the critical cases in the tropics must be watched by observing the changes of the sun's bearing when the altitudes are taken.

If in the morning the azimuth decreases, and in the afternoon increases, the ordinary rule holds good, but if the reverse is observed, the rule must be reversed; and the equation will be additive when the polar distance is decreasing, and subtractive when the polar distance is increasing.

7. The equation of equal altitudes under these circumstances will generally be very small. In the first method both A and B will be small and of contrary signs; and in using the second method S will be nearly a right angle.

Example.—On March 18th, 1858, at Gorden Island, Sydney, in latitude 33° 51′ S, longitude 152° E, the following observations were taken, to find the error of the chronometer:—

A.M.	⊙	P.M.
h. m. s. 12 37 9 38 12·7 39 15·5 40 20·2	° ′ 90 0 20 40 91 0	h. m. s. 16 55 42 54 39·5 53 34·5 52 30·4
Means 12 h. 38 m. 44·35 s.		16 h. 54 m. 6·6 s.

While the double altitude changed 1°, or the altitude half a degree, the time changed 3 m. 11·4 s. = 3·192.

And, dividing this by 2, because 2 minutes correspond to 1°, we have A = 1·595.

Declination at noon 1° 6′ 41″·2 S.	Difference in 1 hour = 59·29
Half the interval	2 h 7 m 41·1 s = 2·128 hours.

Hence the change of declination in the half-elapsed time = 8·126 s., and 1 595 × 8·126 s. = 12·96 s. = B.

A × Cos dec = Cosec S				B × Cos S = *e*.			
A . .	1·595 . .		0·202761	B . .	12·965 .		1·112605
Dec .	1° 7′	Cos.	9·999918	S 38° 50′ . .		Cos	9·891523
S 38° 50′ .		Cosec	10·202679	*e* = 10·15 . . .			1·004128

And the equation of equal altitudes is *plus*, because the polar distance is increasing.

	h	m	s
A.M.	12	38	44 35
P.M.	16	54	6·60
2)	29	32	50·95
	14	46	25·47
		+	10·10
	14	46	35·57

And therefore the chronometer is fast on apparent time 2 h. 46 m. 35·57 s. at the place of observation.

In each of the following examples the equation of equal altitudes is required:—

No.	Date, 1855.	Interval.	Latitude.	Longitude	Equation of equal Altitudes.
		h. m s	° ′	°	s
1	Sept. 1st . .	7 46 35	46 50 N	19 E	+ 16·3
2	March 5th . .	8 29 28	38 34 N	111 W	− 15 5
3	Nov. 27th . .	3 45 4	23 4 N	55 W	+ 5·6
4	April 25th . .	4 58 41	16 0 S	152 E	+ 6·4
5	Sept 4th . .	7 15 21	24 30 S	87 W	− 8·7
6	Sept. 9th . .	5 44 26	61 26 N	29 W	+ 28·1

Sun's declination from the "Nautical Almanac," for these exercises is as follows :—

No.	Declination of Apparent Noon.		Diff in 1 hour.
		° ′ ″	″
1	31st August	8 46 18·3 N	− 54·18
2	5th March	6 8 57·7 S	− 57·95
3	27th November . . .	21 5 52 9 S	+ 27·20
4	24th April	12 46 11·2 N	+ 49·28
5	4th September . . .	7 18 49 N	− 55·47
6	9th September . . .	5 26 46·3 N	− 56·78

Required the Error of the Chronometer at Noon.

1. If on June 25th, 1855, in latitude 13° 9′ S, and longitude 145° E, the sun had equal altitudes A.M. and P.M., when the chronometer showed—

A.M.	P.M.
h. m. s.	h m s
9 52 15	2 1 56

required the errors of the chronometer for mean and apparent time at noon?

Ans. Slow for app. time, 2 m. 55 s.;
Slow for mean time, 4 m. 59·9 s.

Note.—The sun's declination on June 24th, at apparent noon, 23° 26′ 22″·8 N. Diff. in 1 hour − 3″·07.

The equation of time at apparent noon (*add* to apparent time) 1 m. 57·32 s. Diff, in 1 hour + ·532 s.

2. If on May 12th, 1855, in latitude 14° 40′ S, longitude 145° 23′ E, the sun had equal altitudes A.M. and P.M., when the chronometer showed—

A.M.	P.M.
h. m. s.	h. m. s.
9 41 40·2	11 46 10·6

required the errors of the chronometer for mean and apparent time at the place of observation at noon?

Ans. Slow for apparent time, 1 h. 15 m. 59 s.
Slow for mean time, 1 h. 12 m. 7·8 s.

Note.—Sun's declination on May 11th, at app. noon, 17° 48′ 27″·6 N. Diff. in 1 hour + 38″·39.

Equation of time at apparent noon (*sub.* from app. time) 3 m. 50·1 s. Diff. in 1 hour + ·076 s.

3. If on October 8th, 1855, the sun had equal altitudes at the following times by chronometer, required the error of the chronometer at apparent noon for mean and apparent time?

Latitude 32° 3′ 18″ S, longitude 115° 44′ E.

Chronometer	
AM	PM
h m s	h m s
9 55 25·5	15 23 13 7
56 18 5	22 22 5
57 10	21 28 5
58 4	20 36·7
58 56·7	19 42 5

Note.—Sun's declination on Oct. 7th, at app. noon, 5° 23′ 15″·9 S. Diff. in 1 hour + 57″·50.

Equation of time at app. noon (*sub.* from app. time), 12 m. 1·90 s. Diff. in 1 hour − 0·703 s.

Ans. Fast for app. time, 39 m. 11 s.
Fast for mean time, 51 m. 24·6 s.

To find the Error of the Chronometer by equal altitudes of a Fixed Star.

1. Take the mean between the times shown by the chronometer when the star has equal altitudes east and west of the meridian; and thus will be found the time by the chronometer when the star passes the meridian.

2. Compute the time of the star's meridian passage as directed at page 109.

3. And the difference between the time by chronometer found by rule 1 and the correct time found by rule 2 is the error of the chronometer.

Example.

If on June 10th, 1855, in latitude 40° 10′ N, and longitude 30° 40′ W, equal altitudes of Altair were taken when the chronometer showed as undermentioned; required the error for mean time at the place of observation when the star passed the meridian?

	Chron. Time. h. m. s.	
Altair east of meridian	1 54 41	times of equal alts.
„ west „	7 13 48·6	
	2) 9 8 29·6	
Time by chron. of star's transit	4 34 14·8	

To find the Time of the Sun's Transit

		h m s	
Altair's R. A. on June 10th		19 43 44	
Mean ☉'s R. A. June 10th		5 13 6	
Date at place nearly	June 10th	14 30 38	
Longitude in time		+ 2 2 40	W
Greenwich date nearly	June 10th	16 33 18	

Mean Sun's R. A. more accurately

		h m. s.
Mean sun's R. A. June 10th		5 13 6·16
Acceleration for	16 h.	2 37·70
	33 m.	5·42
	18 s.	·05
Mean sun's R. A. corrected		5 15 49·33

Error of the Chronometer.

		h. m s
Altair's R. A.		19 43 44 15
Mean ☉'s R. A.		5 15 49·33
Corrected date of place	June 10th	14 27 54·82
Therefore the chronometer should show		2 27 54·82
But it shows		4 34 14·80
Error of chronometer for mean time at place, when the star passed the meridian		2 6 19·98 fast

To compute the Altitude of a Celestial Object.

The quantities with which the computation is made are the co-latitude of the place, the polar distance, and the meridian distance of the object.

To find the polar distance of the sun or moon a Greenwich date is necessary; and to find the meridian distance, if the object be the sun, the *apparent time* from noon is the meridian distance, and for any other object it is

Mer. Dist. = Mean Time at place + Mean ☉'s R. A. − R. A. of object.

And the rules for calculating the altitude are as follow:—

1. Find the Greenwich date.

2. Take out the declination of the sun and the equation of time, or the right ascension and declination of the moon or star, and the right ascension of the mean sun.

3. Find the meridian distance or hour angle $= h$.

4. Find half the sum of the polar distance and colatitude.

$$\tfrac{1}{2}\,(\textit{polar distance} + \textit{colat.}) = A.$$

5. Compute the angle B from this formula—

$$\sin B = \sqrt{\{\sin \textit{polar dist.} \times \sin \textit{colat.} \times \cos^2 \tfrac{1}{2} \textit{hour} \angle\}}$$

6. Compute half the zenith distance.

$$\sin \tfrac{1}{2} \textit{zenith dist.} = \sqrt{\{\sin (A + B) \,.\, \sin (A - B).\}}$$

Example 1.—Find the altitude of the sun on April 18th, 1855, at 9 h. 25 m. A.M., at a place in latitude 26° 18′ S, longitude 51° 28′ E.

Greenwich Date.		Longitude
	d h m. s	° ′
1855, April	17 21 25 0	51 28 E
Longitude in time .	3 25 52	205·52
1855, April	17 17 59 8	3 h. 25 m 52 s.

Declination N.		Correction
	° ′ ″	″
April 17th	10 22 36·1	+ 52·68
	+ 15 48·2	18
	10 38 24·3	42144
		5268
		948·24
South polar distance .	100 38 24·3	15′ 48″·2

Equation of Time.		Correction.
	m. s.	
+ to M. T.	0 22·42	+ ·584 s.
	+ 10·51	18
	0 32·93	4672
		584
		+ 10·512

Hour Angle=h.		A=½ (co-lat + polar distance)	
	h. m. s.		° ′ ″
Mean Time	21 25 0	Latitude	26 18 0 S
Eq. T.	+ 0 32·9	Colatitude	63 42 0 = l'
App. Time	21 25 32·9	South polar distance .	100 38 24 = p
East hour angle . .	2 34 27·1		164 20 24
	4) 154 27 6		A = 82 10 12
Hour angle	38° 36′ 46″		
Half hour angle . .	19 18 23		

$$\text{Sin B} = \sqrt{\{\sin p \sin l' \cos^2 \tfrac{1}{2} h\}}$$
$$\text{Sin } \tfrac{1}{2} \text{ Z. D} = \sqrt{\{\sin (A+B) \sin (A-B)\}}$$

	° ′ ″		
Half hour angle	19 18 23	cos	9·974863
			2
			19 949726
Polar distance	100 38 24	sin	9·992468
Colatitude	63 42 0	sin	9·952544
A	82 10 12	2)	19·894738
B	62 21 30	sin	9·947369
A + B	144 31 42	sin	9·763653
A − B	19 48 42	sin	9·530109
		2)	19·293762
½ Zenith distance	26 19 36	sin	9·646881
	2		
Zenith distance	52 39 12		
Altitude	37 20 48		

Example 2.—Required the true altitude of Algenib, on Oct. 26th, 1855, at 3 h. 10 m. 24 s. A.M. mean time, in latitude 40° 24′ N, and longitude 32° 18′ W?

Greenwich Date.		Longitude in Time
	h. m. s.	° ′
October 25th	15 10 24	32 18 W
Longitude in time W. .	2 9 12	2 h. 9 m. 12 s.
October 25th	17 19 36	

✱'s R. A.		✱'s Dec	
	h. m s		° ′ ″
R. A.	0 5 49·20		14 22 59·6 N
		North polar distance	75 37 0·4

Right Ascension of Mean ☉

	h m s
Sidereal time at mean noon, October 25th . . .	14 13 14·31
Acceleration for 17 h.	2 47·56
Acceleration for 19 m.	3·12
Acceleration for 36 s.	·10
Right Ascension of mean ☉	14 16 5·09

Hour ∠ = Mean Time + R A Mean ☉ − ✱'s R A

	h m s.
Mean time at place	15 10 24
Mean sun's right ascension	14 16 5·09
Right ascension of meridian	5 26 29·09
Right ascension of Algenib	0 5 49·20
Westerly meridian distance	5 20 39·89
The same reduced to arc	80° 9′ 58″
Half hour angle	40 4 59

To Compute the True Altitude.

	° ′ ″
Latitude . .	40 24 0 N
Colat. . .	49 36 0
Polar distance	75 37 0
	2) 125 13 0
A	62 36 30

	° ′ ″		
½ Hour ∠ .	40 4 59	2 cos	19·767450
N P. D. .	75 37 0	sin	9·986169
Colat. . .	49 36 0	sin	9·881692
A . .	62 36 30		2) 19·635311
B . .	41 4 55	sin	9·817655
A+B . .	103 41 25	sin	9·987483
A−B . .	21 31 35	sin	9·564586
			2) 19·552069
	36 39 40	sin	9 776034
	2		
Zenith distance	73 19 20		
True altitude.	16 40 40		

Example 3.—Required the true altitude of the moon on November 17th, 1854, at 5 h. 16 m. 44 s. A.M. mean time at place of observation, latitude 38° 58′ 30″ N, longitude 175° 16′ 40″ E?

Greenwich Date

	h. m. s.
November 16th . . .	17 16 44
Longitude E	11 41 6·7
November 16th . . .	5 35 37·3

Longitude in Time

175° 16′ 40″ E
11 h. 41 m. 6 s. 40 th.

☽'s Declination.

	° ′ ″
Declination at 5 h. .	0 36 23·4 N
Correction	−8 44·5
	0 27 38·9 N
North polar distance .	89 32 21·1

Correction

	″
Diff in 10 m. . . .	−147·25
Minutes	35·62
$\frac{1}{10}$ Product	524·5
Correction . . .	−8′ 44″·5

☽'s Right Ascension.

	h. m. s
R. A. at 5 h.	12 32 9 14
Correction	1 8·64
R. A.	12 33 17·78

Correction.

	s.
Diff in 1 hour . . .	+ 115·61
„ 1 min. . . .	1·927
Minutes	35·62
Product	+ 1 m. 8·64 s.

Mean Sun's Right Ascension

	h m s.
Sidereal time at mean noon, Nov. 16th .	15 40 55·57
Acceleration for 5 h.	49·28
Acceleration for 35 m.	5·75
Acceleration for 37 3 s.	·10
Mean sun's right ascension	15 41 50·70

Hour ∠ = Mean Time + Mean ☉'s R A − ☽'s R A

	h m s
Mean time at place	17 16 44
Mean ☉'s R. A.	15 41 50·7
R. A. of meridian	32 58 34·7
R. A. of the moon	12 33 17·8
Westerly meridian dist of the moon . .	20 25 16·9
(From 24 hours) East hour angle . . .	3 34 43·1
The same in arc	53°40′ 46″
Half hour angle	26 50 23

To Compute the True Altitude.

	° ′ ″
Latitude .	38 58 30 N
Co-lat . .	51 1 30
Polar dist. .	89 32 21
	2) 140 33 51
A.	70 16 55

	° ′ ″			
½ Hour ∠ .	26 50 23	2	cos	19·900994
Polar distance	89 32 21		sin	9·999986
Co-latitude	51 1 30		sin	9·890656
A . .	70 16 55			2) 19·761636
B . .	51 52 47		sin	9·895818
A+B . .	122 9 42		sin	9·927653
A−B . .	18 24 8		sin	9·499255
				2) 19·426908
	31 7 42		sin	9·713454
	2			
Zenith distance	62 15 24			
True altitude	27 44 36			

Examples for Exercise.

1. January 24th, 1854, at 2 h. 33 m. 14 s. P.M. mean time at place, lat. 49° 30′ S, long. 111° 10′ W; required the true altitude of the sun's centre?

Declination at mean noon S 19° 12′ 45″·3, diff. for 1 hour − 36″ 63
Equation of time „ − 12 m. 24·33 s, „ + ·587 s.

Ans. 48° 38′ 8″.

2. October 17th, 1854, at 9 h. 28 m. 15 s. A.M. mean time at place, lat. 66° 12′ S, long. 114° 00′ E; required the true altitude of the sun's centre?

Declination at mean noon S 8° 52′ 32″·6, diff. for 1 hour + 55″·18
Equation of time „ + 14 m. 20·62 s, „ + ·510 s.

Ans. 28° 16′ 9″.

3. July 20th, 1854, at 11 h. 29 m. 30 s. P.M. mean time at place, lat. 10° 18′ N, long. 48° 27′ W; required the true altitude of Marcab?

*'s declination on the 20th, N 14° 25′ 18″·8, diff. for 10 days + 2″·3
Sidereal time at mean noon, 7 h. 51 m. 45·56 s.
*'s right ascension, 20th, 22 h. 57 m. 31·09 s, diff. for 10 days + ·23 s.

Ans. 37° 43′ 24″.

4. November 8th, 1854, 1 h. 57 m. 10 s. A.M. mean time at place, lat. 28° 50′ S, long. 114° 12′ E; required the true altitude of Sirius?

> ✱'s declination on the 7th, S 16° 30′ 58″ 5, diff. for 10 days + 2″·0
> ✱'s right ascension „ 6 h 38 m 45·20 s., „ + ·27 s.
> Sidereal time at mean noon 15 h. 5 m. 26·57 s.

Ans. 64° 54′ 58″.

5. July 16th, 1854, at 1 h. 58 m. 20 s. A.M. mean time at place, lat. 14° 50′ S, long. 115° 20′ W; required the true altitude of the moon's centre?

> ☽'s declination at 21 hours N 2° 53′ 30″·4, diff for 10 m. + 144″·63
> ☽'s right ascension „ 1 h 3 m. 26 53 s., difference for 1 hour + 1 m. 57·59 s
> Sidereal time at mean noon 7 h. 32 m. 2·77 s.

Ans. 34° 50′ 40″.

6. August 27th, 1854, at 5 h 57 m. 12 s. P.M. mean time at place, lat. 27° 12′ N, long. 54° 13′ E; required the true altitude of the moon's centre?

> ☽'s declination at 2 hours S 4° 25′ 28″ 5, diff for 10 m. + 143″·07
> ☽'s right ascension „ 13 h. 12 m 44·91 s., difference for 1 hour + 1 m. 56·49 s
> Sidereal time at mean noon 10 h. 21 m. 34·68 s.

Ans. 35° 6′ 28″.

To compute the Apparent Altitude of a Heavenly Body when the True Altitude is known.

The corrections must be applied in reverse order and with signs contrary to those with which the true is deduced from the apparent altitude.

(1.) *For the sun.*—1. Subtract the parallax in altitude (Table IV.) from the true altitude.

2. Enter (Table V.) and take out the refraction, and if the altitude is small, *add* it mentally to the altitude and enter the table a second time with the result, and this time *add* the refraction which is taken out to the altitude. The result is the apparent altitude of the sun's centre.

(2.) *For a fixed star.*—1. Add the refraction, using the same precaution as above directed.

(3.) *For a planet.*—1. Find the horizontal parallax in the "Nautical Almanac," and then the parallax in altitude in Table X., subtract the parallax in altitude from the true altitude.

2. Add the refraction.

(4.) *For the moon.*—1. Find her horizontal parallax, and then compute her parallax in altitude, first with the true altitude from this equation.

$$Cos\ true\ alt. \times hor.\ par. = par.\ in\ alt.\ (nearly).$$

Subtract this parallax from the altitude, and re-compute the parallax in altitude;

$$Cos\ corrected\ alt. \times hor.\ par. = par.\ in\ alt.$$

The parallax in altitude thus found must now be ***subtracted*** from the true altitude.

2. With the remainder take out the refraction as above directed and *add* it to the corrected altitude, the result is the apparent altitude.

When the object of the computation of the true and apparent altitudes is to reduce the measured lunar distance to the true lunar distance, or that which would be measured by an observer at the earth's centre, there should in strictness be applied another correction for the spheroidal form of the earth For the computation of this correction, which is generally neglected in nautical practice, the following short summary of rules will suffice —

1. Compute the azimuth.

2. Find the correction of the zenith distances or altitudes from this formula:—

$$Angle\ of\ vertical \times cos\ azimuth = correction.$$

The angle of the vertical will be found in Table XII.

3. *Subtract* the correction, when the azimuth *exceeds* 90°, from the true altitude previous to its correction for parallax, &c, &c, but *add* the correction when the azimuth is less than 90°. And the result is the *reduced* altitude.

To Clear a Lunar Distance from the Effects of Parallax and Refraction.

1. Write under each other in order, the apparent altitude (s) of the sun or star, the apparent altitude (m) of the moon, and the apparent distance (d), divide their sum by 2, and from the half sum (X) subtract the apparent distance, call the remainder ($X - d$).

2. Under ($X - d$) write the true altitude (S) of the sun or star, and the true altitude (M) of the moon.

Also take half the sum of M and S, and mark it A.

3. Compute B from this formula :—

$$\sin B = \sqrt{\{\sec m \cdot \sec s \cdot \cos X \cdot \cos (X - d) \cdot \cos M \cos S\}}.$$

4. Bring B out under A, and take their sum and difference.

5. Compute the true distance from this formula :—

$$\sin \tfrac{1}{2} \textit{ true dist.} = \sqrt{\{\cos (A + B) \cdot \cos (A - B)\}}.$$

Example 1.—Given the quantities as under, it is required to compute the true distance.

Apparent Altitudes.	True Altitudes.	Apparent Distance ☉ – ☾
° ′ ″	° ′ ″	° ′ ″
☉ 18 22 13	☉ 18 19 30	61 19 49
☽ 13 21 21	☽ 14 15 24	

$$\sin B = \sqrt{\{\sec m \sec s \cos X \cdot \cos (X - d) \cos M \cos S\}}$$
$$A = \tfrac{1}{2}(M + S) \,.\, \sin \tfrac{1}{2} D = \sqrt{\{\cos (A + B) \cdot \cos (A - B)\}}$$

		° ′ ″		
Apparent alt. ☉	s	18 22 13	sec	·022716
„ „ ☽	m	13 21 21	sec	·011907
„ dist. ☽ – ☉	d	61 19 49		
		2) 93 3 23		
½ sum	X	46 31 41	cos	9·837588
½ sum – dist.	X – d –	14 48 8	cos	9 985343
True alt. of ☉	S	18 19 30	cos	9·977398
„ ☽	M	14 15 24	cos	9 986414
Sum 32° 34′ 54″ ; ½ sum . . .	A	16 17 27		2) 19 821366
	B	54 29 58	sin	9 910683
	A + B	70 47 25	cos	9·517232
	A – B –	38 12 31	cos	9·895292
				2) 19·412524
½ true distance		30 33 42·2	sin	9·706261
		2		
True distance	D	61 7 24·4		

A good deal of trouble may be saved in the calculation by the following method :—

1. Take away the seconds from the apparent altitudes, deducting at the same time as many seconds from each of the true altitudes.

2. Take away the seconds from the apparent distance, if the sum of the minutes, after adding up the apparent altitudes and apparent distance, will then be an even number; or make that sum even by *adding* as many seconds as the distance wants of the next minute.

3. Then proceed with the logarithms as before directed.

4 Be careful to correct the computed distance by *adding* as many seconds as were *rejected*, or by *subtracting* as many seconds as were *added* to the apparent distance.

The example which follows is the same as that which is given above, and it will be seen that 13″ have been subtracted from the apparent and true altitudes of the sun, and 21″ from the apparent and true altitudes of the moon; and that 49″ have been rejected from the apparent distance, and added again to the computed distance.

The seconds are thus avoided in a considerable portion of the calculation.

To Compute the True Distance

		°	′	″		
Apparent alt. ☉	s	18	22	0	sec	·022707
" ☽	m	13	21	0	sec	·011897
" dist. ☽ − ☉	d	61	19	0		
	2)	93	2	0		
½ sum	X	46	31	0	cos	9·837679
½ sum − dist.	X − d −	14	48	0	cos	9·985347
True alt of ☉	S	18	19	17	cos	9 977407
" ☽	M	14	15	3	cos	9·986425
Sum 32° 34′ 20″, ½ sum	A	16	17	10	2)	19 821462
	B	54	30	30	sin	9 910731
	A + B	70	47	40	cos	9·517141
	A − B −	38	13	20	cos	9·895211
					2)	19·412352
		30	33	18	sin	9·706176
				2		
		61	6	36		
Seconds rejected				+ 49		
True distance	D	61	7	25		

Agreeing with the former calculation, within a few 10ths of a second.

Example 2.—Given the quantities undermentioned to compute the true distance of the sun and moon.

Apparent Altitudes.	True Altitudes.	Apparent Distance.
° ′ ″	° ′ ″	° ′ ″
☉ 22 58 0 s	☉ 22 55 54 S	102 40 0 d
☽ 36 44 0 m	☽ 37 26 57 M	

$$\sin B = \sqrt{\{\sec m \sec s \cos X \cos (X - d) \cos M \cos S\}}$$
$$A = \tfrac{1}{2}(M + S), \ \sin \tfrac{1}{2} D = \sqrt{\{\cos (A + B) \cos (A - B)\}}$$

		° ′ ″		
Apparent alt of ☉	s	22 58 0	sec	·035867
" " ☽	m	36 44 0	sec	·096136
" dist. ☉ − ☽	d	102 40 0		
	2)	162 22 0		
½ sum	X	81 11 0	cos	9·185466
½ sum − dist.	X − d −	21 29 0	cos	9·968728
True alt of ☉	S	22 55 54	cos	9·964246
" ☽	M	37 26 57	cos	9·899762
Sum 60° 22′ 51″	½ sum A	30 11 25	2)	19·150205
	B	22 4 54	sin	9·575102
	A + B	52 16 19	cos	9·786691
	A − B	8 6 31	cos	9·995637
			2)	19·782328
½ true distance		51 6 29	sin	9·891164
		2		
True distance	D	102 12 58		

By commencing with the true altitudes and true distances, and writing the apparent altitudes where the true altitudes now stand in the calculation, the same form will enable us to compute the apparent distance back again, and so verify the work.

Adding 6″ to S and s, and 3″ to m and M, and 2″ to D to be rejected again at the end, the tabulation stands thus:—

		° ′ ″		
True altitude of ☉	S	22 56 0	sec	·035760
" " ☽	M	37 27 0	sec	·100243
" distance ☽ − ☉	D	102 13 0		
	2)	162 36 0		
	X	81 18 0	cos	9·179726
	D − X	20 55 0	cos	9 970394
	s	22 58 6	cos	9 964128
	m	36 44 3	cos	9 903859
Sum 59° 42′ 9″	A	29 51 4·5	cos	2)19·154110
	B	22 11 10 8	sin	9 577055
	A + B	52 2 15·3	cos	9·788977
	A − B	7 39 53·7	cos	9·996102
				2) 19·785079
		51 20 1·8	sin	9·892539
		2		
		102 40 3·6		
Deduct for the 2″ which were added . .		2		
Apparent distance	D	102 40 1·6		

Examples for Practice.

No.	Apparent Altitudes		True Altitudes		Apparent Central Distance.	
		° ′ ″		° ′ ″		° ′ ″
1.	m s	51 37 46 27 34 51	M S	52 13 20 8 27 33 10 True distance	d . . D	53 39 24·2 53 40 52·8
2.	m s	51 28 56 59 18 45	M S	52 18 33 59 16 45 True distance	d . D	32 13 25 31 44 37·4
3.	m s	53 31 17 62 12 24	M S	54 38 20 62 9 51 True distance	d . . D	42 21 30 41 34 32
4.	m s	58 27 16 48 15 36	M S	59 20 19 48 10 29 True distance	d . . D	57 22 34 56 52 20·6
5.	m s	22 56 18 39 21 23	M S	23 51 27 39 18 51 True distance	d . . D	88 21 13 87 45 9 6
6.	m s	12 36 21 32 17 51	M S	13 29 50 32 15 49 True distance	d . D	90 20 35 89 51 47

To Find the Greenwich Date corresponding to a given true Lunar Distance.

The lunar distances are given in the "Nautical Almanac," for *Noon*, III^h, VI^h, IX^h, *Midnight*, XV^h, &c., and so on for every third hour of every day on which it is possible to observe the moon. The tables containing these distances form pages XIII. to XVIII. of every month in the "Nautical Almanac."

1. The day of the month of the Greenwich date being known; enter the table of lunar distances, and note the two distances, between which the given one lies, in other words, the two consecutive distances, the one of which is greater, and the other less, than the given one.

2. Take out that one which corresponds to the earlier hour, together with the proportional logarithm, which stands in the column to the right of it.

3. Write the given distance below that which is taken from the "Almanac," and take the difference.

4. Take from Table XV. the proportional logarithm of this difference, which write under the prop. log. found in the "Nautical Almanac;" take the difference of these logs., and find in Table XV. the corresponding time.

5. Add the time found by Rule 4 to the hours corresponding to the lunar distance, which is taken from the "Nautical Almanac," and after writing the day of the month before this computed time, the result is the *Greenwich date* (on the supposition that the distances change uniformly).

Example I.

By the "Nautical Almanac" for 1855, page 197, XVI. for the month of October, it is seen that on October 17th, the star Antares was W of the moon, and its distance from the moon as under—

Midnight.	P. L of Diff	XV^h	P. L. of Diff.
37° 48′ 32″	2432	39° 31′ 21″	2421

Required the Greenwich date when the distance is 38° 45′ 16″?

At *midnight* on Oct 17th, the dist is . . .	37° 48′ 32″	P. L 2432
And the given distance is . .	38 45 16	
Difference	56 44	P. L. 5014
	h m s	
	1 39 19 . .	2582
Time corres to the dist. taken from the "Naut Almanac"	12 0 0	
Greenwich date nearly Oct 17th	13 39 19	

This is further to be corrected for the want of uniformity in the change of the distances, and for this purpose a table is given in the "Nautical Almanac," entitled, "Correction for Second Differences of Lunar Distances"

Take the difference between the proportional logarithm which follows the distance taken out of the "Nautical Almanac" and that following the next distance, and with this difference and the approximate interval, 1 h. 39 m. 19 s, which has been found by Table XV., enter the table above mentioned, and take out the *correction*, *adding* it to the Greenwich date nearly, when the proportional logarithms are decreasing, and *subtracting* it from that date if the proportional logarithms are increasing.

In the above example,

The proportional log following the dist at midnight is	2432
" " " XV^{h}. . .	2421
Difference (minus)	−11

Whence the correction is found to be 3 s. +, and, therefore, the corrected date is Oct. 17th, 13 h. 39 m. 22 s.

EXAMPLE 2.

From the "Nautical Almanac" for 1856.

The sun's distances from the moon on December 4th, 1856, were as under, at VI^{h}. and IX^{h}. Greenwich mean time.

VI^{h}	P L of Diff	IX^{h}	P L of Diff
84° 58′ 31″	2749	86° 34′ 6″	2737

Required the Greenwich date when the distance is $85^\circ\ 20'\ 54''$?

		° ′ ″	
At VI^h. on Dec. 4th, the distance = ...		84 58 31	P. L. 2749
Given distance		85 20 54	
Difference		22 23	P. L. 9053
		h m s	
		0 42 9 . .	6304
Time over the distance taken from "Nautical Almanac" .		6 0 0	
Greenwich date nearly	Dec 4th	6 42 9	
Correction		+ 3	
Greenwich date	Dec. 4th	6 42 12	

To Find the Altitudes when a Lunar Distance is taken, from Altitudes before and after taking the Distance.

If, in taking lunars, there be only one observer, let him take the altitude of each object both before and after taking the distance, noting by his watch the time of each observation.

Then to find the altitude of either object at the time of taking the distance, if the altitudes be considered as changing uniformly during the short intervals between the observations, we shall have these equal ratios :—

$$\frac{A = \textit{Interval of time between the altitudes.}}{B = \textit{Int. of time between 1st alt and dist.}} = \frac{C = \textit{change of alt, in int. A}}{D = \textit{correction of the first alt.}}$$

$$\text{Whence D} = \text{correction sought} = \frac{B \times C}{A}$$

Example.

	h. m. s			° ′ ″
At	4 10 10	by watch,	the moon's altitude was	34 12 10
At	4 15 29	"	the dist between ☉ and ☽	80 34 20
At	4 19 40	"	the moon's altitude was	34 50 30

Required the moon's altitude at 4 h. 15 m. 29 s., when the distance was taken?

Here the interval of time elapsed between the observations of the moon's altitude is 9 m. 30 s. = 570 s. = A.

And the time between taking the first altitude and the distance = 5 m. 19 s. = 319 s. = B.

The change of altitude $38' \ 20'' = 2300'' = C$.

$$\therefore \ D = \frac{319 \times 2300''}{570} = 1287'' = 21' \ 27''$$

And this correction *added* to the first altitude, because the altitude of the moon is increasing, gives

	° ′ ″
First altitude of the moon	34 12 10
Correction	21 27
Altitude of ☽ when the distance was observed . .	34 33 37

If also the ⊙'s altitude were taken as under, required his altitude at 4 h. 15 m. 29 s?

	h m s		° ′ ″
At	4 12 26	the ⊙'s altitude was	57 16 40
At	4 17 29	" "	56 50 20

Then $A = 5$ m. 3 s. $= 303$ s, $B = 3$ m 3 s, $= 183$ s, and $C = -26' \ 20'' = -1580''$.

$$\therefore D = \frac{183 \times 1580''}{303} = 954''\cdot 2 = 15' \ 54''\cdot 2$$

And this correction must be subtracted from the first altitude of the sun, because the altitude is decreasing.

	° ′ ″
First altitude of the sun	57 16 40
Correction	−15 54
Altitude of ⊙ when the distance was observed .	57 0 46

Examples for Exercise.

Required the altitudes at the time of taking the distance in each of the following examples?

Example 1.

Time by Watch h. m. s.		° ′ ″	Answer	° ′ ″
9 27 34	Marcab's altitude .	21 19 30	☽'s altitude	48 19 1
9 28 47	☽'s altitude . . .	48 40 10	*'s altitude	21 32 52
9 31 2	Distance ☽ and * .	37 29 25		
9 33 14	☽'s altitude . . .	47 58 20		
9 35 8	Marcab's altitude .	21 48 40		

Example 2.

Time by Watch. h. m. s.		° ′ ″	Answer	° ′ ″
15 38 17	☽'s altitude . . .	30 10 0	☽'s altitude	30 34 23
15 40 25	Aldebaran's altitude	60 25 0	*'s altitude	60 38 53
15 42 16	Dist ☽ and * .	48 12 30		
15 44 0	☽'s altitude	30 45 0		
15 45 37	Aldebaran's altitude	61 4 0		

TO FIND THE LONGITUDE BY LUNAR OBSERVATIONS.

1. *When the altitudes are observed.*

1. Find the Greenwich date as nearly as may be.
2. Take from the "Nautical Almanac" the horizontal parallax and semidiameter of the moon, correct them by Tables VI. and VIII.
3. The index error, dip, and semidiameter, having been applied to the observed altitudes, the results are called the *apparent altitudes*—reject the seconds if under 30″, or take the next whole minute if the seconds be more than 30″.
4. Apply the refraction and parallax to the apparent altitudes; taking care to use the altitude of the limb of the sun or moon in taking out the refraction, and not the altitude of the centre. The results are to be considered as the *true altitudes.*
5. Correct the observed distance for the index error, and find the *central* distance by *adding* the semidiameters of the sun and moon to the corrected distance, when the sun is one of the objects observed;—or, if a star be observed, *add* or *subtract* the semidiameter of the moon, according as the distance of the star from the *nearest* or *furthest* limb of the moon is observed.
6. Clear the distance from the effects of parallax and refraction, and then find the *Greenwich date.*
7. Compute the time at the place of observation. And the difference between the date at the place and the Greenwich date, is the longitude of the place in time.

2. *When the altitudes are not observed at the same time that the distance is taken, on account of the obscurity of the horizon.*

In this case, the altitudes for clearing the distance *must be computed;* but, to compute the altitude, it is necessary that the time at the place of observation should be known. Let therefore the error of the watch be found by altitudes taken at some convenient opportunity, before or after the distances are observed. Compute from the log the difference of longitude made in the interval between taking the distances and taking the altitudes for the error of the watch; and if the distances are taken to the *eastward* of the altitudes, *add* the difference of longitude in time to the time of taking the distances, corrected by the error of the watch; but *subtract* it if the distances are taken to the *westward* of the altitudes; and the sum or remainder will be the time at the place where the distances were taken at the instant at which they were observed. Then with this time, the latitude at the same instant deduced from the log, the declinations, &c., of the objects, let their apparent altitudes be computed, and with the altitudes and distance compute the true distance, and thence find the Greenwich time and longitude as before.

EXAMPLE I.

Suppose that on October 24th, 1855, at about 7 h. 10 m. P.M., in longitude by account 52° 16′ W, latitude 17° 41′ S, the following lunar were observed, required the longitude?

	Observed alt. of Fomalhaut East of Meridian	Obs alt ☽	Obs' dist nearest limb
	° ′ ″	° ′ ″	° ′ ″
	67 15 0	21 14 5	57 15 ?
Index error	+ 2 12	− 1 43	− 1 7

Height of the eye 17 feet.

Greenwich Date		Longitude in Time
	h m	° ′
October 24th	7 10	52 16 W
	3 29	3 h. 29 m 4 s
Greenwich date, Oct. 24th	10 39	

☽ 's Semidiameter Noon	Correction	☽ 's Hor Par Noon	Correction.
′ ″ 16 24 3	Diff. in 12^h − ″ 3 2	′ ″ 60 4 9	Diff in 12^h − ″ 11 5
− 2·8	10 65	− 10 2	10 65
16 21·5	2130	59 54·7	5325
Aug. + 6 1	3195	Red − 1·1	11715
16 27·6	12) 34 080	″ 3593 6 = ′ ″ 59 53 6	12) 122 475
	2 84		10 2

To Correct the Moon's Altitude.

	° ′ ″			
Observed alt	21 14 5			
Index error	− 1 43			
	21 12 22			
Dip	− 4 4			
	21 8 18 *			
Semidiameter	+ 16 27·6			
Apparent altitude	21 24 45·6	H P	3593 6	3 555529
Refraction	− 2 26 *			
	21 22 19·6		Cos	9·969059
Parallax in alt.	+ 55 46·5		3346·5	3 524588
True altitude	22 18 6·1			

Adding 14·4 seconds to the apparent altitude, and also to the true altitude, in order to have a whole minute in the apparent altitude

	° ′ ″
M	22 18 20 5
m	21 25 0

To Correct the ✱'s altitude.

	° ′ ″
Observed alt.	67 15 0
Index correction . . .	+ 2 12
	67 17 12
Dip	− 4 4
App alt.	67 13 8
Refraction	− 24
True alt.	67 12 44

Rejecting 8″ from the app. and true altitudes.

S = 67° 12′ 36″
s = 67 13 0

To find the App. Central Dist

	° ′ ″
Observed dist. . . .	57 15 2
Index error	− 1 7
	57 13 55
☽'s semidiameter . .	+ 16 27·6
	57 30 22·6

Rejecting 22″ 6, to be added again to the computed distance, d=57° 30′ 0″

To Clear the Distance

		° ′ ″		
Apparent dist. . . .	d	57 30 0		
„ alt. ☽	m	21 25 0	sec	·031074
„ alt. ✱ . . .	s	67 13 0	sec	·412011
	2)	146 8 0		
	X	73 4 0	cos	9·464279
	X−d	15 34 0	cos	9·983770
	M	22 18 20·5	cos	9·966222
	S	67 12 36	cos	9·588109
	½ (M + S) = A	44 45 28·2	2)	19·445465
	B	58 7 17·4	cos	9·722732
	B+A	102 52 45·6	sin	9·988934
	B−A	13 21 49·2	sin	9·363858
			2)	19·352792
		28 20 17·4	sin	9·676396
		2		
		56 40 34·8		
The seconds rejected before .		+ 22 6		
True distance		56 40 57·4		

It may be observed that in this example the formula employed in *clearing the distance* is slightly different from that which is used in the preceding examples.

To find the Greenwich Date

	° ′ ″		Prop. logs.
Reduced distance as above . .	56 40 57·4		
True dis at IXh by "Naut Alm."	55 41 6	. . . . . .	2430
Difference	59 51 4	(Tab XV.)	4782
	h m s.		
	1 44 44		2352
From the "Nautical Almanac" .	9		
	10 44 44		
Correction for second diff . .	+ 1		
Greenwich date . . Oct. 24th	10 44 45		

To Compute the Time at Place from the Observed Altitude of the Star.

Fomalhaut's Right Ascension	Fomalhaut's Declination and South Polar dist.
h m s	° ′ ″
22 49 40·93	30 23 15 1 S
	South polar distance 59 36 44·9

Right ascension of the mean ☉

	h m. s
Sidereal time at Greenwich mean noon, Oct 24th . .	14 9 17·75
Acceleration for 10 h.	1 38·56
Acceleration for 44 m.	7·23
Acceleration for 45 s.	·12
R. A. of mean sun	14 11 3·66

$$\sin \tfrac{1}{2} \text{Hour} \angle = \sqrt{\{\sec l \;\; \operatorname{cosec} p \;\; \cos S \,.\, \sin (S - a)\}}$$

		° ′ ″		
True alt.	a	67 12 44		
Latitude	l	17 41 0	sec	·021021
Polar distance	p	59 36 45	cosec	·064179
	2)	144 30 29		
	S	72 15 14·5	cos	9·484012
	S − a	5 2 30 5	sin	8·943902
				2) 18·513114
		10 24 3	sin	9·256557
		2		
		20 48 6		
		4		
	6,0)	83 12 24		
		h m s		
Easterly hour angle		1 23 12·4		
Westerly hour angle		22 36 47·6		
✱'s R. A.	plus	22 49 40·93		
R. A. of meridian		21 26 28·53		
Mean ☉'s R A. . .	minus	14 11 3·66		
Date at place Oct. 24th . .		7 15 24·87		

To find the Longitude		
		h m s
Date at Greenwich by lunar	Oct 24th	10 44 45
Date at place by star's alt. . . .	Oct 24th	7 15 25
Difference = Longitude in time, W . . .		3 29 20
Longitude		52° 20′ W

Example 2.

Altitudes to be computed.

A lunar observation taken during one of the recent Northern expeditions, by Mr. R. C. Allen, Master of H M.S. "Resolute."

	h m s.		° ′ ″
1851. August 1st	6 45 4	Distances ☉☾ .	60 19 55
	46 13 4		20 25
	47 7·4		20 45
	48 40		21 35
	50 10		22 35
	5) 237 14 8		5) 105 15
August 1st	6 47 26·9		60 21 3
Error on G. M. T. . .	+ 4 31 40	Index error . .	−15
Greenwich Date, August 1st	11 19 6·9		60 20 48

The latitude was about 74° 36′ N, and longitude about 95° W

By altitudes taken the same day the chronometer was found to be *fast* for the mean time at the place of observation 1 h. 49 m. 7·4 s.

		h m s
Therefore, since the chronometer shows . . .		6·47 26·9
And the chronometer is fast		1 49 7·4
The date at the ship	August 1st	4 58 19·5

To Compute the Sun's Altitude

Greenwich Date, 1851, August 1st, 11 h 19 m 6 9 s

Date of Ship	August 1st, 4 h 58 m 19 5 s

☉'s Declination		Correction	
	° ′ ″		″
On August 1st . .	18 7 49·3	Difference in 1 hour .	−37·9
	7 8·3		11 3
	18 0 41		1137
			4169
North polar distance	71 59 19		428·27

Equation of Time		Correction	
	m s		s
On August 1st . . .	6 3 3	Diff in 1 hour . .	−0 14
	− 1 6	Hours past noon . .	11·3
	6 1·7		42
			154
To be subtracted from M. T.		Correction . .	1·582

Polar Angle = Apparent Time at Ship

	h m s
Mean time at ship . .	4 58 19·5
Equation of time	− 6 1 7
	2) 4 52 17 8
½ Polar angle in time . .	2 26 8 9
" in arc .	36° 32′ 13″

A = ½ (colatitude + polar distance)

	° ′ ″
Latitude	74 36 0 N
Colatitude	15 24 0 = l'
☉'s polar distance	71 59 19 = p
Colatitude + ☉'s polar distance . .	87 23 19
$\frac{1}{2}(p + l') = A$	43 41 39

$$\text{Sin } B = \sqrt{(\sin p \ \sin l' \ \cos^2 \tfrac{1}{2} h)}$$
$$\text{Sin } \tfrac{1}{2} \text{ Zenith Distance} = \sqrt{\{\sin(A+B) \ \sin(A-B)\}}$$

	° ′ ″		
½ hour angle	36 32 13	cos	9·904972
			2
			19·809944
p = polar distance . .	71 59 19	sin	9 978178
l' = colatitude . . .	15 24 0	sin	9·424156
A = ½ ($p + l'$)	43 41 39	2)	19·212278
B.	23 48 52	sin	9·606139

A + B	67 30 31	sin	9·965642
A − B	19 52 47	sin	9·531539
		2)	19·497181
	34 5 30	sin	9·748590
Zenith distance . .	68 11 0		
True altitude . .	21 49 0		
Cor. of altitude . .	+ 2 13	= refraction − parallax	
Apparent altitude .	21 51 13		
Rejecting 13″ from each: S =	21 48 47		
s =	21 51 0		

To Compute the Moon's Altitude.

Greenwich Date, 1851, August 1st, 11 h. 19 m 6·9 s

☽'s Right Ascension.		Correction	
h.	h. m. s		s
At 11	12 42 16·5	Diff in 1 hour . .	+ 134·2
Correction . . .	+ 42·6	„ in 1 min. . .	+ 2·23
			19·11
☽'s R. A.	12 42 59·1		223
			223
			2007
			223
		Correction	42 6153

☽'s Declination		Correction	
At 11h . . .	0° 55′ 55″ N	Diff in 10 min	− 132″ 8
	− 4 14	,, in 1 min	13 28
	0 51 41		19 11
North polar distance	89 8 19		1328
			1328
			11952
			1328
			253 7808
		Correction . .	4′ 13″ 78

Semidiameter		Cor		Hor Par		Cor	
At noon	16′ 22″·5	Diff in 12h	− 4″ 8	At noon	60′ 5″ 4	Diff in 12h	− 17″ 7
Cor .	− 4 5	,, 1	4	Cor .	− 16 6	,, 1	1 47
	16 18 0		11 3		59 48 8		11·3
Aug .	+ 4 3		4 52	Red .	− 11 0		441
	16 22·3			3577 8 =	59 37 8		1617
							16 611

Right Ascension of Mean Sun, and of the Meridian

	h m. s
Sidereal time at mean noon, August 1st	8 37 59 49
Acceleration for 11 hrs . .	1 48 42
Acceleration for 19 m . .	3 12
Acceleration for 7 s	02
Right ascension of mean sun . .	8 39 51 05
Mean time at place	4 58 19 5
Right ascension of the meridian	13 38 10 55

Polar angle of ☽ = R A of Meridian − R A of Moon

	h m s
Right ascension of meridian . . .	13 38 10 55
Right ascension of moon	12 42 59 10
Polar angle in time	0 55 11 45
Polar angle in arc = h	13° 47′ 52″
½ Polar angle	6 53 56

A = ½ (*Colatitude* + *Polar Distance*)

Colatitude	15° 24′ 0″ l'
☽'s Polar distance . . .	89 8 19 p
	104 32 19
	52 16 10 A

$$\sin B = \sqrt{(\sin p \sin l' \cos^2 \tfrac{1}{2} h)}$$

$$\sin \tfrac{1}{2} \text{ Zenith Dist} = \sqrt{\{\sin (A + B) \sin (A - B)\}}$$

½ hour angle	6° 53′ 56″	cos	9 996844
			2
			19·993688
p = Polar distance	89 8 19	sin	9 999951
l′ = co-latitude	15 24 0	sin	9 424156
A = ½ sum	52 16 10		2) 19·417795
B =	30 46 4	sin	9 708897

A + B	83 2 14	sin	9 996786
A − B	21 30 6	sin	9 564107
			2) 19 560893
½ zenith dist	37 5 53	sin	9 780446
	2		

Zenith dist.	74 11 46	Hor. Par	3577·8	3 553616
True alt.	15 48 14		cos	9 983265
Parallax in alt. nearly	− 57 22			3 536881
	14 50 52		cos	9 985251
Parallax in alt.	− 57 38			3 538867
	14 50 36			
Refraction	+ 3 33			
Apparent altitude	14 54 9			

Subtracting 9″ from the true and apparent altitudes —

$$M = 15° \ 48′ \ 5″$$
$$m = 14 \ 54 \ 0$$

TO FIND THE APPARENT CENTRAL DISTANCE

Distance of the limbs	60° 20′ 48″
Moon's semidiameter	16 22·3
Sun's semidiameter	15 47 0
Apparent central distance	60 52 57 3

To Clear the Distance

$$\sin B = \sqrt{\left\{\sec m \sec s \cos X \cos (X - d) \cos M \cos S\right\}}$$

$$A = \tfrac{1}{2}(M + S) \quad \sin \tfrac{1}{2} D = \sqrt{\left\{\cos (A + B) \cos (A - B)\right\}}$$

		° ′ ″		
Apparent alt ☉	s	21 51 0	sec	032376
,, ☽	m	14 54 0	sec	·014854
,, dist. ☽☉	d	60 53 0		
	2)	97 38 0		
	X	48 49 0	cos	9 818536
	d−X	12 4 0	cos	9·990297
	M	15 48 5	cos	9·983270
	S	21 48 47	cos	9 967736
	½ (M + S) = A	18 48 26		2) 19 807069
	B	53 12 30	sin	9 903534
	A+B	72 0 56	cos	9 489619
	A−B−	34 24 4	cos	9·916508
				2) 19 406127
		30 18 49 5	sin	9 703063
		2		
		60 37 39		
To be subtracted, because added before		3		
True distance . .		60 37 36		

To Compute the Greenwich Time and Longitude

		° ′ ″		P L.
True distance as computed above		60 37 36		
Dist. at 9ʰ from "Nautical Almanac" . .		59 19 35	.	2502
Diff		1 18 1	.	3631
		h m s		
		2 18 47·5	.	1129
Time taken from the "Nautical Almanac" . .		9		
		11 18 47·5		
Correction on account of second differences .		− 3		
Greenwich date.	August 1st	11 18 44·5		
Date at ship	August 1st	4 58 19 5		
Longitude in time		6 20 25		
		60		
		4)380 25		
Longitude . . .		95° 6′ 15″ W		

EXAMPLE 3.

On October 12th, 1854, the following lunar was taken at Greenwich, and the longitude computed from it, allowing for the figure of the earth:—

Greenwich Date.		Distance ☉☾
1854, October 11th	23 h. 26 m. 20·1 s.	107° 21′ 2″ 07

The distance is the mean of 14 readings of the sextant corrected for the index error.

To Compute the Sun's Altitude.

☉'s Declination, Page II., "Nautical Almanac"

	° ′ ″
At noon, Oct. 11th .	7 0 26 4 S
Correction	+ 22 5·4
Declination . . .	7 22 31·8
North polar distance.	97 22 31·8

Correction

	″
Diff in 1 hour	+ 56 57
Hours past noon . .	23 43
	16971
	22628
	16971
	11314
	1325 4351
Cor of declination . .	22′ 5″ 4

Equation of Time, P II

	m s.
At noon, Oct. 11th . .	13 10·86
Correction	+ 14·69
Equation of time . . .	13 25·55

+ to mean time.

Correction

	″
Diff. in 1 hour . .	+ ·627
Hours past noon . .	23·43
	1881
	2508
	1881
	1254
Correction . . .	14 69061

Polar Angle = Apparent Time from Noon

	h m s
Mean time at place	23 26 20·10
Equation of time . . .	+ 13 25·55
Apparent time.	23 39 45·65
Before noon	0 20 14 35
Half polar angle in time	10 7 175
" " in arc	2° 31′ 47″ 6 = h

$$A = \tfrac{1}{2}\,(\text{Colatitude} + \text{Polar Distance})$$

	°	′	″	
Latitude of Greenwich	51	28	50 N	
Colatitude	38	31	10	$= l'$
Polar distance of the sun . .	97	22	31·8	$= p$
Sum .	135	53	41·8	
½ sum	67	56	51	= A

$$Sin\ B = \sqrt{(sin\ p\ \ sin\ l'\ \ cos^2 \tfrac{1}{2} h)}$$

$$Sin\ \tfrac{1}{2}\ Zenith\ Distance = \sqrt{\{sin\ A + B),\ sin\ (A - B)\}}$$

	° ′ ″		
½ Polar angle . . .	2 31 47·6	cos	9 999577
			2
			$\bar{}$9·999154
Polar dist	97 22 31·8	sin	9·996392
Colatitude	38 31 10	sin	9 794335
A . . .	67 56 51	2)	19·789881
B . . .	51 43 57	sin	9·894940
A+B . . .	119 40 48	sin	9·938922
A−B . . .	16 12 54	sin	9·445981
		2)	19·384903
	29 30 30	sin	9·692451
	2		
Zenith distance	59 1 0		
True altitude	30 59 0		

To Compute the Sun's Azimuth

		° ′ ″		
True altitude	a	30 59 0	sec	10·066159
Latitude	l	51 28 50	sec	10·205665
Polar dist.	p	97 22 31·8		
	2)	179 50 21·8		
	S	89 55 10·9	cos	7·146473
	p−S	7 27 20·9	cos	9·996312
			2)	17·415309
		2 55 26	sin	8·707654
		2		
True azimuth	S	5 50 52 E		

Correction of Z. D for the Terrestrial Spheroid.

Reduction for latitude ($51\frac{1}{2}°$) (Table VII)	670″	log	2 826075
Azimuth of the sun less than 90° S, 5° 51′ E ..		cos	9 997732
	666·5		2·823807
Correction	− 11′ 6″ 5		

Computation of True and Apparent Alts

	°	′	″
☉'s Zenith distance	59	1	0
Correction found above	−	11	6·5
Reduced zenith distance	58	49	53 5
,, Altitude	31	10	6·5
Parallax .		−	8
	31	9	58 5
Refraction .	+	1	35
Apparent altitude	31	11	33·5

Subtracting 33″·5 from the apparent and also from the true altitude

	°	′	″
S	31	9	33
s	31	11	0

Elements to Compute the Moon's Altitude.

R. A. of Moon		Correction	
	h. m s		s.
At 23 hours	6 5 23·21	Diff. in 1 hour . .	+ 133·84
	+ 58·65	,, 1 min. . . .	2·23
	6 6 21 86		26 3
			669
			1338
			446
			58 649

Declination of Moon		Correction	
At 23 hours . . .	26° 42′ 25″ 6 N	Diff in 10 minutes .	+ 17″·82
	46 9	„ 1 minute . .	1·782
	26 43 12 5		26·3
North polar distance	63 16 47·5		5346
			10692
			3564
			46 8666

Semidiameter Midnight	Correction	Hor Parallax Midnight	Correction
14′ 56″·9	Diff. 12 h. — 2″ 9	54′ 45″·0	Diff. 12 h — 10″·7
— 2 7	11 4	— 10 2	11 4
14 54·2	116	54 34·8	428
Aug + 3·6	319	Red. — 6 5	1177
14 57·8	12) 33·06	54 28 3	12) 121 98
	2·7	3268·3″	10 16

To find R A of Mean Sun and of the Meridian

	h m s
Sidereal time at mean noon, Oct 11th	13 18 59·59
Acceleration for 23 h	3 46·70
26 m.	4 27
20 s.	·05
Right ascension of the mean sun	13 22 50 61
Mean time at place.	23 26 20 10
Right ascension of the meridian	12 49 10·71

Hour Angle = R. A. of Meridian — R. A of Moon

	h m s
Right ascension of meridian	12 49 10 71
Right ascension of moon	6 6 21·86
Hour angle in time	6 42 48·85
Half-hour angle in time	3 21 24·42
„ „ in arc	50° 21′ 6″·3

To Compute the Moon's Altitude

Half-Polar angle		50° 21′ 6″·3	cos	9·804870
				2
				19 609740
Polar distance		63 16 47·5	sin	9·950955
Colatitude		38 31 10	sin	9·794335
	A	50 53 59		2) 19 355030
	B	28 25 5	sin	9 677515
	A + B	79 19 4	sin	9·992408
	A − B	22 28 54	sin	9 582504
				2) 19·574912
		37° 48′ 22″ 5	sin	9·787456
		2		
Zenith distance		75 36 45		
True altitude		14 23 15		

To Compute the Moon's Azimuth

Altitude	a	14° 23′ 15″	sec	·013839
Latitude	l	51 28 50	sec	205665
Polar distance	p	63 16 47 5		
	2)	129 8 52·5		
	S	64 34 26 2	cos	9·632807
	S − p	1 17 38·7	cos	9·999889
				2) 19·852200
		57° 30′ 53″	sin	9 926100
		2		
		S 115 1 46 W		

Correction of Z D for the Terrestrial Spheroid

Reduction for latitude 51½° (Table VII) 670 log		2·826075
Azimuth of moon greater than 90° S, 115° 2′ W cos		9 626490
	283·5 log	2·452565
Correction		+ 4′ 43″·5

Computation of True and Apparent Altitudes of ☽

	° ′ ″		
☽'s zenith distance	75 36 45		
Correction computed above	+ 4 43 5		
	75 41 28 5		
Altitude	14 18 31 5	cos	9·986314
Hor Parallax 3268″ 3	. . . .	log	3 514322
Parallax in alt nearly	− 52 47		3 500636
	13 25 44 5	cos	9·987961
Parallax in alt	− 52 59 ·		3 502283
	13 25 32·5		
Refraction	+ 3 54		
Apparent altitude	13 29 26 5		

And subtracting 26″·5 from the true and apparent altitudes

$$M = 14^\circ\ 18'\ 5''$$
$$m = 13\ \ 29\ \ 0$$

To Clear the Lunar Distance

	° ′ ″	
Distance of limbs	107 21 2 1	
☽'s semidiameter	+ 14 57 8	
☉'s semidiameter	+ 16 4 4	
App central dist.	107 52 4 3	d

		° ′ ″		
App dist.	d	107 52 0	rejecting 4 3″	
Alt. ☉	s	31 11 0	sec	012138
Alt ☽	m	13 29 0	sec	067772
	2)	152 32 0		
	X	76 16 0	cos	9·375487
	d − X	31 36 0	cos	9 930300
	S	31 9 33	cos	9 932338
	M	14 18 5	cos	9·986328
	½ (S + M) = A	22 43 49	2)	19 304363
	B	26 40 31	sin	9·652181
	A + B	49 24 20	sin	9 813381
	A − B	3 56 42	cos	9·998970
			2)	19 812351
		53° 40′ 41″ 6	sin	9·906175
		2		
		107 21 23 2		
Rejected above		+ 4 3		
True distance		107 21 27·5		

To Find the Greenwich Time and Longitude.

True distance	° ′ ″ 107 21 27 5		P. L.	
Dist. at XXI. hours	108 28 30		3397	
	1 7 2·5		4289 5	
21 hours +	h m s. 2 26 33·5		0892 5	
Oct. 11th	23 26 33 5			
	— 2	Cor for second differences.		
Oct. 11th	23 26 31·5	Greenwich date by lunar		
Oct. 11th	23 26 20 1	Known date at place of observation.		
	11 4	W long in time found by lunar		

EXAMPLES (with altitudes observed).

1. If on September 3rd, 1855, at about 5 h. 10 m. A.M. mean time at place, lat. 55° 20′ N, long. by account 106° E, the following lunar observations were taken; required the longitude?

	Obs alt. α Arietis West of Meridian	Obs alt. ☽'s L L	Obs dist F L.
	° ′ ″	° ′ ″	° ′ ″
	51 16 14	57 24 19	30 5 20
Index error .	— 2 23	— 56	+ 2 20

Height of the eye 11 feet. Ans. Longitude 106° 10′ E.

2. If on February 3rd, 1855, at 8 h 45 m P.M. mean time at place nearly, lat. 47° 50′ N, long. by account 176° 40′ E, the following lunar was observed; required the longitude?

	Obs alt. Saturn West of Meridian	Obs. alt ☽'s L. L	Obs dist N L.
	° ′ ″	° ′ ″	° ′ ″
	59 19 0	26 59 4	78 57 23
Index error .	— 50	— 1 57	+ 1 35

Height of the eye 16 feet. Ans. Longitude 176° 34′ E.

3. On October 2nd, 1855, about 4 h. A.M. mean time at place, lat. 35° 20′ S, longitude by account 126° E, the following lunar was observed; required the longitude? Aldebaran and the moon being both too near to the meridian for finding time, the time at the place of observation was determined by other stars to be 15 h. 58 m. 36·2 s.

	Obs. alt. Aldebaran	Obs alt ☽'s L L	Obs dist F. L
	° ′ ″	° ′ ″	° ′ ″
	38 28 52	24 40 37·5	20 25 20
Index error .	— 1 35	+ 3 10	- 3 44

Height of the eye 9 feet. Ans. Longitude 126° 9′ E.

Requisites for the preceding questions from the "Nautical Almanac" for 1855.

For *Question* 1. On August 29th. Declination of *a* Arietis 22° 46′ 41″ 0 N, and diff. in 10 days + 1″ 8. Right ascension 1 h. 59 m. 2 64 s, diff. in 10 days + 0·26 s Sidereal time at mean noon on September the 2nd, 10 h. 44 m. 16 95 s.

Also the semidiameter and horizontal parallax, &c., of the moon as under.

Date	Semidiameter ☽		Hor Parallax		Date	Lunar Distances			
	Noon	Midnight	Noon	Midnight		IX	P L of Diff	Midnight	P L of Diff
Sept. 2nd	′ ″ 15 38 0	′ ″ 15 30 7	′ ″ 57 15 5	′ ″ 56 48 8	Sept 2nd	° ′ ″ 29 15 27	2943	° ′ ″ 30 46 51	2935

For *Question* 2. On February 2nd, the declination of Saturn at noon 20° 13′ 5″ 6 N, diff in 1 hour + 0″·3; and the right ascension 4 h. 30 m. 4 65 s., diff in 1 hour − 0 13 s. The sidereal time at mean noon 20 h. 48 m. 27 09 s.

Also the semidiameter and horizontal parallax, &c., of the moon as under.

Date.	Semidiameter		Hor Parallax		Date	Lunar Distances			
	Noon.	Midnight	Noon	Midnight		XVIII	P L of Diff	XXI	P L of Diff
Feb 2nd	′ ″	′ ″ 14 45 6	′ ″	′ ″ 54 3 5	Feb 2nd	° ′ ″ 77 4 42	3056	° ′ ″ 78 33 45	3053
Feb 3rd	14 46 7		54 7·8						

For *Question* 3. On September the 28th, the declination of Aldebaran is 16° 13′ 4″ N, diff. in 10 days + 0″·4, the right ascension 4 h. 27 m. 38·71 s., diff. in 10 days + 0 28 s. The sidereal time at mean noon on October the 1st, 12 h. 38 m. 37·01 s.

Also the semidiameter and horizontal parallax, &c., of the moon as under.

Date	Semidiameter.		Hor Parallax		Date	Lunar Distances			
	Noon	Midnight.	Noon	Midnight		VI	P L of Diff	IX	P L of Diff
Oct 1st	′ ″ 15 26·4	′ ″ 15 19 6	′ ″ 56 33 0	′ ″ 56 8 3	Oct 1st	° ′ ″ 18 45 56	3221	° ′ ″ 20 11 40	3173

TO FIND THE RATE OF A CHRONOMETER BY LUNARS.

At a time favourable for taking lunars, let a set of distances with the altitudes be carefully taken, and the corresponding times noted by a chronometer. Take the mean of the times, altitudes, and distances respectively, clear the mean distance from the effects of parallax and refraction; and find the Greenwich mean time to which the corrected distance corresponds; the difference between which time and the mean of the times shown by the chronometer will be the error of the chronometer for Greenwich mean time, as deduced from that set of lunars. Continue for a day or two at convenient opportunities to take similar sets of lunars, and find from each set the error of the chronometer for Greenwich mean time, and the mean of all the several errors as determined may be taken as the error at the mean of the times when the different sets of lunars are observed.

It will contribute to greater accuracy in the result, if an equal number of sets of distances from objects *east* and *west* of the moon be obtained. But when this cannot be done, take the mean of the errors found from objects *east*, and also the mean from objects *west*, of the moon; and consider half the sum of these two means as the error for Greenwich time resulting from all the observations A few days afterwards, let the error for mean Greenwich time be found from a series of independent sets of lunar distances in the same manner, and the difference of the two mean errors so found divided by the days, &c., between the mean of the times of each series, will be the *rate* of the chronometer. This rate, with the last-found error, may be used in finding the longitude by the chronometer, till another error and rate can be obtained either by the same or some other method.

The following example is given in the tables of Mendoza Rios, and it contains the results of actual observations made at sea by Captain Huddart in 1788:—

No	Date 1788			By Distances of ☽ from	Chronometer fast on G M T	
		h	m		m	s
1	April 11th	1	30	Sun	2	41
2	,, 11th	5	45	Regulus	6	0
3	,, 12th	1	0	Sun	4	34
4	,, 12th	1	45	Sun	4	23
5	,, 12th	5	30	Regulus	4	10
6	,, 13th	5	30	Aldebaran	5	5
7	,, 13th	5	30	Regulus	4	25
8	,, 15th	3	15	Sun	3	42
9	,, 15th	7	0	Pollux	5	55
10	,, 15th	7	15	Spica	2	57
11	,, 17th	6	15	Regulus	4	58
12	,, 17th	6	45	Spica	4	12
	12) 165	9			12) 53	2
	April 13th	18	45		4 25	5

Therefore on April 13th, at 18 h. 45 m., the chronometer was too fast for mean time at Greenwich 4 m 25 s.

By subsequent observations it was found, in the same manner, that on April 23rd, at 18 h., the chronometer was too fast for mean time at Greenwich 4 m. 43 s.

Thus in about 9 d. 23 h., the chronometer has gained 17·5 s., whence the daily rate will be found to be + 1·8 s.

The interval between the second epoch, April 23rd at 18 h., and the following noon is 6 h; and the proportional part to it of the rate +1·8 s will be found to be +0·3 s., consequently the error of the chronometer on April 23rd at noon at Greenwich was 4 m. 43·3 s.

On the next page will be found convenient blank forms for registering and preserving observations, and their computed results.

Name of observer . .
Sextant . .
Chronometer . . . .

LUNAR OBSERVATIONS.

Place	Astronomical Date at Place.	Apparent Altitude.		Reading of Sextant, corrected for Index Error	Time by Chronometer	True Distance	Corresponding Mean Time at Greenwich	Mean Time at Place.	Longitude	Number of Observations	Greenwich Errors.	Lunar Rates.	
		☉'s Centre.	☽'s Centre.										

LATITUDE.

Place	Astronomical Date at Place	Observations corrected for Index Error	Observations corrected for Index Error	Latitude by Observation	

LONGITUDE BY CHRONOMETER.

Place	Astronomical Date at Place.	Number of Observations.	Mean of Times	Mean Altitude corrected for Index Error	Error of Chronometer	Longitude by Chronometer

VARIATION OF COMPASS.

Place	Astronomical Date at Place	Altitude corrected for Index Error	Or the Hour ∠	Observed Azimuth	Variation	

ON TIDES.

TIDES are the daily rising and falling of the waters of the ocean, and they are produced by the attractions of the sun and moon, but chiefly by the attraction of the moon.

The attractive force of the moon, like that of every other body, varies in the inverse proportion of the square of its distance from the object which it attracts; and, consequently, the particles of the earth immediately under the moon are *more*, and those on the opposite side *less* attracted by her, than the intermediate parts are. And as the attraction of the moon, in the former case, acts in opposition to the gravity of the particles towards the earth's centre, their tendency towards the centre will be diminished; and, consequently, if they were at liberty to move freely among themselves, they would rise above the level of the place which they would otherwise occupy, and form a wave, which would follow the moon in her diurnal circuit round the earth. And although the moon's attractive force is in the same direction with the gravitating force of the particles on the opposite side of the earth, yet as she exerts a greater force in the same direction, on the central parts, the relative gravitation of the central parts, and the particles on the opposite side towards each other, will also be diminished; and, therefore, if at liberty to move freely, these particles will also rise above the general level, and form a wave or tide on the side opposite the moon.

The tide on the side next the moon, or that which happens when the moon is above the horizon, is called the *superior*, and the other the *inferior* tide.

Now it is only the particles of fluids that can be sensibly affected by such small variations in the gravitating forces; and it is only in the ocean, and large seas, that there is sufficient water to admit the effect, even in fluids, to be distinctly observable.

The time of full tide, however, even if the earth were covered with water, would not be at the time at which the moon is on the meridian of any place: for the waters, having been once put in motion, would continue to rise for some time, even if the moon's action were to cease altogether; and they, of course, continue longer to rise when her force is only a little diminished.

The waters of the ocean are similarly affected by the attraction of the sun: but though his attractive force on the earth is immensely greater than the moon's, yet, from his great distance, the effect is more nearly equal upon every particle, and therefore the tides which he produces are, with respect to the moon's, comparatively small; and, in

fact, they are only perceived in the modifications which they produce in the times and the heights of those which are primarily regulated by the moon.

In the interval between two successive transits of the moon she produces two tides; and the sun, in every twenty-four hours, produces two tides also. The tides produced by the sun and moon coincide at the times of full and new moon; and the consequence is, that the tides at those times are higher. At the quarter of the moon, the effect of the solar is to diminish the lunar tide; and hence, at those times, we find the tides are below the average height. The tides at full and change are called *spring tides*, and those at the quarters *neap tides*; but the highest and lowest tides are generally about the the *third tide* after the full and change, and the quarters.

As the action of the sun increases or diminishes the height of the lunar tide, so it also accelerates or retards the times at which high water happens.

When the moon is in the first and third quarters, the observed tide, or that compounded of the solar and lunar ones, is to the westward, and in the second and fourth quarters to the eastward of that raised by the moon alone; hence the action of the sun makes high water earlier in the former, and later in the latter case. This explanation will be received with a slight modification, arising from the consideration, that the solar and lunar tides are eastward of the places of the sun and moon, and consequently this acceleration and retardation, like the times of the highest spring and the lowest neap tides, will take place a little after the moon enters the quarters mentioned.

When the moon is at her least distance from the earth, the tides of course are greater than usual; and when the full or change of the moon happens about the beginning of January, when the sun also is nearest the earth, the tides are the greatest of all. The nearer the moon passes the zenith of any place, the greater are the tides which she produces at that place.

In small seas which are much enclosed by land, such as the North Sea, the observed tides are supplied from those raised in the adjoining ocean; and the Baltic, the Mediterranean, and such other seas as communicate with the ocean by very small mouths, cannot receive a sufficient supply of water in a tide to produce a material elevation in their surfaces. in these seas, in consequence, the tides are found to be very trifling.

The times of high water at any individual place are greatly influenced by its local situation; but there is at every place a mean relation between the time of high water and that of the moon's passing the meridian, which relation is subject to periodical variations, depending on the distances and relative positions of the sun and moon. The time of high water, too, often is materially affected by the wind; but it may be found, with sufficient exactness for any practical purpose in navigation, by means of the following problems:—

Problem I.

To find the time of high water on a given day at any place where the time of high water at full and change is known.

Let the time of the moon's passing the meridian of the given place be found, and to this time apply the correction from the following table, corresponding to her meridian passage and semidiameter, and to the result add the time of high water at full and change at the given place, and the sum will be the time of high water on the afternoon of the given day. If this sum exceed 12 h. 24 m., or 24 h. 49 m., subtract those times from it, and the remainder will be nearly the time of high water on the afternoon of the given day.

Corrections to be applied to the time of the Moon's meridian passage in finding the time of high water.

☽'s Mer Pass. App. Time	☽'s Semidiameter			☽'s Mer Pass App Time	☽'s Mer Pass App Time	☽'s Semidiameter			☽'s Mer Pass App Time.
	14′ 30″	15′ 30″	16′ 30″			14′ 30″	15′ 30″	16′ 30″	
h. m	h m.	h m	h m	h m	h m.	h. m.	h m.	h m.	h. m
0 0	− 0 4	0 0	+ 0 5	12 0	6 0	− 0 55	− 1 2	− 1 12	18 0
0 30	− 0 10	− 0 8	− 0 5	12 30	6 30	− 0 46	− 0 51	− 0 58	18 30
1 0	− 0 17	− 0 16	− 0 15	13 0	7 0	− 0 32	− 0 34	− 0 37	19 0
1 30	− 0 24	− 0 25	− 0 25	13 30	7 30	− 0 17	− 0 16	− 0 14	19 30
2 0	− 0 31	− 0 34	− 0 36	14 0	8 0	− 0 1	+ 0 3	+ 0 9	20 0
2 30	− 0 38	− 0 41	− 0 46	14 30	8 30	+ 0 8	+ 0 15	+ 0 24	20 30
3 0	− 0 44	− 0 49	− 0 55	15 0	9 0	+ 0 14	+ 0 21	+ 0 32	21 0
3 30	− 0 50	− 0 56	− 1 4	15 30	9 30	+ 0 16	+ 0 24	+ 0 36	21 30
4 0	− 0 55	− 1 2	− 1 12	16 0	10 0	+ 0 15	+ 0 23	+ 0 34	22 0
4 30	− 0 58	− 1 6	− 1 16	16 30	10 30	+ 0 12	+ 0 19	+ 0 29	22 30
5 0	− 1 0	− 1 8	− 1 19	17 0	11 0	+ 0 7	+ 0 14	+ 0 23	23 0
5 30	− 0 59	− 1 7	− 1 18	17 30	11 30	+ 0 2	+ 0 7	+ 0 15	23 30
6 0	− 0 56	− 1 2	− 1 12	18 0	12 0	− 0 4	+ 0 0	+ 0 5	24 0

Problem II.

From the observed time at high water at any place on a given day, to find the time of high water at full and change.

To the time at which the moon passes the meridian of the given place, apply the correction from the preceding table, and the result, subtracted from the observed time of high water, will leave the time of high water on the afternoon of the days of full and change.

If the time to be subtracted exceed the observed time of high water, let 12 h. 24 m., or 24 h. 49 m. (whichever is necessary to make it greater), be added to the observed time before the subtraction is made.

Problem III.

From the mean height of the spring and neap tides at any place, to find the height of any other tide at that place.

With the moon's semidiameter at the top, and the apparent time of her meridian passage in the side column of the following table; take out the numbers below A and B; and multiply the number below A by the mean height of the spring tide; and that below B by the *mean* height of the neap tide, and the sum of the products will be nearly the height of the required tide, independent of winds, freshes, &c., of which it is impossible to estimate the effects. This table is due to Bernoulli.

Apparent Time of Moon's Mer. Pass.		Moon's Semidiameter					
		16′ 30″		15′ 30″		14′ 30″	
		A	B	A	B	A	B
h. m.	h. m.						
0 0	24 0	0·99	0·15	0·88	0 12	0 79	0·08
0 40	23 20	1·10	0·04	0·97	0 03	0·87	0·02
1 20	22 40	1·14	0 00	1·00	0 00	0·90	0·00
2 0	22 0	1·10	0 04	0 97	0·03	0·87	0 02
2 40	21 20	0·99	0·15	0 88	0·12	0·79	0·08
3 20	20 40	0·85	0·32	0 75	0·25	0·68	0·18
4 0	20 0	0·67	0 53	0·59	0 41	0·53	0·29
4 40	19 20	0 46	0 75	0 41	0·59	0·37	0·41
5 20	18 40	0·28	0 96	0·25	0·75	0·23	0·53
6 0	18 0	0·13	1 13	0·12	0·88	0·11	0·62
6 40	17 20	0·03	1·21	0 03	0·97	0·03	0·68
7 20	16 40	0·00	1·28	0·00	1·00	0·00	0·70
8 0	16 0	0·03	1 24	0·03	0·97	0·03	0·68
8 40	15 20	0·13	1 13	0·12	0·88	0·11	0·62
9 20	14 40	0·28	0 96	0·25	0 75	0·23	0·53
10 0	14 0	0·46	0·75	0·41	0·59	0·37	0·41
10 40	13 20	0·67	0·53	0·59	0 41	0·53	0·29
11 20	12 40	0·85	0·32	0·75	0·25	0·68	0·18
12 0	12 0	0·99	0·15	0·88	0·12	0·79	0·08

Example.

If the height of the spring tide at a certain place be 18 feet, and that of the neap tide 14 feet, required the height of the tide when the time of the moon's meridian passage is 8 h. 40 m., and her semidiameter 15′ 30″?

Here we have A = ·12 and B = ·88
And ·12 × 18 = 2·16
·88 × 14 = 12·32

Answer. . 14·48

ON WINDS.

The chief causes of winds are the expansion and contraction of the air from heat and cold; and, though in our climate nothing is more proverbial than the inconsistency of the winds, in some parts of the earth they appear to be governed by laws which operate with considerable regularity.

The most remarkable of these winds are the *trade winds*, which in a zone extending in general about 30° on each side of the equator, blow from the eastward nearly quite round the globe; inclining towards the north in north latitude, and towards the south in south latitude, and forming what are called the N E and S E trade winds. In the intermediate space the wind is variable, but it in general blows from the eastward. In this space sudden squalls and heavy rains frequently occur.

When the sun has his greatest north declination, the S E trade wind extends several degrees N of the equator; and, in the opposite season, the N E trade wind extends in like manner to the south side of the equator; but in all seasons these winds are found to be greatly modified by local circumstances, particularly near land. In the vast expanse of the Pacific Ocean they prevail almost uninterruptedly, out at sea; but in the Atlantic, on the coast of Brazil, they blow from N E to E N E from September to March, and from S S E to E S E during the other half of the year. Near the African coast, the winds in general tend towards the coast; but in the Gulf of Guinea there is sometimes found a moderate breeze from the N E. Off this coast, about 7° N latitude, and 20° W longitude, there is a considerable space, where almost continual calms are found, attended with thunder and lightning; and the rains are so frequent and heavy, that the space has acquired the name of the *Rains*.

In the Indian Ocean, from about latitude 28° S to the equator, the S E trade wind blows pretty constantly; but in the Arabian Sea and the Bay of Bengal there are certain periodical winds, called *monsoons*, which, from April till October, blow from S W, and from the opposite point from October till April. The S W monsoon is frequently attended with tempests and rain; but during the prevalence of the N E monsoon the weather is in general dry and pleasant. About the change from one monsoon to another very violent storms of wind are usually met with.

Monsoons are found also in the Mozambique Channel; but the fair season there is during the S W, and the rainy season during the N E monsoon.

On the coasts of Sumatra and Java, and along the coast of China, monsoons also prevail; but they blow more nearly from the north and

south than those in the Arabian Sea; and they are, besides, much less regular, and are frequently interrupted by violent hurricanes, called *typhoons*. Off the western coast of New Holland there are regular monsoons, which blow from N W from October to April, and from S E during the remainder of the year.

Beyond the limits to which the trade winds extend, the winds are so variable, that all attempts to deduce the laws by which they are governed have hitherto been unsuccessful; westerly winds, however, are observed to be, on the whole, most frequent.

From what has been said, it will readily be perceived that a mariner, bound to the westward, will avail himself of the trade wind, when he can reach it without going too far out of his way; but this wind, so favourable in running towards the west, is directly adverse in sailing eastward, and it would therefore be a mere waste of time to attempt to sail to the eastward in the trade winds.

Indiamen, both outward and homeward bound, generally cross the equator between 18° and 23° west longitude, and thus avoid the coast of America, as well as the calms off the coast of Africa; and steering to the south-westward across the S E trade winds, till they reach the latitude where variable winds prevail, they then make towards the east. In sailing outwards in the Indian Ocean, they generally run down their longitude south of the parallel of the Cape of Good Hope, and then steer across the S E trade winds towards India.

Ships bound from Europe for the West Indies, and the southern parts of North America, avail themselves of the trade wind, which they endeavour to reach as soon as possible; and, in returning, they steer towards the north, till they get without the limit of the trade wind, where they find the winds variable, and they then work their way towards the east.

Ships from the coast of Guinea sail from S to E S E, as the wind will permit, till they reach the Island of St. Thomas; and with the wind which is generally found in that quarter they run to the westward till they meet the S E trade wind.

Near the shore, in tropical climates, there are daily land and sea breezes, the wind blowing from the sea during the heat of the day, and from the land during the night. In very warm weather these breezes are often observed to blow pretty regularly, even in temperate climates.

CYCLONES.

Cyclones, or circular storms, consist of a violent rotatory motion of the air extending over a very great space of from 50 to 500 miles and upwards in diameter. It is observed that the rotatory motion is different in the storms which occur on opposite sides of the equator. In the Northern hemisphere the whirl is from right to left, or in a direction contrary to the motion of the hands of a clock; and in the Southern hemisphere it is from left to right, or in the same direction as that in which the clock-hands move.

Perhaps the easiest rule for remembering in which direction the rotation takes place is this: *the wind is westerly in that part of the cyclone which lies nearest to the equator in either hemisphere.*

It is also observed that the centre of the cyclone has a progressive motion; those which arise in the tropics travelling first towards the westward, and gradually turning to the northward in the Northern hemisphere, and to the southward in the Southern hemisphere, and after arriving in lat. 20° or 25° turning back again towards the eastward.

To find the bearing of the centre of a circular storm.

A more or less rapid *veering* of the wind is one of the indications of a cyclone, and the bearing of the centre of the storm is always at right angles to the direction of the wind, and will be found by allowing 8 points to the *right* of the direction of the wind in the Northern hemisphere, and to the *left* of that direction in the Southern hemisphere. For example, in a northern storm, when the wind is N, the centre of the storm bears E; but in a southern storm, when the wind is N, the centre is due W of the ship.

Again, in a northern cyclone, when the wind is N E by N, the centre bears S E by E; but in the Southern hemisphere, with the wind N E by N, the bearing of the centre would be N W by W.

Or, turn your back to the wind, and point sidewise with your *left* hand in the Northern hemisphere, or with your *right* hand in the Southern hemisphere, and your hand will indicate the direction of the centre of the cyclone.

Of the veering of the wind in a cyclone.

The direction in which the wind veers round will also indicate the direction of the general motion of the storm. Thus if the wind changes from N to N E and then to E, the bearings of the centre will also change in the Northern hemisphere from E to S E and then to S; if the observer be supposed to look towards the centre of the

storm, it will be passing to the southward of him, and from left to *right* or towards the westward. The wind was supposed to veer from N to N E and E, or towards the *right;* and it may be remarked that, supposing the observer's face towards the centre of the storm, then when the wind is veering to the right, the storm is moving to the right; and when the wind is veering to the left, the storm is moving to the left; but if no veering takes place, but an increase in the force of the wind, the ship is directly in the track of the storm.

As another example of this phenomenon, suppose that in the Southern hemisphere the gale veers from W to S W and S, or to the *left,* the centre of the storm will change its bearing from S to S E and E, and will be observed to be moving towards the *left* hand of a person who looks towards its centre.

Of heaving-to in a circular storm.

When it is considered desirable to heave-to in a storm, it is of importance that it should be done so that the ship may *come up* to the wind instead of *falling off* from it; and the rule is this, when the wind is veering to the right, or the centre of the storm is moving to the right, the ship should be laid-to on the starboard tack. If the wind be veering to the left, or the centre of the storm be moving to the left, lay the ship to on the port tack.

Of sailing from the centre of the gale.

Captain Andrews, Commander of one of the Royal mail steamers, has observed, "that by keeping the wind on the starboard quarter when in a revolving storm, in the Northern hemisphere, ships gradually sailed from the storm's centre. And by keeping the wind on the port quarter, when in the Southern hemisphere, ships gradually sailed from the centre of a revolving storm. This rule applies to three-quarters of the storm's circle. But care should be taken in its application, lest the ship be carried into what has been called the quadrant of greatest danger, and before the centre of the advancing storm" *

For further information on this subject, see the valuable works of Sir W. Reid and Mr. Piddington, &c.

* The "Progress of the Development of the Law of Storms," &c, by Sir William Reid, K.C.B.

PART III.

INVESTIGATION OF THE RULES IN

NAVIGATION AND NAUTICAL ASTRONOMY.

PART III.

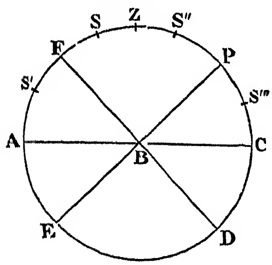
NAVIGATION AND NAUTICAL ASTRONOMY.

CONTAINING THE DEMONSTRATIONS OF THE RULES, ETC., GIVEN IN THE PRACTICAL PORTIONS OF THE WORK.

TO DEDUCE THE LATITUDE FROM THE OBSERVED MERIDIAN ALTITUDE OF A KNOWN CELESTIAL OBJECT.

LET, in the annexed figure, A C be the intersection of the plane of the meridian and that of the rational horizon, and F D the intersection of the meridian with the plane of the equator; Z the zenith, P the north and E the south pole; if S be the true place of a celestial object on

Fig. 20



the meridian, S A is its meridian altitude, S Z its meridian zenith distance is the complement of S A the meridian altitude, and is denoted as *north* or *south*, according as it is estimated from the object towards the north or south pole, and S F its north declination; and if S′ be the place of the object, S′ A is its altitude, S′ Z its zenith distance, north, and S′ F its south declination: if S″ be the place of the object, S″ C is its altitude, S″ Z its zenith distance, south, and S″ F its north declination.

Now Z S + S F, or S′ Z − S′ F, or S″ F − S″ Z = Z F the latitude.

Hence, for any object, from its observed meridian altitude, find its

true meridian altitude, and the complement of this is the meridian zenith distance; to be called north when the observer's zenith is north of the object, and south when the zenith is south of the object.

Find the Greenwich time of the object's meridian passage; and for that time find its declination.

Then if the zenith distance and declination be of the same denomination, that is, if they be both north or both south, their sum is the latitude; but if they be of different denominations, their difference is the latitude, and the latitude is always of the same denomination as the greater.

The declinations of the sun, moon, planets, and principal fixed stars are given in the "Nautical Almanac," the sun's both for apparent and mean noon, the moon's for every hour, the planets' for every day, and the fixed stars for the Greenwich time of their meridian passage for every tenth day in the year.

To find the Latitude from the Observed Altitude of a known Celestial Object when on the Meridian below the Pole.

In the last figure F P and Z C are quadrants: if therefore the common part Z P be omitted from each, P C, the elevation of the pole above the horizon, will be left equal to F Z, the latitude. Hence, if S‴ be an object on the meridian, below the pole; S‴ C, its altitude, added to S‴ P, the complement of its declination, will be equal to the latitude, and of the same name with the declination.

Now the sun is on the meridian below the pole 12 hours after noon; therefore if the longitude in time be added to or subtracted from 12 hours, according as it is west or east, the Greenwich time at which the sun will be below the pole will be obtained. If to the time of the moon's passing the meridian, 12 hours, and half the daily difference of her meridian passage be added, the sum will be the time at which she will be below the pole; and if 12 hours be added to the time at which a planet is on the meridian, the sum will be the time at which it is below the pole, exactly enough to compute its declination for finding the latitude at sea.

Let then the declination of the object be taken out for the instant of Greenwich time at which it passes the meridian below the pole, and to the complement of its declination add its true altitude, and the sum will be the latitude, of the same name with the declination.

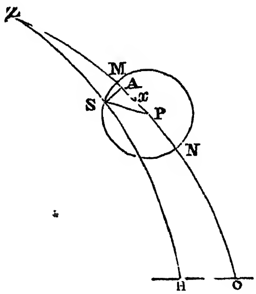
To find the Latitude by the Altitude of the Pole-Star.

Let P represent the north pole, M S N the circle which the pole-star appears to describe round it in each sidereal day.

Let Z P O be the meridian of the observer, Z his zenith, H O the horizon, then Z O and Z H are each 90°.

Now when the star is at M, its altitude is M O, and greater than the latitude which is measured by P O, by the polar distance M P.

Fig 21.



Again, when the star is at N, P O can be found by adding the polar distance P N to N O the altitude of the star.

Half the sum of M O and N O is also equal to P O, the latitude of the place of observation.

Next suppose the star to be observed at S, out of the meridian; its *meridian distance* Z P S can be computed, by subtracting the star's right ascension from the right ascension of the meridian (*see* page 127). P S, the polar distance, is also obtained by subtracting the star's declination from 90°; and the altitude S H is got by observation. Let Z P S $= h$; P S $= p$; S H $= a$, and Z S $= 90° - a = z$.

Draw S A perpendicular on the meridian; then Z A is less than Z S; for Z A is less than a quadrant, and therefore the opposite angle Z S A is less than a right angle; and therefore Z S opposite to the right angle A, is greater than Z A. Then, A being nearer to the zenith than S is, A has a greater altitude than S. This being the case, let x be supposed to have the same altitude as the star S. Then P x is the quantity to be subtracted from the altitude of x, or of the star S, in order to find P O or the latitude.

Now P x is evidently the same as P A $-$ A x, and therefore subtracting this from the altitude a, we shall have

$$\text{the } \textit{latitude} = a - PA + Ax.$$

And it remains to assign values to P A and to A x, in terms of the known quantities h, p, and a.

S P A may be considered as a plane triangle, its longest side P S being only about 90 minutes; and therefore

$$PA = PS \, . \cos SPA; \text{ or}$$

$$1st \textit{ Correction} = \textit{polar distance} \times \cos \textit{meridian distance}, \text{ or } p \, . \cos h.$$

$$\therefore \textit{The latitude} = a - p \, . \cos h + Ax.$$

The little term A x still remains to be determined.

$Z\,S = Z\,x$, and therefore $\cos Z\,S = \cos Z\,x$; but by the spherical triangle Z S A; $\cos Z\,S = \cos Z\,A \,.\, \cos A\,S$.

$$\therefore \cos Z\,x = \cos Z\,A \,.\, \cos A\,S \quad . \; . \; . \; . \; . \; . \; . \quad (1);$$

but

$$Z\,x = Z\,A + A\,x.$$

$$\therefore \cos Z\,x = \cos Z\,A \,.\, \cos A\,x - \sin Z\,A \,.\, \sin A\,x;$$

but since $A\,x$ is very small, its cosine may be taken as unity:

$$\therefore \cos Z\,x = \cos Z\,A - \sin Z\,A \,.\, \sin A\,x. \quad . \quad (2).$$

And equating these two values of $\cos Z\,x$, given in (1) and (2);

$$\cos Z\,A \,.\, \cos A\,S = \cos Z\,A - \sin Z\,A \,.\, \sin A\,x.$$

And therefore by transposition—

$$\sin Z\,A \,.\, \sin A\,x = \cos Z\,A - \cos Z\,A \,.\, \cos A\,S,$$

$$= \cos Z\,A \,.\, 2\sin^2 \frac{A\,S}{2}.$$

and dividing by $\sin Z\,A$,

$$\sin A\,x = \cot Z\,A \,.\, 2\sin^2 \frac{A\,S}{2}.$$

In this formula, Z x may be taken for Z A, from which it differs very little, and then as $\cot Z\,x = \tan x\,O$, or $\tan a$,

$$\sin A\,x = \tan a \,.\, 2\sin^2 \frac{A\,S}{2} \quad . \; . \; . \; . \; . \; . \; . \quad (3).$$

But since $A\,x$ and $\frac{A\,S}{2}$ are small quantities,

$$\sin A\,x = (A\;x)''\sin\,1'' \text{ and } \sin \frac{A\,S}{2} = \left(\frac{A\,S}{2}\right)'' \sin\,1''.$$

Where $(\text{A}\;x)''$ and $\left(\frac{\text{A S}}{2}\right)''$ are written for the number of seconds in A x and $\frac{\text{A S}}{2}$ respectively;

$$\therefore \sin^2 \frac{A\,S}{2} = \left(\frac{A\,S^2}{4}\right)'' \sin^2\,1'',$$

and

$$2\sin^2 \frac{A\,S}{2} = \frac{(A\,S^2)''}{2} \sin^2\,1''.$$

And by the triangle A P S, $A\,S = S\,P\,.\,sin\,S\,P\,A = p\,.\,sin\,h$;

$$\therefore\ 2\,sin^2\frac{A\,S}{2} = \tfrac{1}{2}\,(p\ sin\ h)^2\,sin^2\ 1''.$$

Therefore, substituting for $sin\,A\,x$, and for $2\,sin^2\dfrac{A\,S}{2}$ in equation (3), and dividing by $sin\ 1''$,

$$(A\,x)'' = \tfrac{1}{2}\,tan\ a\,.\,(p\ sin\ h)^2\,.\,sin\ 1''.$$

And therefore, finally, the equation with which we set out, *viz.*—

$$latitude = a - A\,P + A\,x,$$

becomes

$$latitude = a - p\,.\,cos\ h + \tfrac{1}{2}\,tan\ a\,.\,(p\,.\,sin\ h)^2\,.\,sin\ 1''.$$

When h, which is the star's westerly meridian distance, is under 90° or over 270°, $cos\ h$ is *positive*; but when it is between 90° and 270°, $cos\ h$ is *negative*, and $p\,.\,cos\ h$, or the first correction, becomes additive instead of subtractive.

The values here given of the first and second corrections of the *pole-star's* altitude, are the same as those given in the "Preface to the Nautical Almanac," and there employed in the computation of Tables I. and II.

The second term of the correction $\frac{1}{2}\,tan\ a\,.\,(p\ sin\ h)^2\,.\,sin\ 1''$, may be put into a form rather more convenient for computation If p be estimated in minutes instead of seconds, it becomes

$$\tfrac{1}{2}\,tan\ a\,.\,(p\ sin\ h)^2\,.\,sin\ 1'.$$

And this may now be reduced to seconds by multiplying by 60, when it becomes

$$30\ tan\ a\,.\,(p\ sin\ h)^2\,.\,sin\ 1'.$$

But $30\ sin\ 1' = \frac{7}{800}$ very nearly;

$$\therefore\ \frac{7}{800}\,.\,tan\ a\,.\,(p\ sin\ h)^2,$$

will now represent the second correction in seconds, but again

$$\begin{aligned}(p\,.\,sin\ h)^2 &= (P\,S\,.\,sin\ \angle\ P)^2\\ &= A\,S^2\\ &= S\,P^2 - P\,A^2\\ &= (S\,P + P\,A)\,.\,(S\,P - P\,A)\\ &= (polar\ distance + first\ cor)\,.\,(polar\ dist - 1st\ cor).\end{aligned}$$

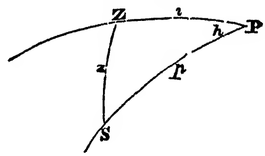
therefore the expression becomes

$$\frac{7}{800}\,.\,tan\ alt.\,(pol.\ dist. + 1st\ cor)\ \ (pol\ dist - 1st\ cor.)$$

Method of finding the Latitude by Altitudes of any Celestial Object when it is near the Meridian.

In the annexed figure, Z P is the meridian, Z the zenith, P the pole, Z P the colatitude, S the place of any celestial object at the

Fig. 22.



time of observation; z its zenith distance, p its polar distance, h the hour angle or meridian distance; then by spherical trigonometry—

$$\cos z = \cos p \cos l' + \sin p \sin l' \cos h;$$

or, if l = latitude,

$$\cos z = \cos p \sin l + \sin p \cos l \cos h \quad . \quad . \quad . \quad (1).$$

When the object is on the meridian, the hour angle vanishes, and therefore its *cosine* = 1.

Let $(z - r)$ = the meridian zenith distance; less than the zenith distance z by a small quantity r.

$$\therefore \cos (z - r) = \cos p \sin l + \sin p \cos l \quad . \quad . \quad (2).$$

Therefore, taking the difference between equations (1) and (2),

$$\cos (z - r) - \cos z = \sin p \cos l \,.\, 2 \sin^2 \tfrac{1}{2} h \quad . \quad . \quad (3),$$

for $1 - \cos h = 2 \sin^2 \frac{1}{2} h$.

Also $\cos (z-r) - \cos z = 2 \sin\left(z - \dfrac{r}{2}\right) . \sin \dfrac{r}{2}$;

$$= 2 \sin (z - r) \,.\, \sin \frac{r}{2}, \text{ nearly,}$$

writing $z - r$ for $z - \dfrac{r}{2}$ which may be done with impunity, the quantity $\dfrac{r}{2}$ being very small, as the object is supposed to be near the meridian.

Substituting this in equation (3), and dividing by 2,

$$\sin (z - r) \cdot \sin \frac{r}{2} = \sin p \cdot \cos l \cdot \sin^2 \frac{h}{2},$$

$$\sin \frac{r}{2} = \sin p \cdot \cos l \cdot \operatorname{cosec} (z - r) \cdot \sin^2 \frac{h}{2},$$

whence the *seconds in* $r = \sin p \cdot \cos l \cdot \operatorname{cosec} (z - r) \times \dfrac{2 \sin^2 \frac{1}{2} h}{\sin 1''}$,

nearly, r being a small quantity.

The *reduction* computed from this formula is to be subtracted from the zenith distance of the object at the time of observation, when the object is near its upper transit, or added to the zenith distance deduced from observation near the lower transit, to obtain the zenith distance at the time when the object passes the meridian.

The factor $\dfrac{2 \sin^2 \frac{1}{2} h}{\sin 1''}$ is given in Table XI.

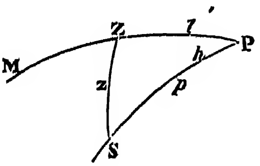
The declination has not been considered as changing, and is, therefore, to be taken out for the *Greenwich date of the observation;* and when several observations are taken, the declination must be taken for the *Greenwich date*, corresponding to the *mean of the times* of observation.

If the meridian altitude be not observed, the meridian zenith distance may be computed, by adding the latitude and declination together, when one is north and the other south, and by taking the difference between the latitude and declination, when they are both north or both south.

To find the Hour Angle or Meridian Distance of any Celestial Object from the observed Altitude.

(1.) In the adjoining figure, let M Z P be the meridian of the observer, Z the zenith, P the elevated pole, S any celestial object, Z S the zenith distance, Z P the colatitude, P S the polar distance of the object $= p$, $\angle$ Z P S the hour angle or meridian distance.

Fig 23.



Let the altitude $= a$, the latitude $= l$, the $\angle$ ZPS $= h$. Then by the spherical triangle ZPS,

$$\cos \text{ZPS} = \frac{\cos z - \cos p \,.\, \cos l'}{\sin p \,.\, \sin l'}$$

$$\therefore \cos h = \frac{\sin a - \cos p \,.\, \sin l}{\sin p \,.\, \cos l} \quad . \quad . \quad . \quad . \quad \text{A.}$$

$$\therefore 1 - \cos h = 1 - \frac{\sin a - \cos p \,.\, \sin l}{\sin p \,.\, \cos l}$$

$$2 \sin^2 \tfrac{1}{2} h = \frac{\sin p \,.\, \cos l + \cos p \,.\, \sin l - \sin a}{\sin p \,.\, \cos l}$$

$$= \frac{\sin (p + l) - \sin a}{\sin p \,.\, \cos l} \quad . \quad . \quad . \quad (1).$$

But $\sin (p + l) - \sin a = 2 \cos \frac{1}{2} (p + l + a) \,.\, \sin \frac{1}{2} (p + l - a)$, and if $\frac{1}{2} (p + l + a)$ be represented by S,

$$2\,\text{S} = p + l + a.$$

$$2\,\text{S} - 2\,a = p + l - a$$

$$\text{S} - a = \tfrac{1}{2} (p + l - a).$$

$$\therefore \sin (p + l) - \sin a = 2 \cos \text{S} \,.\, \sin (\text{S} - a)$$

And this substituted for the numerator in equation (1), dividing by 2 and taking the square root,

$$\sin \textit{half hour angle} = \sqrt{\frac{\cos \text{S} \,.\, \sin (\text{S} - \textit{altitude})}{\cos \textit{lat.} \sin \textit{polar dist.}}}$$

The method of finding the time at any place from the hour angle computed by means of this formula is explained in the practical portion of the course, at page 127.

On the Rising and Setting of Celestial Objects, Twilight, &c.

Resuming the equation (A) of the preceding demonstration,

$$\cos h = \frac{\sin a - \cos p \,.\, \sin l}{\sin p \,.\, \cos l} \quad . \quad . \quad . \quad . \quad (\text{A}).$$

At the time of the rising or setting of any object its altitude is 0, and therefore 0 must be substituted for a in the formula; and if H be taken to represent the value of the hour angle or meridian distance at the time of the rising or setting, formula (A) becomes

$$\cos \text{H} = -\frac{\cos p \,.\, \sin l}{\sin p \,.\, \cos l} = -\cot p \,.\, \tan l \quad . \quad . \quad . \quad (1).$$

The negative sign which is found attached to the second side of

the equation is to be explained in the following manner: if p, or the polar distance, be less than 90°, H, the hour angle, is greater than 90°, and so *cos* H, negative; otherwise, the anomaly is presented of a positive quantity equal to a negative one. And again, if the polar distance p be greater than 90°, *cot* p becomes a negative quantity, and the second side of the equation is then positive, and therefore the hour angle H is then less than 90°, or 6 hrs.

Hence, when the polar distance is *greater* than 90°, the object rises and sets at *less* than 6 hrs. distance from the meridian; and conversely, when the polar distance is less than 90°, the object rises and sets at a greater distance than 6 hrs. from the meridian. From this, it might be inferred that when the polar distance $= 90^\circ$, the meridian distance will be $=$ 6 hrs.; and this is also borne out by the formula (1); for if $p = 90^\circ$, *cot* $p = 0$: hence the second side of the equation vanishes, and therefore also *cos* $\text{H} = 0$, and consequently $\text{H} = 90^\circ$, or 6 hrs.

The polar distance of a celestial object is estimated from the pole of the heavens which is elevated above the horizon of the observer, that is, from the North Pole in north latitude, and from the South Pole in south latitude; and hence the polar distance is less or greater than 90°, according as the object and the zenith of the observer are on the same side or on different sides of the equator; in other words, as the latitude of the observer and the declination of the object are of the same or different denominations.

Thus in north latitude, when the sun has south declination, his north polar distance is greater than 90°, and then the sun will rise and set at less than 6 hrs from the meridian, and therefore remains less than 12 h. above the horizon. So, when in north declination the sun continues visible for more than 12 hrs. When his declination is 0, as it is at the time of the equinoxes, the day (abstracting the effect of refraction which will extend its duration) is 12 h. in length, in whatever latitude the observer may be placed. The day and night are equal at these times at every place on the earth

We may now inquire what effect will be produced by varying the latitude in the same formula: in the first place, if the latitude $l = 0$, then *tan* l also $= 0$, whence *cos* $\text{H} = 0$, and consequently $\text{H} = 90^\circ$, or 6 h., whatever the polar distance may be.

If the object be the sun, then it follows that, to a person situate at the equator, the sun will remain above the horizon 12 h and below the horizon 12 h., or the days and nights are equal at all times of the year, or whatever be the sun's place in the ecliptic Moreover, it appears that at the equator every parallel of declination is bisected by the horizon, for every star remains as long above as below the horizon. This is also accounted for by the fact, that both the poles of the heavens are in the horizon; the elevation of one pole and the depression of the other being always equal to the latitude of the place of the observer, which is in this case *nothing*; hence the horizon will be an hour circle or meridian of the heavens, and is, moreover, the

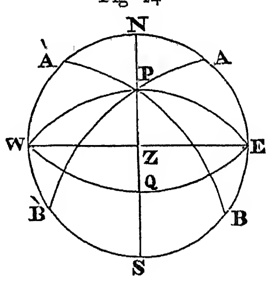
six o'clock hour circle, for it is at right angles to the meridian of the observer.

Lastly, if $cot\ p \cdot tan\ l$ should be greater than 1 (which is always the case when the latitude l exceeds the polar distance p), the value of cos H is impossible, for cos H cannot be greater than 1: and therefore the *hour angle* at rising or setting could not be computed from the formula: and this is the way in which the equation indicates the simple fact, that any star whose polar distance is less than the latitude of the place of observation never meets the horizon of that place, and therefore can never be said to rise or set. This is the case with the stars of the constellation Ursa Major and others viewed in the latitude of London, and with the sun himself in the polar regions. The sun's greatest declination is about $23° 28'$, and therefore his least polar distance about $66° 32'$; and therefore in a higher latitude than this, the sun may be seen to complete his daily course without sinking beneath the horizon.

One of the objects of the preceding remarks has been to show how a concise mathematical formula such as equation (1) embraces all the geometrical features of the problem to which it belongs, and the method of translating such expressions so as to extract from them all the various conditions and forms under which the problems connected with them may be presented.

Some of the results which have been deduced may be illustrated by the annexed diagram, which represents the horizontal projection of the sphere described at page 72. Let P represent the North Pole; N is the north, S the south, E the east, and W the west, points of the horizon; W Q E the equator. The half of the horizon W N E is north of the equator, and the other half W S E is south of the equator; and therefore any object A which rises in the quadrant N E, or sets in N W, must have north declination, and its north polar distance must be less than 90°, but its meridian distance at rising or setting ∠ S P A, or ∠ S P A′, is greater than 90°.

Fig 24



Again, any object rising at B, or setting at B′, has for its polar distance P B, or P B′, each more than 90°, and for its meridian

distance ∠ S P B, or ∠ S P B′, which are each less than 90°; for the six o'clock hour circle W P E is at right angles to the meridian.

EXAMPLE.—Required the time of sunset on May 16th, 1855, when the sun's declination is about 19° 2′ N, in latitude 18° 50′ N, longitude 160° E.

Here the polar distance p = 70° 58′ and l = 18° 50′ N.

$$\cos H = -\cot p \cdot \tan l.$$

	° ′		
p =	70 58	cot	9·537792
l =	18 50	tan	9·532853
	83 15	cos	9·070645
	96 45	*(greater than 90° because p is less)*	
	4		
6,0)	387 0		
	6 h 27 m		

Hence the time of sunset is about 6 h. 27 m apparent time, and the time of sunrise as much before noon, or May 16th, 5 h. 33 m. A.M. apparent time. However, this must only be considered as approximate, for the declination given is that which the sun has at Greenwich apparent noon. But with the time just found and the longitude, the Greenwich date may be nearly ascertained and the declination of the sun taken out more correctly, thus:—

		h m	
Date at place nearly May 16th	.	6 27	
Longitude 160° E	.	10 40	
Greenwich date	. May 15th	19 47	app. time.

Declination. Page I	Equation of Time.
° ′ ″	m. s
18 48 3·4 N	3 53 87 − to app time.
+ 11 38·9	− 42
18 59 42·3	3 53 45
p = 71 00 17·7 N. P D.	

	p =	71° 00′	cot	9·536972
	l =	18 50	tan	9 532853
		83 15	cos	9·069825
		96 45 (greater than 90° because p is less		
		387 00		
		h m s		
Hour angle West	.	6 27 0		
Equation of Time .	.	3 53 45		
	May 16th	6 23 6 55	Mean time of setting	

The time of sunrise on the same day would be about 6 h. 27 m. nearly before noon, or the astronomical date of sunrise would be May 15th, 17 h. 33 m.; the Greenwich date corresponding to this is May 15th, 6 h. 53 m, and for this date the polar distance and equation of time taken out from the "Nautical Almanac," are,

Polar distance, 71° 7′ 53″. Equation of time, 3 m 53·64 s.

	p =	71° 8′	cot	9 533679
	l =	18 50	tan	9 532853
		83 18	cos	9·066532
		96 42		
		h m. s		
		6 26 48		
May 15th .		17 33 12		
Equation of Time .		− 3 54		
May 15th .	.	17 29 18		

Or May 16th, 5 h 29 m. 18 s A M civil date.

It must now be said that the times here computed are those at which the sun's centre is on the rational horizon, not the times of its appearance above or disappearance below the horizon at sea

By reason of refraction, celestial objects appear about 33′ above the horizon when they are really upon it, and the elevation of the eye above the sea on board ship enables us also to see a few minutes of space below the true horizon; for instance, when an object appears in the horizon to a person whose eye is at the level of the sea, it will appear to have an elevation of 3′ 56″ to a person who is raised 16 feet above the sea. Besides, the sun has a disc whose semidiameter is 16′, and consequently, if we understand by its rising the first appearance, and by its setting the vanishing, of the *upper limb*, the circumstances here mentioned will accelerate the first appearance and

retard the final disappearance of the sun by as much time as it will take for him to rise or sink through a space equal to the sum of the dip, refraction, and semidiameter.

§. Supposing the height of the eye to be 16 feet, and therefore the dip 3′ 56″, the semidiameter 16′, the refraction 33′, the sum of these will be 53′ nearly, and when the sun's upper limb just appears at the sea horizon, the sun's centre will really be 53′ below the rational horizon, or will have a depression of 53′.

For a star, the dip and refraction only must be taken into account.

The formula for computing the meridian distance, and thence the time at which a celestial body will have any given depression below the horizon may be readily deduced from what has already been done, for we have only to write $-\ d$ for $+\ a$, when the formula

$$sin^2 \tfrac{1}{2} h = \frac{sin\ (p + l) - sin\ a}{sin\ p\ .\ cos\ l}$$

becomes

$$sin^2 \tfrac{1}{2} h = \frac{sin\ (p + l) + sin\ d}{sin\ p\ .\ cos\ l}$$

and reducing this by the principles of trigonometry, and putting S for $\frac{1}{2}\ (p + l + d)$

$$sin\ \tfrac{1}{2} h = \sqrt{\frac{sin\ S\ .\ cos\ (S - d)}{sin\ p\ .\ cos\ l}}$$

in which d represents the depression below the horizon.

This formula will serve not only to compute more correctly the time of the rising and setting of the heavenly bodies, but also the times of the beginning and ending of twilight, and other interesting problems of the same class.

With respect to the *twilight*, it is generally agreed that the atmosphere ceases to refract light when the sun has sunk 18° below the horizon, hence writing 18° for d in the formula last given, the sun's meridian distance and the time may be found when twilight begins or ends.

As, however, the declination is required in these computations, and therefore a Greenwich date, it is necessary to assume a date: or to compute the time approximately first, with the declination at noon, and then having found the corresponding Greenwich date, correct the declination and recompute the whole. When the object to which the calculation relates is a fixed star, the process is more simple, since the star's declination may be considered as constant; and it will be sufficient to take it out for the day of the month.

Example.—At what time did Altair rise on August 19th, 1855, in latitude 51° 29′ N, longitude 18° 00′ W? Height of eye 18 feet.

Dip	′ ″ 4 11	Declination	° ′ ″ 8 29 24 8 N
Ref.	33 0	North polar distance	81 30 35·2
Actual depression	37 11		

Depression	$d =$	° ′ ″ 0 37 11		
Latitude	$l =$	51 29 0	sec	·205692
Polar distance	$p =$	81 30 35	cosec	·004786
	2)	133 36 46		
	S	66 48 23	sin	9·963400
	$S - d$	66 11 12	cos	9·606122
			2)	19·780000
	½ hour ∠	50 55 6	sin	9·890000
	Hour ∠	101 50 12		
		407 20 48		
	Eastern hour angle	6 h. 47 m 20 8 s.		
	Western hour angle	17 12 39 2		

		h m s
Western hour angle		17 12 39·2
Altair's R A.	add	19 43 44 9
(Rejecting 24 h) R. A of meridian		12 56 24·1
R A of mean sun at Greenwich mean noon		9 49 5·2
Time at place (nearly)	Aug. 19th	3 7 18·9
Longitude in time W.	add	1 12 0 0
Corresponding Greenwich date	Aug. 19th	4 19 18·9

		h. m s
On Aug 19th, at noon, mean ☉'s R A		9 49 5 2
Acceleration for	4 h	39·4
	19 m	3·1
	18·9 s.	·1
		9 49 47 8

		h m. s
R A of meridian		12 56 24·1
R. A of mean ☉		9 49 47 8
Date at place, of star's rising	Aug. 19th	3 6 36·3

To find the Ratio between a Small Error of Altitude and the Error in the Hour Angle computed with the Altitude.

Commencing with the fundamental equation,

$$\cos z = \cos p \,.\, \sin l + \sin p \,.\, \cos l \,.\, \cos h \quad . \quad . \quad . \quad (1).$$

Where z = zenith distance, p polar distance, l latitude, h hour angle.

Then if with a zenith distance $z + x$, the computed hour angle change to $h + y$, where x and y are supposed to be both small, the problem proposed is reduced to the inquiry as to the value of $\frac{x}{y}$, or $\frac{y}{x}$, y being the error of the hour angle, and x the error in the altitude.

And in the first place equation (1) becomes

$$\cos (z + x) = \cos p \,.\, \sin l + \sin p \,.\, \cos l \,.\, \cos (h + y) \quad \ldots . (2).$$

Next expanding $\cos (z + x)$ and $\cos (h + y)$, and noticing that $\cos x$ and $\cos y$ will be very nearly = 1,

$$\cos z - \sin z \,.\, \sin x = \cos p \,.\, \sin l + \sin p \,.\, \cos l \,.\, \cos h - \sin p \,.\, \cos l \,.\, \sin h \quad \sin y \ (3).$$

And now subtracting each side of this equation from the corresponding sides of equation (1),

$$\sin z \,.\, \sin x = \sin p \quad \cos l \,.\, \sin h \,.\, \sin y;$$

$$\therefore \quad \frac{\sin y}{\sin x} = \text{very nearly } \frac{y}{x} = \frac{\sin z}{\sin p \,.\, \cos l \,.\, \sin h} \quad \ldots . (4).$$

But $\qquad \sin p \,.\, \sin h = \sin z \,.\, \sin azimuth$ (see figure, page 258;)

$$\therefore \frac{y}{x} \text{ or } \frac{error\ of\ hour\ angle}{error\ of\ altitude} = \frac{1}{\sin azimuth \,.\, \cos latitude}$$

$$= cosec\ azimuth \,.\, sec\ latitude$$

or $\quad error\ of\ hour\ angle = (error\ of\ alt.) \,.\, cosec\ az.\ sec.\ lat \quad \ldots . (5).$

But the result here obtained can be much more readily deduced by means of the differential calculus, for differentiating equation (1) with respect to z and h,

$$- \sin z = - \sin p \cdot \cos l \cdot \sin h \frac{d\,h}{d\,z},$$

$$\frac{d\,h}{d\,z} = \frac{\sin z}{\sin p \cdot \cos l \quad \sin h},$$

and this equation agrees with that marked (4), $d\,h$ and $d\,z$ here representing the small errors denoted by y and x above, and the reduction to the form (5) being completed as before.

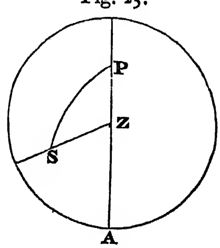
From the equation (5) it may be seen that any small error of the altitude will produce a smaller error in the hour angle, the smaller the factor *cosec azimuth* is. Now *cosec azimuth* is least when the *azimuth* is 90°, for then *cosec azimuth* = 1. When the azimuth is 90°, the object is upon the prime vertical, and bears E or W, and we learn that the nearer that any object is to the prime vertical at the time of observation the better will that observation be for the computation of time.

Method of Computing the Azimuth.

1. *With the Altitude.*

Let A Z P be the meridian of the place of observation, P the elevated pole, Z the zenith, S the object; then A Z S is the azimuth from the north in south latitude, or from the south in north latitude.

Fig. 25.



$$\text{Now } \cos S Z P = - \cos A Z S,$$

and therefore

$$- \cos A Z S = \frac{\cos p - \cos z \cdot \cos l'}{\sin z \cdot \sin l'}.$$

Where $p = P\,S$, $z = Z\,S$, and $l' = P\,Z$.

Or if a = the altitude of S; and l the latitude of the place of observation,

$$-\cos AZS = \frac{\cos p - \sin a \,.\, \sin l}{\cos a \,.\, \cos l};$$

$$\therefore\ 1 - \cos AZS = \frac{\cos a \,.\, \cos l + \cos p - \sin a \,.\, \sin l}{\cos a \,.\, \cos l}$$

But $1 - \cos AZS = 2 \sin^2 \tfrac{1}{2} AZS$,

and $\cos (a + l) = \cos a \cos l - \sin a \sin l$

$$\therefore\ 2 \sin^2 \tfrac{1}{2} AZS = \frac{\cos (a + l) + \cos p}{\cos a \,.\, \cos l} \quad . \quad . \quad . \quad . \quad (1).$$

Again $\cos (a + l) + \cos p - 2 \cos \tfrac{1}{2} (a + l + p) \,.\, \cos \tfrac{1}{2} (a + l - p)$,

and if $2S = a + l + p$; $2S - 2p = a + l - p$,

and $S = \tfrac{1}{2} (a + l + p)$; $S - p = \tfrac{1}{2} (a + l - p)$;

Therefore $\cos (a + l) + \cos p = 2 \cos S \,.\, \cos (S - p)$; and this substituted for the numerator of the fraction on the second side of equation (1), gives

$$Sin\ \tfrac{1}{2} AZS = \sqrt{\frac{\cos S \,.\, \cos (S - polar\ dist.)}{\cos alt \,.\, \cos lat}} \quad . \quad . \quad (2)$$

2. When the altitudes are taken in the morning or evening for time, if the bearing of the sun be also observed, then after the hour angle P is computed, the azimuth may be computed thus:—

$$\frac{Sin\ (AZS\ or\ SZP)}{\sin P} = \frac{\sin PS}{\sin ZS};$$

$$\therefore\ Sin\ (AZS\ or\ SZP) = \frac{\sin p \,.\, \sin P}{\sin z}$$

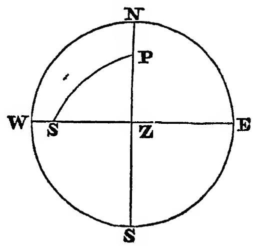
$$= \frac{\sin polar\ dist \times \sin hour\ angle}{\cos alt.}$$

3. When the object is near the prime vertical, some doubt may exist as to whether the azimuth A Z S computed from this formula is greater or less than 90°. This doubt may be resolved by computing the altitude of the object when on the prime vertical, and if this be less than the true altitude deduced from the observation, the azimuth will be less than 90°; but if the altitude deduced from the observation be less than the altitude on the prime vertical, the azimuth will be greater than 90°.

4. *To compute the Altitude of the Object when on the prime vertical.*

Let S be the position of the object on the prime vertical, then PS is its polar distance, WS its altitude; and in the right-angled triangle PZS.

Fig 26.



$$Cos\ SZ = \frac{cos\ SP}{cos\ PZ},\ \text{or}\ sin\ WS = \frac{cos\ SP}{sin\ PN}.$$

PN is the elevation of the pole = latitude of place.

$$\therefore\ sin\ (alt.) = \frac{cos\ polar\ distance}{sin\ latitude},$$

$$\text{or}\ sin\ (\pm\ alt.) = \frac{sin\ declination}{sin\ latitude}.$$

The sign + to be taken when the declination and latitude are both N or both S, and the sign − (signifying depression below the horizon), when the declination and latitude are of contrary denominations.

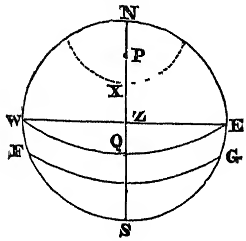
5. When the declination is greater than the latitude,

$$\frac{sin\ declination}{sin\ latitude}\ \text{is greater than 1},$$

and, therefore, there cannot be found any altitudes on the prime vertical; in other words, the object does not pass the prime vertical at all.

This analytical result may be graphically illustrated by the annexed figure: NESW again represents the horizon, WZE, the prime vertical: WQE, the equator.

Fig 27.



ZQ is the latitude; XQ, the declination of an object at X, is greater than the latitude ZQ. The object X is carried by the diurnal rotation round the dotted circle, and therefore never meets the prime vertical at all

Again, the circle FG represents a parallel of declination on the other side of the equator from the elevated pole, and a body moving in this circle rises at G and sets at F, and does not meet the prime vertical *above* the horizon N E S W.

The sun at the equinoxes is in the equator W Q E, and then rises at E the east point, and sets at W the west point of the horizon.

If another parallel of declination were drawn cutting the meridian between Z and Q, it will be readily seen that such a parallel would cut the prime vertical W Z E, in the diagram, which represents the visible hemisphere above the horizon; and therefore any object apparently moving on such a parallel might be *observed* on the prime vertical. The declination of this parallel would be less than ZQ; that is, less than the latitude of the place of observation; and hence this is the test of the visibility of a celestial object on the prime vertical. *The declination must be less than the latitude, and the latitude and declination must be of the same denomination.*

In the example at page 129, the altitude = 49° 10′, latitude 32° 40′ S, polar distance 67° 21′, and computed hour angle = 45° 3′.

	°	′		
Hour angle . .	45	3	sin	9·849864
Polar distance .	67	21	sin	9 965143
				9·815007
Altitude	49	10	cos	9·815485
	87	19	sin	9·999522
or	92	41		

Now one of these is the azimuth from the north, and the other from the south point of the horizon. But which is the azimuth from the N: the acute angle 87° 19′, or the obtuse angle 92° 41′?

	°	′		
Polar distance	67	21	cos	9·585574 = sin dec
Latitude	32	40	sin	9·732193
Altitude on prime vertical	45	31	sin	9·853381
Altitude at time of obs. .	49	10		

The sun is therefore past the prime vertical, and the azimuth N 87° 19′ E.

No doubt can arise when using the formula (2) at page 265, and therefore in cases like the one here given it is to be preferred.

Method of Computing the Azimuth without the Altitude.

1. With the time at place and Greenwich date the meridian distance of the object can be found, and then the *polar distance, latitude,* and *meridian distance* are the data with which the computation is to be made.

2. See figure, page 254, SP, PZ, and the contained angle P are given; therefore by Napier's analogies,

$$\tan \tfrac{1}{2}(Z - S) = \sin \tfrac{1}{2}(SP - PZ) \,.\, \operatorname{cosec} \tfrac{1}{2}(SP + PZ) \,.\, \cot \tfrac{1}{2} P.$$

$$\tan \tfrac{1}{2}(Z + S) = \cos \tfrac{1}{2}(SP - PZ) \,.\, \sec \tfrac{1}{2}(SP + PZ) \,.\, \cot \tfrac{1}{2} P.$$

Whence Z and S can be found; noticing that the greater angle is opposite to the greater side; that Z is the azimuth from the north in north latitude, and from the south in south latitude; and that $\frac{1}{2}$ (Z + S) is always of the same affection as $\frac{1}{2}$ (SP + PZ).

To Compute the Amplitude of a Heavenly Body.

The amplitude is the angular distance of any celestial body from the east point when rising, or from the west point when setting. The amplitude is therefore the complement of its azimuth at these times.

Now referring to the investigation at page 265,

$$- \cos AZS = \frac{\cos p - \sin a \,.\, \sin l}{\cos a \,.\, \cos l};$$

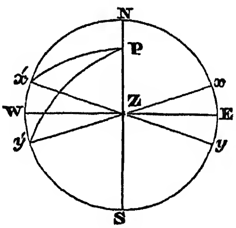
But at the rising or setting of the body the altitude $(a) = 0$, and therefore $\sin a = 0$ and $\cos a = 1$, and therefore if A represent the azimuth at this time,

$$- \cos A = \frac{\cos p}{\cos l} = \cos p \,.\, \sec l \quad . \quad . \quad . \quad (1).$$

The negative sign indicates that when the polar distance is greater than 90°, the azimuth at rising or setting is less than 90°; and conversely, when p is less, A is greater than 90°. Therefore in north

latitude, when the declination of the object is north, the object rises in the N E and sets in the N W quarter of the horizon, as at x and x', and in north latitude, when the declination of the object is south, the

Fig. 28.



object rises and sets in the S E and S W quarter respectively as at y and y'. Therefore the amplitude E y, Wy', E x, Wx', will be northward or southward of E or W according as the declination is N or S.

Again, if $p = 90°$, since $\cos p$ will then $= 0$, so also $\cos A = 0$, and A $= 90°$, or the object will rise and set at the east and west points the horizon respectively.

Now A = the complement of the amplitude, and therefore by equation (1),

$$\textit{Sin amplitude} = \textit{sin declination} \times \textit{sec lat.}$$

or in the figure, since P y' = the polar distance, and P N the elevation of the pole or the latitude of the place of observation, and N P y' is a right-angled spherical triangle,

$$\cos N y' = \frac{\cos P\, y'}{\cos P N}$$

$$\sin W y' = \cos P\, y' \,.\, \sec P N,$$

or,

$$\textit{sin amplitude} = \textit{sin declination} \times \textit{sec latitude.}$$

METHOD OF FINDING THE LATITUDE AND LONGITUDE BY MEANS OF TWO ALTITUDES, AND THE CORRECT GREENWICH DATE DEDUCED FROM A CHRONOMETER.

About thirty-five years ago the late Mr. Ivory gave, in the "Philosophical Magazine," a solution of the double Altitude Problem, showing how both the latitude and middle time at the place might be computed from the altitudes, elapsed time, and the mean of the decli-

nations at the times of observation; and he gave also the first term of the correction of both for the change of the sun's declination in the interval. In a subsequent number of the same work, the problem for latitude was much simplified by my father, and in the form in which it was put by him, it has since been published in many works on Nautical Astronomy, both in this country and elsewhere.

About midsummer, 1849, it occurred to my father, that besides the latitude, to the finding of which the problem had been exclusively applied in practice, the longitude might readily be determined by it, if the times were marked by a chronometer, whose error for Greenwich time was known; and he proposed to effect this by comparing the middle time at Greenwich, deduced from the chronometer, with the computed middle time at the place of observation.

Having taken much interest in this extension of the problem, and daily witnessed the successful application of it by the boys of the Nautical School, I was induced to examine whether the first term of the *corrections* just referred to would give results which would satisfy the requirements both of theory and practice; and I found that for *latitude* the first term is sufficient for this purpose, but with respect to the middle time, the second term of the correction may have a value which ought not to be neglected. The investigation of these corrections which follows was first given in a paper printed in 1850. —J. R.

METHOD OF DOUBLE ALTITUDES OF THE SUN.

Let A be the position of the sun at the first, and B its place at the second observation; P A and P B, the polar distances are considered as equal to each other, and in the practical solution of the problem are

Fig. 29

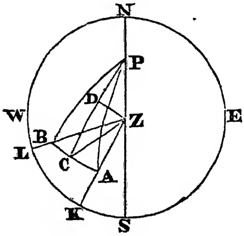
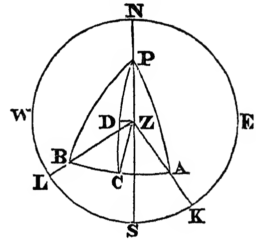


Fig 30.



taken out for the middle time; thus one is always greater, and the other less than the truth. A B is an arc of a great circle joining the points A and B. P C is drawn bisecting the angle B P A, and it therefore bisects the arc B A also; and Z D is drawn perpendicular to P C.

A P B is the measure of the elapsed apparent time, A P C or B P C half the elapsed apparent time.

A Z and B Z are the true zenith distances, A K and B L the true altitudes, Z P the colatitude, P N the latitude.

In the following demonstration the polar distance is denoted by the letter p, the angle A P C by H, half the sum and half the difference of the altitudes by S and D respectively.

And the arcs A C, C P, Z D, C D, and D P by the letters **A**, **E**, **I**, **O**, and (**E** − **O**), and they are found in the order here mentioned.

1. In the triangle A P C, $\sin AC = \sin AP \,.\, \sin APC$,

or, $\sin \mathbf{A} = \sin p \,.\, \sin H$ . . . (1).

2. In the same triangle, $\cos AP = \cos AC \,.\, \cos PC$;

$\therefore \cos PC = \cos AP \,.\, \sec AC$,

or $\cos \mathbf{E} = \cos p \,.\, \sec \mathbf{A}$ . . . (2).

3. The third and fourth arcs, **I** and **O**, require a little more discussion.

Since P C A and P C B are right angles

$$\therefore \cos ZCA = \sin ZCD \text{ and } \cos ZCB = -\sin ZCD.$$

And therefore in the spherical triangles Z C A and Z C B

$$\cos ZA \,(=\sin AK) = \cos ZC \,.\, \cos CA + \sin ZC \,.\, \sin CA \,.\, \sin ZCD \quad . \; . \quad (3)$$

and $\cos ZB \,(=\sin BL) = \cos ZC \,.\, \cos CB - \sin ZC \,.\, \sin CB \,.\, \sin ZCD$. (4)

Therefore by subtraction, observing that C B = C A,

$$\sin AK - \sin BL = 2 \sin ZC \,.\, \sin CA \,.\, \sin ZCD \quad . \; . \quad (5).$$

But $\sin ZC \,.\, \sin ZCD = \sin ZD$, and by substituting this in the second side of equation (5),

$$\sin AK - \sin BL = 2 \sin CA \,.\, \sin ZD;$$

$$\therefore \sin ZD = \frac{\sin AK - \sin BL}{2 \sin CA} \quad . \; . \; . \; . \quad (6).$$

And expanding the numerator of the fraction, and denoting $\frac{1}{2}(AK + BL)$ by S, and $\frac{1}{2}(AK - BL)$ by D

$$\sin ZD = \frac{\cos S \,.\, \sin D}{\sin CA},$$

$$\text{or} \quad \sin \mathbf{I} = \cos S \,.\, \sin D \,.\, \operatorname{cosec} \mathbf{A} \quad . \; . \; . \quad (7)$$

which is the formula for computing the third arc **I**.

4. Taking the sum of (3) and (4),

$$\sin AK + \sin BL = 2 \cos ZC \,.\, \cos CA\,;$$

but $\cos ZC = \cos CD \,.\, \cos DZ$,

and therefore $\sin AK + \sin BL = 2 \cos CD \,.\, \cos DZ \,.\, \cos CA$,

$$\text{and} \qquad \cos CD = \frac{\sin AK + \sin BL}{2 \cos DZ \,.\, \cos CA};$$

And again expanding the numerator, and proceeding as with equation (6),

$$\cos CD = \frac{\sin S \,.\, \cos D}{\cos \mathsf{l} \,.\, \cos A};$$

l and A being put for DZ and CA as agreed upon at the outset. CD is also to be denoted by O.

$$\therefore \cos \mathsf{O} = \sin S \,.\, \cos D \,.\, \sec \mathsf{A} \,.\, \sec \mathsf{l} \quad . \quad . \quad (8).$$

5. And in the case illustrated by the two diagrams,

$$PD = PC - CD = (\mathsf{E} - \mathsf{O})$$

$$\text{and } \cos PZ = \sin \text{lat} = \cos PD \,.\, \cos DZ$$

$$= \cos (\mathsf{E} - \mathsf{O}) \,.\, \cos \mathsf{l} \quad . \quad . \quad (9).$$

6. The formulæ numbered 1, 2, 7, 8, 9, are those given in the rules at page 173, for the practical solution of the problem, and they enable us to compute the latitude (nearly).

7. For the computation of the *middle hour angle*, ZP and DZ are known;

$$\therefore \sin DZ = \sin ZP \,.\, \sin DPZ,$$

$$\text{or } \sin \mathsf{l} = \cos \text{lat} \,.\, \sin H',$$

where $H' = \angle DPZ$, the hour angle at the middle time;

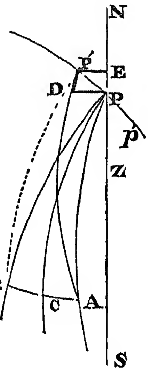
$$\therefore \sin H' = \sin \mathsf{l} \;\; \sec \text{lat} \;\; . \; . \; . \; . \; . \; . \; . \; . \quad (10).$$

Which is the formula given for the computation at page 174.

If the object observed be the sun, the latitude and middle time will both require to be corrected for the false assumption of the equality of the polar distances at the times of observation.

Let A P and B P represent the equal polar distances which have been employed in the previous calculation, and let A P′ and B P′ be the actual polar distances at the times of observation, the corrected place of the pole will be P′ or p, according as A P′ is greater or less

Fig 31.



than B P′. P P′ joining the mean place of the pole P to its true place P′ may also be considered as a straight line at right angles to the bisector P C.

Make A D = A P, then D P′ = error in each polar distance used in the calculation, the one, A P, being as much less than A P′ as the other, B P, exceeds B P′. Hence D P′ is half the difference between A P′ and B P′, or half the change of the polar distance in the time elapsed between the observations, and can be computed thus—

$$c = (\textit{diff. of declination in 1 hour}) \times \tfrac{1}{2}\,(\textit{elapsed time, in hours}) \quad (1).$$

If P′ E be drawn perpendicular to the meridian, the distance of E from P will measure how much the corrected position of the pole, P′, is to the northward or southward of its place found by assuming the polar distances equal: in the case chosen for illustration, P′ is north of P by the quantity P E, and, therefore, if N represent the north point of the horizon, N P will be the first computed latitude, and N E the corrected latitude.

Join D P, which may be considered as perpendicular both to A P and A P′.

$$\therefore \angle\, CPP' = \angle\, APD;$$
and omitting $\angle$ C P D from each of these angles,
$$\angle\, APC = \angle\, DPP';$$

but A P C is the measure of the half-elapsed time,

and, therefore, D P P′; is also = half-elapsed time in arc = H. . (2).

The angle Z P C, between the meridian P Z and the middle-hour circle P C, is the hour angle at the middle time, or middle-hour angle = H′;

$$\begin{aligned}
\text{But } \angle ZPC + \angle CPP' + \angle P'PE &= 180^\circ \\
\text{and } \qquad \angle CPP' &= 90^\circ \\
\therefore \angle ZPC + \angle P'PE &= 90^\circ \\
\text{and } \angle PP'E + \angle P'PE &= 90^\circ \\
\therefore \angle ZPC &= \angle PP'E
\end{aligned}$$

Consequently $\angle PP'E$ also $= H'$ . . . . (3).

By the right-angled triangle D P P′,

$$PP' = DP' \,.\, cosec \angle DPP' = c \,.\, cosec\ H.$$

And by the right-angled triangle P′ P E,

$$PE = PP' \,.\, sin \angle PP'E;$$

But $\angle PP'E = H'$ by equation (3) and $PP' = c \,.\, cosec\ H$, as has just been shown;

$\therefore$ *Correction of computed lat.* $= c \,.\, cosec\ H \,.\, sin\ H' = C$. (4).

1. In the case to which the diagram applies, the sun is further from the meridian at the second observation than the first; and, therefore, the second altitude is *less* than the first

2. Also the polar distance B P′, at the time of the second observation, is *less* than A P′, the sun's polar distance at the time of the first observation; the sun is, therefore, advancing towards the pole of that hemisphere in which the observer is situate, and therefore the days are increasing in length.

3. In the third place, since the corrected latitude, N E, is less than that which has been found with the equal polar distances, P N, the correction P E is *subtractive* from P N.

Hence the Rule. (1) *If the second altitude be less than the first, and* (2) *the days be increasing in length, the correction of the computed latitude is subtractive.*

If either of the conditions (1) or (2) be changed, the correction becomes additive; but if both be changed, it is still subtractive.

The method adopted to investigate the effect of the erroneous elements upon the computed value of H′, the hour angle at the middle time, is briefly this: the effect upon the hour angle at each observation is represented both with respect to the error in the computed latitude

and the error in the polar distance of the sun; the expressions are then summed up with due regard to the signs, and half the result is the measure of the error on the computed hour angle at the middle time. For the sake of completing the subject, the investigation is appended; but the learner had better omit the reading of it until he has mastered the article upon equal altitudes, where some of the principal formulæ here used are explained

Let the colatitude be denoted by l, the mean of the polar distances by P, the hour angles by h and h', and the change of the declination in the half-elapsed time by c; and let the correction of latitude determined by formula (4) be denoted by C.

Now as A P′ is greater than A P by the small quantity c, and Z P′ greater than Z P by the small quantity C, the angle Z P′A will be less than Z P A by the quantities

$$e_1 = c \cdot \frac{\cot l}{\sin h} - c \cdot \frac{\cot p}{\tan h}$$

$$\text{and } e_2 = \mathrm{C} \cdot \frac{\cot p}{\sin h} - \mathrm{C} \cdot \frac{\cot l}{\tan h}$$

which are the measures respectively of the effects of the small differences c and C.

And similarly the corrections on the other hour angle h' are

$$e_3 = c \cdot \frac{\cot l}{\sin h'} - c \cdot \frac{\cot p}{\tan h'}$$

$$\text{and } e_4 = \mathrm{C} \cdot \frac{\cot p}{\sin h'} - \mathrm{C} \cdot \frac{\cot l}{\tan h'}$$

e_3 being additive to Z P B, and e_4 subtractive. Hence the total effect of these corrections on the half sum of Z P A and Z P B, or on the computed middle time (H′), will be expressed by

$$\tfrac{1}{2}(-e_1 - e_2 + e_3 - e_4) = -\tfrac{1}{2}(e_1 - e_3) - \tfrac{1}{2}(e_2 + e_4) = x.$$

And representing, as before, the middle time and half-elapsed time by H′ and H, easy reductions give

$$-\tfrac{1}{2}(e_1 - e_3) = -c \cdot \sin \mathrm{H} \cdot \frac{\cot l \cdot \cos \mathrm{H}' - \cot p \cdot \cos \mathrm{H}}{\sin h \cdot \sin h'}$$

$$-\tfrac{1}{2}(e_2 + e_4) = \mathrm{C} \cdot \sin \mathrm{H}' \cdot \frac{\cot l \cdot \cos \mathrm{H}' - \cot p \cdot \cos \mathrm{H}}{\sin h \cdot \sin h'};$$

and substituting for C its value $\frac{c \,.\, sin\, H'}{sin\, H}$, formula (4), and adding,

$$x = c \,.\, \frac{sin^2 H' - sin^2 H}{sin\, H} \,.\, \frac{cot\, l \,.\, cos\, H' - cot\, p \;\; cos\, H}{sin\, h \,.\, sin\, h'},$$

But $sin^2 H' - sin^2 H = (sin\, H' + sin\, H) \,.\, (sin\, H' - sin\, H)$

$$= sin\,(H' + H) \,.\, sin\,(H' - H) = sin\, h \,.\, sin\, h'\,;$$

And, therefore, putting l' for the latitude,

$$x = c \,.\, tan\, l' \,.\, cos\, H' \,.\, cosec\, H - c \,.\, cot\, p \,.\, cot\, H \quad \ldots (5),$$

which is therefore the quantity to be added to the computed middle time to reduce it to the true middle time when the days are lengthening, but to be subtracted when the days are shortening.

It is to be observed, however, that the sign of the second term becomes *plus* when the polar distance is greater than 90°.

It is worthy of remark that when the computed middle time (H′) = 0 or 24 hours, as in the case of equal altitudes east and west of the meridian, the value of x is reduced to

$$c \,.\, tan\, l' \,.\, cosec\, H - c \,.\, cot\, p \,.\, cot\, H,$$

which is the well-known formula for the equation of equal altitudes, and that problem is thus shown to be only a particular case of the more general one we have here been discussing, and hence the formula (5) would not be inappropriately named the equation of the middle time.

The first term in the value of x can be put into a form more convenient for calculation,

$$C = c \,.\, cosec\, H \,.\, sin\, H'\,;$$
$$\therefore\; c = C \,.\, cosec\, H' \,.\, sin\, H\,;$$

and this value of c being substituted in the first term of equation (5),

$$x = C \,.\, tan\, latitude \,.\, cot\, H' - c \,.\, cot\, polar\ distance \,.\, cot\, H.$$

The Direct Method of Double Altitudes.

In this method, as was stated in the practical part of this Treatise, two observations are requisite, and it is a matter of indifference whether the same object be observed twice, allowing some time to elapse between the observations, or the altitudes of different objects be employed, pro-

vided that the places occupied by the sun or stars, at the two epochs differ sufficiently, and not too much, in azimuth. Say from 60° to 120°.

The data used in the calculation are then the zenith distances, polar distances; and the polar angle, included between the polar distances, or circles of declination, which pass over the *points of the sky* which were occupied by the objects when their altitudes were taken.

The zenith distances are deduced from the observed altitudes, and the polar distances are obtained with the aid of a Greenwich date, from the "Nautical Almanac."

The method of finding the *polar angle* varies with the character of the observation, and must be discussed under the following heads:—

1. When the sun is the only object observed
2. When two altitudes of the *same* star are taken.
3. When the altitudes of two stars are taken at the same instant by two observers
4. When the altitudes of two stars are taken by the same observer, who must, of course, allow some time, however short, to elapse between his observations.

For the first of these cases, let A and B, in the first of the figures at page 270, represent the positions occupied by the sun at the times of observation; then, as the westerly meridian distance of the sun is the measure of the apparent time, the angles Z P A and Z P B are the measures of the apparent times of observation at the meridian N P Z S; and the polar angle A P B, which is the difference of those times, must be found by converting the elapsed time shown by the chronometer, *first*, into mean time, by allowing a proportional part of the daily *rate*, and *then* into apparent time, by taking account of the change which takes place in the equation of time during the time elapsed between the observations.

For the next case, viz., when the same star is observed twice. Time enough should be permitted to elapse to allow the bearing of the star to change about 90°. The star performs a complete revolution in a sidereal day, and, therefore, if A and B in the figure now represent the two positions of the star when observed, the angle A P B will be the same part of a sidereal day as the interval of mean time shown by the chronometer is of a mean solar day. And, consequently, all that remains to be done is to convert the interval of mean solar time into the equivalent interval of sidereal time, and the result is the measure of the angle A P B. Ivory's method may also be used with advantage, and will not require any correction.

In the third place, if A and B represent different stars at the same instant of time, the angle included between their declination circles is equal to the difference of their right ascensions.

And for the fourth case, that of two stars, when the altitudes are taken by a single observer. Having taken one star, some time must necessarily elapse before the arrangements are completed for getting the altitude of the second star, and, during this time, the first star has

changed *its place in the visible hemisphere of the sky*, by reason of the general westward movement of the stars, and the *point of the sky* which was occupied by that star, is now to the eastward of it, and has consequently, a greater right ascension than the star by just so much sidereal time as corresponds to the star's westward movement. Hence the precept—*add the sidereal interval to the right ascension of the star first observed*, gives the right ascension of the *point of the sky* whose altitude was taken, for the same instant of time at which the second star is observed And then the difference between this corrected right ascension and the right ascension of the second star will be the polar angle.

The angle A P B having been thus determined. In the triangle, A P B, with A P, P B, and angle A P B, find A B and the angle P B A.

In the triangle Z B A, with Z B, Z A, and A B, find the angle Z B A, the difference between ∠ Z B A and ∠ P B A is ∠ P B Z, when, as in the figure, the great circle A B does not pass between P and Z, and the other case may be avoided by a proper selection of stars.

And in the triangle P B Z, with P B, B Z, and the angle P B Z, find P Z the colatitude of the place of observation.

Of Equal Altitudes.

1. If the times be noted when a fixed star has equal altitudes, east and west of the meridian, on a given night, then, as the elements with which the meridian distance of the star would be computed, are the same in both cases, the star will be equidistant from the meridian at the times of observation, and, therefore, on the meridian at the middle time.

2. If the actual time of the star's transit be computed, and compared with the mean of the times shown by a chronometer, when the star has equal altitudes, the error of the chronometer will be known

3. But if equal altitudes of the sun be taken, when he is east and when he is west of the meridian, on the same day, the meridian distances will not be equal, on account of the change of the sun's declination in the interval of time between the observations, which will retard or accelerate the time of his second arrival at any given altitude which he was observed to have before noon

4. If the times at which the sun has equal altitudes be noted, it follows, that the middle time will not coincide with that shown by the chronometer when the sun passed the meridian: and the *correction* to be applied to the middle time, to obtain the time by the chronometer of the sun's transit, is called the *equation of equal altitudes*.

5. The middle time will be later or earlier than the time of transit, according as the western hour angle is greater or less than the eastern hour angle.

6. We will now proceed to a closer examination of the problem. Let h and h' represent the eastern and western hour angles (reckoned in time) when the sun had equal altitudes upon a certain day, then (24 *hours* $- h$) and (24 *hours* $+ h'$) will also represent the apparent times of observation, and their mean $\left(24\, hours + \frac{h' - h}{2}\right)$ will exceed the true time of noon by $\frac{h' - h}{2}$, or by half the difference of the hour angles, provided h' be greater than h, or the westward hour angle be the greatest. But the expression for the mean may also be written thus—

$$24\, hours - \frac{h - h'}{2},$$

if the eastward hour angle be the greatest; and in this case the middle time is before the time of noon by half the difference of the hour angles, or $\frac{h - h'}{2}$.

Half the difference of the hour angles is called the *equation of equal altitudes.*

7. We will next proceed to investigate the expression for the ratio of the corresponding small differences in the polar distances and in the hour angles of the sun. Commencing with the formula,

$$\cos z = \cos p \cos l' + \sin p \sin l' \cos h \quad . \quad . \quad . \quad . \quad . \quad (1),$$

z, p, l', h, being respectively the zenith distance of the sun, his polar distance, the colatitude of the place of observation, and the hour angle or meridian distance of the sun.

Or if a and l be taken to represent the altitude of the sun and the latitude of the place,

$$\sin a = \cos p \sin l + \sin p \cos l \cos h \quad . \quad . \quad . \quad . \quad . \quad (2).$$

And from this it is required to find what change takes place in h for a given small change in p. And as, in general, h is less when p is greater, suppose $p + c$ and $h - e$ to be corresponding values of the polar distance and hour angle, differing from p and h by small quantities denoted by $+ c$ and $- e$.

$p + c$ and $h - e$ being substituted for p and h in equation (2),

$$\sin a = \cos (p + c) \sin l + \sin (p + c) \cos l \cos (h - e) \quad . \quad . \quad . \quad (3).$$

This, being now expanded, is reduced to a simple form by observing that $\cos c$ and $\cos e$ may each be considered $= 1$, and that the product of the small fractions $\sin c$ and $\sin e$ may be neglected altogether.

This done, and the result subtracted from the corresponding sides of equation (2), it will be found that

$$\sin p \,.\, \cos l \,.\, \sin h \times \sin e = \sin p \quad \sin l \times \sin e - \cos p \,.\, \cos h \,.\, \cos l \times \sin c.$$

Whence, by division,

$$\sin e = \sin c \,.\, \left(\frac{\tan l}{\sin h} - \frac{\cot p}{\tan h}\right);$$

or, since e and c are small,

$$e = c \,.\, \left(\frac{\tan l}{\sin h} - \frac{\cot p}{\tan h}\right) \quad . \quad . \quad . \quad (4).$$

Or separating the two terms,

$$\left.\begin{array}{l} A = c \,.\, \tan latitude \,.\, cosec\ h \\ B = c \,.\, \tan declination \,.\, \cot\ h \end{array}\right\} \quad . \quad . \quad (5),$$

and $A \underline{+} B = e$.

8. If c represent half the change of the sun's polar distance between the morning and afternoon observations when the sun has equal altitudes, and h half the interval of time elapsed, then e is half the difference between the morning and afternoon hour angles caused by the change of the polar distance; or e is half the difference between h and h'. In the investigation it has been assumed that the western hour angle is diminished when the polar distance is increased, and therefore e is to be added (using the word in its *algebraic* sense) to the middle time when the polar distance is increasing, and to be subtracted from it when the polar distance is decreasing, to find the time shown by the chronometer at noon.

9 When the polar distance exceeds 90°, the sign of the second term of equation (4) becomes positive, and hence the rule, *add the parts A and B when the declination and latitude are of contrary denomination, and take the difference when they are of the same denomination.*

10. On formula (5) it may be remarked that cosec h always exceeds cot h, and therefore, whenever the latitude equals or exceeds the declination, A is greater than B

When the sun's declination = 0, B = 0, and when the latitude = 0, A = 0.

In high latitudes, A becomes considerable and the results less and less to be relied upon; indeed, the best method is to compute the hour angles separately with the correct polar distances, and then deduce the error of the chronometer from each observation, and if the observations be made when the sun is nearly equidistant from the meridian, without the limitation of exact equality, the mean of the errors will be very nearly the error at noon.

For there is no instrument which can be called a good equal altitude instrument. The change of temperature affects not only the

refractions, but the very form of the sextant, and therefore the equality of the apparent altitudes cannot be accepted as a proof of the equality of the true altitudes.

No observation should be taken without determining the index error; and curious fluctuations in its amount will be found at different temperatures.

11. The error in the hour angle resulting from a small error in the latitude with which it is computed may be investigated in the same manner, or it may be deduced directly from expression (4) by writing polar distance for colatitude, and colatitude for polar distance, from whence results

$$e' = c' \cdot \left(\frac{cot\ p}{sin\ h} - \frac{tan\ l}{tan\ h}\right) \quad . \quad . \quad . \quad (6),$$

e' and c' representing the errors in the hour angle and latitude respectively. It will be seen that these expressions, (4) and (6), are used in the preceding investigation of the correction to be applied to the computed middle time in Ivory's "Method of Double Altitudes."

It is evident that the error e' vanishes when

$$\frac{cot\ p}{sin\ h} = \frac{tan\ l}{tan\ h};$$

or reducing this to its simplest form,

$$cot\ p = cos\ h \cdot tan\ l;$$

a condition which only holds when the azimuth or $\angle$ Z (see figure, page 264) is 90°. Therefore when the object is upon the prime vertical, the error of the hour angle arising from any small error in the latitude may in general be considered as evanescent; and in addition to this, it has been shown that at this time any small error in the observed altitude vitiates less than at any other time the hour angle or time computed with it.

12. *Having observed the altitude of an object when it is east of the meridian, to find the time nearly when the corresponding equal altitude may be observed, the star being then west of the meridian.*

Let T = mean time of the transit of the object, and E = error of the chronometer on mean time (supposed fast), then T + E = the time by chronometer, of the star's transit.

Let T′ = chronometer time of the first observation, then T + E − T′ = the hour angle of the object nearly, and this added to the chronometer time of transit, will give the time by chronometer nearly, of the second observation, which is therefore

$$2\,T + 2\,E - T'.$$

Observing that the hours are supposed to be counted continuously from T′ to T, and that if the result of the final formula exceed 12 hours, 12 hours must be deducted, in order to find the time which the chronometer will show.

If the chronometer be slow, the formula is

$$2\,T - 2\,E - T'.$$

Investigation of the Second Method of computing the Equation of Equal Altitudes.

By equation (4) of the preceding article,

$$e = c \cdot \left\{\frac{\tan l}{\sin p} - \frac{\cot p}{\tan h}\right\}.$$

Where l = latitude, p = polar distance of the sun, and h = the meridian distance.

Let the colatitude $= q$,

$$\text{Then } \frac{\tan l}{\sin h} - \frac{\cot p}{\tan h} = \frac{\cot q}{\sin h} - \frac{\cot p}{\tan h}$$

$$= \frac{\cot q \cdot \sin p - \cos p \cdot \cos h}{\sin p \cdot \sin h}$$

$$= \frac{\cot S}{\sin p}$$

$$\therefore e = c \cdot \frac{\cot S}{\sin p} = \frac{c \cdot \cos S}{\sin S \cdot \sin p} \quad . \quad . \quad (1).$$

Where S = the angle between the zenith distance and the polar distance of the sun.

Again, we have

$$\cos z = \cos p \cdot \cos q + \sin p \cdot \sin q \cdot \cos h.$$

And considering z and h, or the altitude and time only to change, during the short period employed in making the A.M. or P.M. observations,

$$\cos z' = \cos p \cdot \cos q + \sin p \cdot \sin q \cdot \cos h'$$

$$\therefore \cos z - \cos z' = \sin p \cdot \sin q \quad (\cos h - \cos h')$$

$$2 \sin \frac{z' + z}{2} \cdot \sin \frac{z' - z}{2} = \sin p \cdot \sin q \cdot 2 \cdot \sin \frac{h' + h}{2} \cdot \sin \frac{h' - h}{2}.$$

$$\text{or} \qquad a \cdot \sin z = t \cdot \sin p \cdot \sin q \cdot \sin h, \text{ very nearly.}$$

Which is obtained by writing a and t for $z' - z$ and $h' - h$; and z and h for $\frac{z' + z}{2}$ and $\frac{h' + h}{2}$, which may be done when the difference between z' and z, and between h' and h are small.

Hence $$a \cdot \sin z = t \cdot \sin p \cdot \sin q \cdot \sin h,$$

and $$\frac{t}{a} = \frac{\sin z}{\sin p \cdot \sin q \cdot \sin h}.$$

But $$\frac{\sin z}{\sin q} = \frac{\sin \angle ZPS}{\sin \angle ZSP},$$ (see figure, page 254),

$$= \frac{\sin h}{\sin S}$$

$$\therefore \frac{t}{a} = \frac{1}{\sin p \cdot \sin S} \quad . \quad . \quad . \quad . \quad . \quad (2).$$

Let $\frac{t}{a}$ be represented by A: it is the ratio of the change of time to the change of altitude; then

$$A \cdot \sin p = A \cdot \cos \textit{declination} = \operatorname{cosec} S \ldots . (3).$$

And by equation (1),

$$e = A \cdot c \cdot \cos S.$$

Or multiplying A by c, and denoting the product by B,

$$e = B \cdot \cos S \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4).$$

And on the equations (3) and (4) the rules given at page 197 are founded.

To compute the Altitude of a given Celestial Body for a given Time.

By the equation frequently referred to before,

$$\cos h = \frac{\cos z - \cos p \cdot \cos l'}{\sin p \cdot \sin l'}$$

$$\therefore \cos z = \cos p \cdot \cos l' + \sin p \cdot \sin l' \cdot \cos h$$

But $\cos h = (2 \cos^2 \tfrac{1}{2} h - 1)$, and this being substituted for $\cos h$,

$$\cos z = \cos p \cdot \cos l' - \sin p \cdot \sin l' + 2 \sin p \cdot \sin l' \cos^2 \tfrac{1}{2} h,$$

$$= \cos (p + l') + 2 \sin p \cdot \sin l' \cdot \cos^2 \tfrac{1}{2} h;$$

$$\therefore 1 - \cos z = 1 - \cos (p + l') - 2 \sin p \cdot \sin l' \cdot \cos^2 \tfrac{1}{2} h;$$

$$\therefore 2 \sin^2 \tfrac{1}{2} z = 2 \sin^2 \tfrac{1}{2} (p + l') - 2 \sin p \cdot \sin l' \cdot \cos^2 \tfrac{1}{2} h \quad . \quad (1).$$

Let $\frac{1}{2}(p+l') =$ A, and let angle B be computed from this equation,

$$\sin^2 B = \sin p \cdot \sin l' \cdot \cos^2 \tfrac{1}{2} h \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2).$$

Substituting $sin^2 B$ for the last term in equation (1), and dividing by 2,

$$sin^2 \tfrac{1}{2} z = sin^2 A - sin^2 B,\text{*}$$
$$= sin (A + B) . sin (A - B).$$

$$\left\{\begin{array}{c} \therefore \text{ If } A = \tfrac{1}{2} (polar\ distance + colatitude), \\ \text{and } sin\ B = \sqrt{} (sin\ polar\ distance \times sin\ colat \times cos^2 \tfrac{1}{2}\ hour \angle), \\ sin \tfrac{1}{2}\ zenith\ distance = \sqrt{} \{sin (A + B) . sin (A - B)\}. \end{array}\right\}$$

Another method,—

$$cos\ z = cos\ p . cos\ l' + sin\ p . sin\ l' . cos\ h.$$

But $cos\ h = 1 - 2\, sin^2 \frac{h}{2}$;

$$\therefore cos\ z = cos\ p . cos\ l' + sin\ p . sin\ l' - 2\ sin\ p . sin\ l' . sin^2 \frac{h}{2}$$

$$= cos (p - l') - 2\ sin\ p . sin\ l' . sin^2 \frac{h}{2};$$

adding 1 to both sides, and reducing as before; writing A for $\frac{1}{2}(p - l')$, and putting

$$sin^2 B = sin\ p . sin\ l' . sin^2 \frac{h}{2}.$$

The result is—

$$cos^2 \frac{z}{2} = cos^2 A - sin^2 B,\ \dagger$$
$$= cos (A + B) . cos (A - B).$$

$$\left\{\begin{array}{c} \text{Therefore, if } A = \tfrac{1}{2} (polar\ dist. - colat.) \\ \text{and } sin\ B = \sqrt{sin\ polar\ dist.\ sin\ colat.\ sin^2 \tfrac{1}{2}\ hour \angle}, \\ cos \tfrac{1}{2}\ zenith\ dist. = \sqrt{cos (A + B) . cos (A - B)}. \end{array}\right\}$$

The zenith distance having been computed by either of these methods, the true altitude is found by subtracting the zenith distance from 90°.

* *See* Note 2.

† *See* Note 2.

ON THE PRINCIPLES OF THE METHODS OF FINDING THE LONGITUDE AT SEA BY CELESTIAL OBSERVATION.

As the longitude of any place is measured by the arc of the equator, or the angle at the pole, included between the meridian of that place and the first meridian; and the difference between the time at any place and that at the first meridian is measured by a like arc of the celestial equator, or a like angle at the celestial pole, the longitude of any place would be known, if the mean, the sidereal, or the apparent time at the place, and at the first meridian, could be found at the same instant. Now, during the apparent diurnal revolution of the heavens, the distances of celestial objects from the horizon are continually varying, increasing from the time at which they rise till they pass the meridian, and then decreasing in like manner till they set. Hence, at a given place, any proposed altitude of a known celestial object, eastward or westward of the meridian, corresponds to a determinate instant of time; and the time at any given place may therefore be inferred from the observed altitude of a known celestial object. But an altitude for determining the time should not be observed when the object is near the meridian, as the altitude then varies so slowly, that a small mistake in measuring it will produce a considerable error in the computed time, and the nearer the bearing of an object is to the east or west, the less effect will any mistake in measuring its altitude produce in the time computed from it.

Now, if a chronometer, keeping mean time, were set to the time at the first meridian, it would continue to show the time at that meridian to whatever place it might afterwards be carried; and therefore the difference between the time shown by such an instrument, and the mean time at any other meridian, determined by observation, or otherwise, *would be the longitude of that meridian in time*, twenty-four hours of time corresponding to the circumference of the equator, or to 360° degrees of longitude.

The simplicity of this method of finding the longitude at sea, and the perfection to which the construction of chronometers have been brought, have combined to introduce it into very general practice, and its usefulness has been amply proved. But so delicate a machine as a chronometer must be peculiarly liable to be put out of order, even by causes which are difficult to detect and impossible to avoid; it is therefore desirable that, if possible, we should have some independent method of ascertaining the time at the first meridian.

Now the moon revolves round the earth, or appears to revolve among the stars, from west to east, with an angular velocity so considerable, that the instant of time when she is at a given distance from a celestial object lying in the direction of her motion, may be determined with considerable precision; and from the principles of Physical Astronomy, aided by observations, her place in the heavens can

now be *predicted* with sufficient exactness for the practical purposes of navigation; and, in fact, in the "Nautical Almanac," the distances of her centre from that of the sun, four of the planets, and some of the principal fixed stars that lie in the direction in which she moves, are given, and published for several years in advance, for every three hours of mean Greenwich time, except near the change, when she cannot be seen.

Hence, if an observer can determine by observation the moon's distance from the sun, or any of these stars, he may easily find the mean time at Greenwich, by comparing the observed distances with the distances given in the "Nautical Almanac."

But the distances there given are those which would be seen at the centre of the earth; and therefore before any comparison can be instituted between them, and distances observed upon the surface, for the purpose of determining the Greenwich time, the distances observed on the surface must be reduced to what they would have been if seen at the centre. Now the places of celestial objects, as seen at the surface, differ from their places as seen from the centre by the effects of parallax and refraction, which vary with the altitude of the objects; the moon's place, as seen from the centre being *above* and that of any other celestial object *below* its place as seen from the surface.

Hence, before the true distance can be computed, the *altitudes* of the objects, as well as their *apparent distance*, must be known.

In practice the altitudes and distances of the objects are generally measured at the same instant, by three different observers, while a fourth notes, by a watch, the times at which the observations are taken. Several sets of observations should, if possible, be taken, and a mean of the whole used as a single observation.

Such an observation is called a *lunar observation;* and this method of finding the longitude, by the distance of the moon from the sun or a star is called *the method of finding the longitude by lunar observations.*

In altitudes used only for computing the true distance, no great precision is necessary; but an altitude for computing the time ought to be taken as exactly as possible. The greatest care, however, is required in measuring the distance, as an error of 1′ in it will generally produce an error of about 2 m. of time, or about half a degree in the longitude deduced from it. The distance of the nearest limbs of the sun and moon is always measured, and their semidiameters added to obtain the apparent distance of their centres. The distance of a star is measured from the round or enlightened edge or limb of the moon, whether that limb be the nearest to or furthest from the star; and the moon's semidiameter is added to the observed distance when it is measured from the nearest limb, but subtracted when from the furthest limb, to obtain the apparent distance of the star from the moon's centre

A dexterous observer may himself obtain both the altitudes and distance by taking the altitudes of the objects both before and after he measures the distance, noting the time of each observation, and then

computing by proportion, from the change of the altitudes, what they must have been at the time at which the distance was observed.

Various other methods have been proposed for finding the longitude by observations at sea. The methods of which we have here sketched the principles are, however, those to which the attention of the practical mariner ought chiefly to be directed.

We may restate that, to find the longitude, we must be able to do two things, which are perfectly distinct in themselves; viz., *to find the time at the place at which we are, and to find the time at the same instant at a place whose situation we know.* The former of these is found at sea from the observed altitudes of celestial objects, and the latter by the aid of a chronometer, or by the distance of the moon from the sun or a fixed star, from which her distance is computed in the "Nautical Almanac."

When the apparent motion of a planet is contrary to that of the moon, the longitude can be more correctly deduced by a lunar distance from it than by one from a fixed star; and, besides, Venus, Jupiter, Mars, and Saturn (the planets whose distances from the moon are given) can often be seen when there is daylight enough to take their altitudes with every requisite degree of exactness, either for clearing the distance or computing the time.

Altitudes can seldom be obtained at night at sea with sufficient exactness for computing the time; it will therefore be generally found preferable to find the error of the watch from altitudes of the sun during the day, and then to find the time at the ship at which a lunar distance is taken, by allowing for the error of the watch and the difference of longitude between the places where the altitude for the time and the lunar distance are measured. It will often indeed be found difficult to take altitudes at night with sufficient precision for the purpose of clearing the distance; but as the time may be inferred from the altitudes of celestial objects, so, conversely, their altitudes may be inferred from the time; and it will often be found that, at sea, the altitudes of stars can be determined by computation with greater correctness than they can be observed.

Lunar Distances.

1. The true altitude of the sun is found by subtracting the refraction and then adding the parallax to the apparent altitude of its centre. The parallax of the sun is very small, and is always exceeded in amount by the refraction; the true altitude of the sun's centre is therefore *less* than the apparent altitude by as much as the refraction is greater than the parallax.

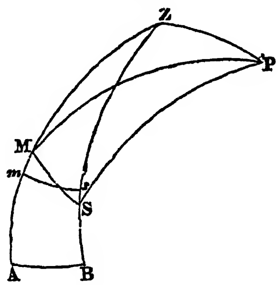
2. A fixed star has no appreciable parallax, and therefore its true altitude is *less* than the apparent altitude by the refraction. The moon's parallax in altitude always exceeds the refraction, and there-

fore her true altitude is *greater* than her apparent altitude by the difference between the parallax in altitude and the refraction at the time of observation.

3. The moon's bright limb is always turned towards the sun, and therefore when the distance between the sun and moon is taken, it is always the distance between the nearest limbs which is observed, and the apparent distance of the centres is found by adding the semidiameters of the sun and moon; having first applied the index error of the sextant.

4. When the distance between the moon's bright limb and a star east or west of her is taken, it must be noticed whether the distance is measured from the *nearest* or *furthest* limb of the moon. The semidiameter of the moon must be *added* to the distance, if the distance of the star from the *nearest* limb is measured, and it must be *subtracted* when the distance of the star is measured from the furthest limb. In all cases the index error of the sextant should be first allowed for.

Fig 32.



Let Z be the zenith, M and S the places of the moon and sun corrected for refraction and parallex, and *m* and *s* their apparent places; MS the true distance, *m s* the apparent distance of their centres, and let these be denoted by D and *d* respectively.

Now in the triangle *m* Z *s*, the three sides *m* Z, *s* Z, and *ms* being known,

$$\cos \angle Z = \frac{\cos m\,s - \cos m\,Z \,.\, \cos s\,Z}{\sin m\,Z \,.\, \sin s\,Z}.$$

If the letters *m* and *s* be now used to denote the altitudes of the points *m* and *s*, then *m* and *s* are the complements of *m* Z and *s* Z, and therefore

$$\cos \angle Z = \frac{\cos d - \sin m \,.\, \sin s}{\cos m \,.\, \cos s.};$$

and adding 1 to both sides and reducing,

$$\cos^2 \tfrac{1}{2} Z = \cos \tfrac{1}{2}(m + s + d) \,.\, \cos \tfrac{1}{2}(m + s - d) \,.\, \sec m \,.\, \sec s;$$

and if X be written for $\frac{1}{2}(m + s + d)$ and $(X - d)$ for $\frac{1}{2}(m + s - d)$,

$$\cos^2 \tfrac{1}{2} Z = \cos X \,.\, \cos (X - d) \,.\, \sec m \,.\, \sec s \quad . \quad . \quad . \quad . \quad (1)$$

$\angle$ Z may therefore be considered as determined.

Next, in the triangle M Z S, knowing M Z, Z S, and $\angle$ Z, the distance M S may be found from these formulæ,

$$\left.\begin{array}{l} \text{Let } \sin \theta = \sqrt{\sin MZ \,.\, \sin SZ \,.\, \cos^2 \tfrac{1}{2} Z} \quad . \quad . \quad . \quad . \\ \text{then } \sin \dfrac{D}{2} = \sqrt{\sin\{\tfrac{1}{2}(MZ+SZ)+\theta\} \,.\, \sin\{\tfrac{1}{2}(MZ+SZ)-\theta\}} \end{array}\right\} a^*$$

Or writing M and S for the true altitudes which are the complements of the zenith distances M Z and S Z,

$$\sin \theta = \sqrt{\cos M \,.\, \cos S \,.\, \cos X \,.\, \cos (X - d) \,.\, \sec m \,.\, \sec s} \;.(2),$$

where it will be seen that the value found for $\cos^2 \frac{1}{2}$ Z in formula (1) is also substituted. By means of this formula (2) the auxiliary angle θ can be computed, after which the distance M S = D is found thus,

$$\sin \frac{D}{2} = \sqrt{\sin\{\tfrac{1}{2}(MZ+SZ)+\theta\} \;\; \sin\{\tfrac{1}{2}(MZ+SZ)-\theta\}} \; (b).$$

Now $\frac{1}{2}(MZ + SZ) = 90° - \frac{1}{2}(M + S)$ for,

$$MZ = 90° - M$$

$$\text{and } SZ = 90° - S;$$

$$\therefore \; MZ + SZ = 180° - (M + S);$$

$$\therefore \tfrac{1}{2}(MZ + SZ) = 90° - \tfrac{1}{2}(M + S);$$

and this being substituted in the value of $\sin \dfrac{D}{2}$, gives

$$\sin \frac{D}{2} = \sqrt{\cos\{\tfrac{1}{2}(M+S)+\theta\} \,.\, \cos\{\tfrac{1}{2}(M+S)-\theta\}} \quad (3).$$

Equations (2) and (3) complete the solution of the problem; they may be thus expressed:—

(1.) Let $\sin^2 \theta$ = *sec (app. alt.* ☽) . *sec (app. alt.* ☉)
cos (true alt. ☽) × *cos (true alt.* ☉)
× *cos* ($\frac{1}{2}$ *sum of app. alts. and app. distance, or X*)
× *cos (X − app. dist.)*

(2.) Then $\sin^2 \frac{1}{2}$ *(true dist.)* = *cos* ($\frac{1}{2}$ *sum of true alts.* + θ) .
× cos ($\frac{1}{2}$ *sum of true alts.* − θ).

* These are convenient formulæ, when two sides and the contained angle of a spherical triangle are given, and it is required to find the remaining side.

The methods of clearing the lunar distance from the effects of parallax and refraction are very numerous; that which is given above is simple and direct, and gives the corrected distance without any embarrassment of algebraic signs, and moreover has the advantage of requiring no *special* tables in its application.

To find the Longitude by the Eclipses of Jupiter's Satellites.

By the immersions and emersions of these satellites are meant the instants of their vanishing in or reappearing from the shadow of the planet.

To the mean Greenwich time given in the Almanac for the expected immersion or emersion, apply the longitude by account in time, adding it if east, and subtracting it if west, and the result is the mean time at the place of observation when the phenomenon may be looked for.

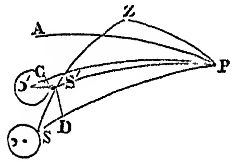
Begin to observe a few minutes before that time, and note the instant of mean time at which the immersion or emersion actually takes place, and the difference between that mean time, and the Greenwich time given in the "Nautical Almanac," is the longitude of the place in time.

Thus, if I observe the immersion of Jupiter's first satellite at 11 h. 2 m. 12 s. mean time, and I find by the "Nautical Almanac" that it takes place at 9 h. 40 m. 15 s. Greenwich mean time, my longitude in time must be 1 h. 21 m. 57 s. east.

Investigation of the Method of Computing the Moon's Right Ascension from an Occultation of a Fixed Star.

1. In the annexed figure let P A be the hour-circle passing through the first point of Aries; Z, the reduced place of the zenith; S, the

Fig 33



place of the star at the time of the occultation; ☽′, the corrected place of the moon's centre, and S′ that of the point of the moon's limb at which the occultation took place. Then S S′ is the moon's parallax-in-altitude; P S the star's polar distance; P ☽′ the moon's polar distance; A P S the star's right ascension, and A P ☽′ that of the moon.

2. Z P S is the *easterly* meridian distance of the star, for S is east of the meridian P A, by its right ascension A P S, and, therefore, east of the meridian P Z, by the angle Z P S.

3. The star S, is represented on the eastern limb of the moon, and as the moon moves eastward amongst the stars, she will pass over the star; and the figure relates to an *immersion*, or *disappearance* behind the moon.

4. Having taken out from the "Nautical Almanac" the right ascension of the *mean* or *true* sun for the estimated Greenwich date, the meridian distance of the star Z P S is computed from the formula,

mer. dist. = *mean time at place* + *R.A. of mean sun* − ✱'s *R.A.*

or, *mer. dist.* = *app. time at place* + *R.A. of sun* − ✱'s *R.A.*

5. The meridian distance having been found, the angle Z S P, at the star, must be computed by Napier's analogies, with the reduced colatitude Z P, the star's polar distance P S, and the meridian distance Z P S. This angle shall be denoted by the letter S.

6. Let S′ D be drawn perpendicular to P S, and S′ C perpendicular to P ☽′.

S S′ the parallax-in-latitude of the moon = hor. par. × sin Z S;

or, $\quad S S' = \Pi \,.\, \sin Z S,$

if Π be taken to represent the moon's horizontal parallax.

7. For the computation of S′D we have, then,

$$
\begin{aligned}
S'D &= S'S \,.\, \sin S,\\
&= H \,.\, \sin ZS \,.\, \sin S,\\
&= H \,.\, \sin ZP \,.\, \sin ZPS,\\
&= \text{☽'s (hor. par.)} \times \sin \text{colat.} \times \sin \text{✳'s hour angle} \quad (1).
\end{aligned}
$$

8. For the computation of S D,

$$SD = S'D \,.\, \cot S,$$

$$\text{and thence } PD = PS - SD,$$

$$\text{or } PD = \text{✳'s polar distance} - SD. \quad . \quad (2).$$

In the figure ∠ S is acute. When ∠ S is obtuse, S D changes its sign, and then

$$PD = \text{✳'s polar distance} + SD \quad . \quad (2^*).$$

9. In the right-angled spherical triangle S′P D,

$$\cos PS' = \cos PD \,.\, \cos DS'.$$

P D is greater or less than P S′, according as P S′ is greater or less than 90°, suppose P S′ less than 90°, and let x represent the small difference between P S′ and P D, so that P S′ = (P D + x),

$$\text{then} \qquad \cos (PD + x) = \cos PD \,.\, \cos DS'.$$

Expanding this, and putting 1 for the *cosine* of the small difference x,

$$\cos PD - \sin PD \,.\, \sin x = \cos PD \,.\, \cos DS';$$

$$\text{whence } \sin x = \cot PD \,.\, 2 \sin^2 \frac{DS'}{2},$$

$$\text{and} \qquad x = \tfrac{1}{2} \,.\, \cot PD \,.\, (DS')^2 \,.\, \sin 1'' \,. \quad . \quad . \quad . \quad . \quad (3).$$

This quantity x must be added to P D when P D is less than 90°, but subtracted from it when it is greater than 90°.

$$PD \pm x = S'P \quad . \quad . \quad . \quad . \quad . \quad . \quad (4).$$

10. Now take out from the "Nautical Almanac" the moon's declination, and find her polar distance P ☽′, and take the difference between this and P S′. This difference is ☽′ C.

Take also the moon's semidiameter, and then compute the small arc S′ C by this formula,

$$S'C = \sqrt{(\text{☽'s semidr.} + \text{☽}'C)\,(\text{☽'s semidr.} - \text{☽}'C)} \,. \quad . \quad (5).$$

11. Next, the angles S′P D and S′P C are to be computed,

$$\angle S'PD = S'D \,.\, \operatorname{cosec} S'P,$$

$$\angle S'PC = S'C \,.\, \operatorname{cosec} S'P.$$

The quantities necessary for the computation being found as directed in (7), (9), and (10).

12. The angle S′ P D, reckoned in time, is to be subtracted from the right ascension of the star A P S, when the star is east of the meridian, as shown in the figure: but it must be added to A P S if the star be west of the meridian; and the result is A P S′.

13. The angle S′ P C must next be subtracted, if an immersion be observed, as shown in the figure; but it must be added if the phenomenon observed be an emersion of the star.

The result of this second correction is the angle A P ☽′, or the right ascension of the moon.

14. The Greenwich time is then found by comparing the computed right ascension of the moon with those which are tabulated in the "Nautical Almanac."

15 When the Greenwich time thus found differs considerably from that by account, take the moon's polar distance for a minute or two earlier or later, and repeating the computations into which it enters, find the corresponding Greenwich time again; and then, by proportion, the true Greenwich time, or that to which the declination employed and the resulting right ascension both correspond, will readily be determined.

Summary.

(*a.*) With the estimated Greenwich date, take out the star's right ascension and declination, and the moon's declination, horizontal parallax, and semidiameter.

(*b.*) Find also the star's meridian distance, and with the reduced colatitude, the star's polar distance and meridian distance, compute the angle S by Napier's analogies.

(*c.*) The order of computation is then as follows:—

1. . . . S by rule (*b*) above.
2. . . . A = ☽'s hor. parallax . sin colat. . sin ✱'s hour ∠.
3. . . . B = A . cot S.
4. . . . { C = ✱'s polar dist. − B, B is additive when S is obtuse. }
5. . . . D = $\frac{1}{2}$ A². cot C . sin 1″.
6. . . . { E = C + D, D is subtractive when C is greater than 90°. }
7. . . . F = moon's polar dist. − E.
8. . . . G = √ (☽'s semidr. + F) . (☽'s semidr. − F).
9. . . . X = B . cosec E.
10. . . . Y = G . cosec E.

X, reckoned in time, is to be *added* to the star's right ascension, when the star is west of the meridian, and to be *subtracted* when the star is east of the meridian.

Y, reckoned in time, is *additive* at an emersion, and *subtractive* at an immersion.

These corrections having been applied to the star's right ascension, the result is the moon's right ascension, with which the Greenwich date is to be obtained.

A Problem.

To find the position of an unknown star or comet by its distances from two known stars.

Having taken a series of altitudes of the unknown star or comet, and also of the distances between it and the stars with which it is to be compared, reduce these measurements to the same instant of time by interpolation. The altitudes of the stars may either be found in the same manner from observations, or they may be computed. The distance between the comparison stars must next be computed from their right ascension and declinations. Let p and p' be their polar distances, and α the difference of their right ascensions.

$$\text{Let also } \sin^2\theta = \sin p \,.\, \sin p' \,.\, \cos^2\frac{\alpha}{2}$$

$$\text{then } \sin^2\frac{\text{dist.}}{2} = \sin\left(\frac{p+p'}{2}+\theta\right).\,\sin\left(\frac{p+p'}{2}-\theta\right) \quad . \quad (1).$$

Clear the distances between the comet and each of the stars from the effects of refraction, by means of the apparent and true altitudes, in the same manner as for lunar distances.

With the polar distances of the stars p and p', and their distance d, which form a triangle having the pole at its vertex, compute the angle at one of the stars by the formula—

$$\cos^2\frac{A}{2} = \frac{\sin S \,.\, \sin(S-p)}{\sin d \,.\, \sin p'}$$

$$\text{where } S = \tfrac{1}{2}(p+p'+d).$$

Again in the triangle whose angular points are the comet and the two stars, let δ and δ' represent the distances of the comet from the stars, and in this triangle compute the angle at the same star as before.

$$\cos^2\frac{A'}{2} = \frac{\sin S' \,.\, (\sin S' - \delta)}{\sin\delta \,.\, \sin\delta'}$$

$$\text{where } S' = \tfrac{1}{2}(\delta+\delta'+d).$$

Next when the comet and the pole are on the same side of the great circle joining the two stars, take the difference between A and A′, otherwise take their sum. Let this difference or sum = Q.

Then with δ', p' and Q find the comet's polar distance, thus:—

$$\text{Let } \sin^2 \phi = \sin \delta' \,.\, \sin p' \,.\, \cos^2 \frac{Q}{2},$$

$$\text{then } \sin^2 \tfrac{1}{2} \textit{ polar dist.} = \sin\left(\frac{\delta' + p'}{2} + \phi\right) . \sin\left(\frac{\delta' + p'}{2} - \phi\right).$$

Let this polar distance = X, then the angle contained between the polar distances X, and p' is the difference between the right ascension of the comet and the star whose polar distance is p', and the difference may be computed from the three sides p', δ', and X, by the formula—

$$\cos^2 \tfrac{1}{2} \,(\textit{diff. of R. A s.}) = \frac{\sin S'' \,.\, \sin\,(S'' - \delta')}{\sin X \,.\, \sin p'}.$$

It should be noted whether the comet is eastward or westward of the star from which this difference of right ascension is estimated; if the comet is eastward, *add* the difference to the right ascension of the star, but if the comet is westward subtract the difference, adding 24 hours to the star's right ascension if necessary, and the sum or remainder is the comet's R A.

Great Circle Sailing.

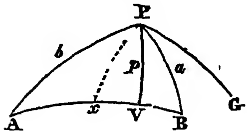
The shortest distance between two places is that which is measured on the great circle which passes through them; it is represented on the globe by a thread tightly stretched between them. It may also be exhibited on a globe, by turning the globe about until the places are both seen to lie evenly with the wooden horizon, which is itself a great circle. Indeed this, with an eight-inch globe, is as good a method as can be devised for observing how the great circle track lies between the given places, and it offers very little more difficulty than the drawing of the straight rhumb-line track on a Mercator's chart.

First Method.

1. Call the place which is nearest to the equator A, and the other B. Let P be the pole nearest to B; then the ∠ A P B is the

difference of longitude between A and B, and A P and B P are the colatitudes of A and B found by subtracting the latitudes from 90°

Fig 34



when the places are in the same hemisphere, but adding 90° to A's latitude when they are on different sides of the equator.

Then P A will always be greater than P B. Now, in the triangle A P B, if the distance A B be denoted by d,

$$\cos d = \cos a \cdot \cos b + \sin a \cdot \sin b \cdot \cos \angle \mathrm{APB};$$

but $\cos \mathrm{P} = 1 - 2 \sin^2 \frac{\mathrm{P}}{2}$.

Hence $\cos d = \cos a \cdot \cos b + \sin a \cdot \sin b - 2 \sin a \cdot \sin b \cdot \sin^2 \frac{\mathrm{P}}{2}$,

$$= \cos (b - a) - 2 \sin a \cdot \sin b \cdot \sin^2 \frac{\mathrm{P}}{2}.$$

And adding 1 to each side,

$$1 + \cos d = 1 + \cos (b - a) - 2 \sin a \cdot \sin b \cdot \sin^2 \frac{\mathrm{P}}{2}.$$

But $1 + \cos d = 2 \cos^2 \frac{d}{2}$, and $1 + \cos (b - a) = 2 \cos^2 \frac{b - a}{2}$;

$\therefore$ substituting and dividing by 2,

$$\cos^2 \frac{d}{2} = \cos^2 \frac{b - a}{2} - \sin a \cdot \sin b \cdot \sin^2 \frac{\mathrm{P}}{2}.$$

Let $\sin a \cdot \sin b \cdot \sin^2 \frac{\mathrm{P}}{2} = \cos^2 x$ . . . . (1),

$$\therefore \cos^2 \frac{d}{2} = \cos^2\left(\frac{b - a}{2}\right) - \cos^2 x \;.\;.\quad *$$

$$= \sin \left\{x + \frac{b - a}{2}\right\} . \sin \left\{x - \frac{b - a}{2}\right\} \ldots (2).$$

Equations (1) and (2), may be expressed thus:—

$$\cos^2 x = \cos (\textit{latitude } \mathrm{A}) \cdot \cos (\textit{latitude } \mathrm{B}) \cdot \sin^2 \frac{\textit{difference of longitude}}{2},$$

$$\cos^2 \frac{\textit{distance}}{2} = \sin (x + \tfrac{1}{2} \textit{ diff. of latitude}) \cdot \sin (x - \tfrac{1}{2} \textit{ diff. of latitude}).$$

These formulæ are employed in finding the distance.

* See Note 2.

2. The distance being found, it is necessary in the next place to find the latitude of the highest point or V, which is evidently the point where the perpendicular P V meets the circle A B.

$$\sin A \,.\, \sin A'B = \sin P \,.\, \sin P B;$$

or, $$\sin A \,.\, \sin d = \sin P \,.\, \sin a;$$

$$\therefore \quad \sin A = \frac{\sin P \,.\, \sin a}{\sin d};$$

multiplying by $\sin b$,

$$\sin b \,.\, \sin A = \frac{\sin a \,.\, \sin b \,.\, \sin P}{\sin d};$$

but $\sin b \,.\, \sin A = \sin p;$

$$\therefore \sin p = \frac{\sin a \,.\, \sin b \,.\, \sin P}{\sin d};$$

or, $$\cos(\mathit{latitude}\ \mathrm{V}) = \frac{\cos(\mathit{latitude}\ \mathrm{A}) \,.\, \cos(\mathit{latitude}\ \mathrm{B})\ \sin(d\ \mathit{of\ longitude})}{\sin\ \mathit{distance}},$$

and this is the formula for finding the lat. of the vertex.

3. Again $\angle$ A P V is the difference of longitude between A and V, and by the spherical triangle A P V.

$$\mathit{Cos}\ \angle\ A P V = \tan p \,.\, \cot b;$$

or, $\cos$ *diff. long. between* A *and* $V = \cot$ *lat. of vertex* $\times \tan$ *lat.* V.

4. Take any point x on A B, then $\angle$ V P x is the diff. long. between V and x.

and $$\cos \angle\ V P x = \tan p \,.\, \cot P x$$

and $\therefore$ $$\cot P x = \cos \angle\ V P x \,.\, \cot p;$$

or $\tan$ *lat.* $x = \cos$ (*diff. long. from* V) $\times \tan$ *lat.* V.

And this is the formula for computing the latitude of any point between A and B.

5. And having found the positions of a number of such points, the course from each to the next must be computed, and the ship steered from point to point.

Second Method.

In the right-angled spherical triangles A P V and B P V, by Napier's rules,

$$\cos \angle APV = \tan PV . \cot PA$$

and $$\cos \angle BPV = \tan PV . \cot PB$$

$$\therefore \quad \frac{\cos \angle BPV}{\cos \angle APV} = \frac{\cot PB}{\cot PA}$$

whence $$\frac{\cos \angle BPV + \cos \angle APV}{\cos \angle BPV - \cos \angle APV} = \frac{\cot PB + \cot PA}{\cot PB - \cot PA}$$

and reducing this to its simplest form,

$$\cot \tfrac{1}{2}(\angle APV + \angle BPV) . \cot \tfrac{1}{2}(\angle APV - \angle BPV) = \frac{\sin(PA + PB)}{\sin(PA - PB)},$$

Now P A and P B are the colatitudes of the points A and B, and these with the latitudes of the same point will make 180°,

$$\therefore \sin(PA + PB) = \sin \text{ (sum of the latitudes)}$$

and $PA - PB$ is equal to the *difference of latitude.*

Moreover $\angle$ A P V + $\angle$ B P V = $\angle$ A P B or the *difference of longitudes* between the meridians P A and P B, and $\frac{1}{2}$ ($\angle$ A P V − $\angle$ B P V) is the quantity denoted by X in the rules.

Therefore denoting half the difference of longitude by L, the sum of the latitudes by S, and the difference of latitude by D, the equation above becomes

$$\cot L . \cot X = \frac{\sin S}{\sin D};$$

whence $$\tan X = \cot L . \operatorname{cosec} S . \sin D.$$

Now if P x in the figure be the middle meridian between P A and P B, the angle A P x or B P x is L, and the angle x P V is X.

Also the angle x P V is the difference of longitude between the vertex V and the point of *middle longitude* x: and the angle x P V lies on the same side of P x as P B does, that is, *eastward* when P B is *eastward* of P x and P A as in the figure; and *westward* when P B is *westward* of P x and P A.

And hence this rule, *X is to the difference of longitude between the vertex and the middle meridian, and is to be considered east when B is east of A, and west when B is west of A: and applied with its proper sign to the middle longitude it will give the longitude of the vertex.*

If P G be the meridian of Greenwich, G P V is the longitude of the vertex, and G P x the longitude of the middle meridian, and

$$GPV = GPx - xPV;$$

Or, in this case, the longitude of the vertex is the difference between the middle longitude and the quantity X.

The longitudes of V and A being now both known, their difference A P V is known. And in the right-angled triangle A P V,

$$\text{Cos } \angle \, APV = \tan PV \,.\, \cot PA$$

or, $\cos$ (*difference of longitude A & V*) $=$ $\cot$ (*latitude of V*) . $\tan$ (*latitude of A*)

$\therefore$ $\cot$ (*latitude of vertex*) $=$ $\cos$ (*difference of longitude A & V*) . $\cot$ (*latitude of A*)

which is the rule given at page 59.

The latitude and longitude of the vertex having been thus determined, the latitudes and longitudes of as many intermediate places as may be convenient can next be found as before directed.

Indeed, in actual practice, after the latitude and longitude of the vertex are found, it will be only necessary to assume *one* longitude a few degrees ahead of the ship, and compute the latitude of that point of the great circle to which this longitude belongs; and having thus ascertained the place of *one* point of the great circle, the ship is to be steered for it as well as circumstances will permit. If she attain it, another point may then be sought, towards which the ship may next be steered, if not, it will be necessary to commence on another great circle track.

NOTE I

MERCATOR'S CHART.

MERCATOR's chart is so called from its inventor, Gerard Mercator, who, in 1569, first attempted to remedy the defects of the plane charts then in use, by representing meridians and parallels of latitude by parallel straight lines as in a plane chart, but gradually increasing the length of the degrees of latitude in advancing from the equator to the poles, so that the rhumb-lines might be extended into straight lines, and that a straight line, drawn from one place to another on the chart, should make an angle with the meridians, expressing the course between them.

Mercator seems to have employed some empirical method in the division of the meridians, the correct principle having been explained in the subsequent work of Mr Edward Wright, published in 1599, entitled "The Correction of certain Errors in Navigation" The second edition of this work, in 1610, was dedicated by Mr Wright to his royal pupil, Henry Prince of Wales That the meridional parts were related to the *logarithmic tangents* of $45^\circ + \frac{1}{2}$ *the latitude*, was shown by Mr Bond in 1645, and the problem afterwards occupied the attention of Mr James Gregory in 1668. Dr. Halley in 1695—whose paper is printed in No 219 of the "Philosophical Transactions of the Royal Society"—and of Mr. Roger Cotes, in his "Logometria," first published in the "Philosophical Transactions" for 1714, No 388.

It has been shown that the leading principle of Mercator's projection of the surface of the globe upon a plane is, that, at every point of the surface, the increased breadth consequent upon drawing the meridians parallel to each other, instead of permitting them to converge as they do upon the globe, is compensated by increasing the portions of the meridian in the same ratio as the meridian distances are increased.

So that if $d\,q$ represent a very small portion of the equator, and $d\,p$ the corresponding portion of any parallel, in latitude m,—then upon the globe,

$$d\,p = d\,q \,.\, \cos m \qquad \ldots \qquad (1),$$

and upon Mercator's projection of the globe,

$$d\,p' = d\,q \qquad \ldots \qquad (2),$$

$d\,p'$ being the representation on Mercator's chart of $d\,p$ upon the globe; and if $d\,m$ represent an infinitesimal part of the meridian on the globe equal to $d\,q$ and adjacent to $d\,p$,

$$d\,p' = d\,m$$

But the portion of the meridian upon the chart which represents $d\,m$ must be larger than $d\,m$, in the same ratio that $d\,p'$ exceeds $d\,p$, or if $d\,m'$ represent $d\,m$ upon the chart,

$$\begin{aligned}
\frac{d\,m'}{d\,m} &= \frac{d\,p'}{d\,p} \\
&= \frac{1}{\cos m} \quad \ldots \quad \text{by equations (1) and (2).} \\
&= \frac{1}{\sin (90^\circ - m)} \\
&= \frac{1}{2\,.\,\sin \frac{1}{2}(90^\circ - m)\ \cos \frac{1}{2}(90^\circ - m)} \\
&= \frac{1}{2} \cdot \frac{1}{\sin^2 \frac{1}{2}(90^\circ - m)} \cdot \frac{1}{\cot \frac{1}{2}(90^\circ - m)} \\
&= \frac{\frac{1}{2}\ \operatorname{cosec}^2 \frac{1}{2}(90^\circ - m)}{\cot \frac{1}{2}(90^\circ - m)}
\end{aligned}$$

Therefore the integral is

$$\begin{aligned}
m' &= \log_e \cot \tfrac{1}{2}(90^\circ - m) \\
&= \log_e \cot \tfrac{1}{2}\ \text{colatitude}
\end{aligned}$$

That is, the sum of all the elements $d\,m'$ or the length of the meridian on the chart from lat. 0° to lat. m = Naperian log cot $\frac{1}{2}$ colatitude.

To adapt this to the common logarithms, it must be multiplied by 2.302585, the Napierian log of 10, and to express its value in nautical miles, it must be multiplied by the number of such miles in the radius of the terrestrial *sphere*; the logarithm of the product of these factors is 3.8984895.

Whence the common rule for computing the meridional parts,

$$\log m' = 3.8984895 + \log (\log \cot \tfrac{1}{2} \text{colat} - 10).$$

Now M. Delambre has shown in the *Connoissance des Temps* for 1805, page 342,* that taking account of the spheroidal figure of the earth, and neglecting the cube and higher powers of the small eccentricity of the meridians, the formula for the meridional parts may be expressed by

$$\log \tan (45^\circ + \tfrac{1}{2} \lambda'),$$

where λ' is the geocentric latitude, or this formula may be written thus—

$$\dot{m} = \log \cot \tfrac{1}{2} (\text{geocentric colat.}),$$

and is therefore of exactly the same form as that which is deduced for the *sphere*.

Hence a common table of meridional parts may be employed to obtain the meridional parts for a given spheroid of small ellipticity, by subtracting the *angle of the vertical* from the true latitude, using the remainder as the argument of the table: this rule is given by Mendoza Rios in his nautical tables.

M. Delambre attributes the original discussion of the problem with reference to the spheroid to Maclaurin, in whose "Treatise of Fluxions" it will be found, at page 719 of the second volume (edition of 1742).

Maclaurin mentions a tract on the same subject, which was published in 1741 by the Rev. Dr. Murdoch.

Maclaurin gives a rule identical with that employed by Mendoza Rios, for determining the meridional parts for the spheroid from those which are computed for the sphere.

The celebrated Gauss has, however, exhausted the subject of such projections, in his answer to the general problem *to represent the parts of a given surface on another given surface, so that the smallest parts of the representation shall be similar to the corresponding parts of the surface represented*—a prize question proposed by the Royal Society of Copenhagen. A translation of the paper is given in the August and September Numbers of the "Philosophical Magazine" for 1828.

One of the purposes to which Mercator's chart is applied in the modern practice of Navigation is the projection of great circles upon it, indicating the shortest route from one place to another.

Several years ago the Rev. Mr. Fisher drew my attention to the fact, that, although the projected curves are not really circles, yet for the purposes of Navigation, large portions of them might be very fairly represented by arcs of circles, differing in the lengths of their radii with the maximum latitude attained by the circumferences of the circles. Availing himself of this, Mr. Fisher arranged the convenient rules which he has kindly given to me for this work; and after a careful examination I feel in a position to say that nothing else so simple nor so generally applicable has yet been proposed for the same purpose, including, as these rules do, those cases in which the direct employment of the great circle would be inconvenient and dangerous.

In the "Monthly Notices" of the Royal Astronomical Society for March, 1858, the Astronomer Royal has given a method and its demonstration, for approximating very closely to the curve of a projected great circle founded upon the observation above mentioned, namely, that the arcs of the projected curve may be represented by arcs of circles. The method without the demonstration has since been copied into the meteorological papers of the Board of Trade, nor is the investigation sufficiently elementary for these pages.

For the exact practice of great circle sailing the second method at page 58, is especially commended to the attention of students of Navigation.

The geometry of Mercator's chart offers many points of interest to the mathematician, a classification of some of which I was tempted for a moment to introduce here, but omit on the consideration that they do not possess a direct bearing on the science of Navigation, and therefore would be out of place.

* Traité de Navigation, par J. B. E. de Bouguer, à Paris, 1808, page 328, where the problem is fully discussed.

NOTE 2.

The *forms* in the text from which reference is made to this note are of such frequent occurrence in trigonometrical investigations, that a concise and general method of reducing them to others more convenient for logarithmic computation is given. Without a knowledge of these equivalent expressions, the succeeding step of the demonstration in which these *forms* occur cannot be understood.

1. To prove $\sin^2 A - \sin^2 B = \sin(A+B) \,.\, \sin(A-B)$

$$\begin{aligned}\sin^2 A - \sin^2 B &= \sin^2 A - \sin^2 A \sin^2 B + \sin^2 A \sin^2 B - \sin^2 B\\ &= \sin^2 A\,(1-\sin^2 B) - \sin^2 B\,(1-\sin^2 A)\\ &= \sin^2 A \cos^2 B - \cos^2 A \sin^2 B\\ &= \sin(A+B) \,.\, \sin(A-B).\end{aligned}$$

2. To prove $\cos^2 A - \cos^2 B = \sin(B+A) \,.\, \sin(B-A)$

$$\begin{aligned}\cos^2 A - \cos^2 B &= \cos^2 A - \cos^2 A \cos^2 B + \cos^2 A \cos^2 B - \cos^2 B\\ &= \cos^2 A\,(1-\cos^2 B) - \cos^2 B\,(1-\cos^2 A)\\ &= \cos^2 A \sin^2 B - \cos^2 B \sin^2 A\\ &= \sin(B+A) \,.\, \sin(B-A).\end{aligned}$$

3. To prove $\sin^2 A - \cos^2 B = -\cos(A+B) \,.\, \cos(A-B)$

$$\begin{aligned}\sin^2 A - \cos^2 B &= \sin^2 A - \sin^2 A \cos^2 B + \sin^2 A \cos^2 B - \cos^2 B\\ &= \sin^2 A\,(1-\cos^2 B) - \cos^2 B\,(1-\sin^2 A)\\ &= \sin^2 A \sin^2 B - \cos^2 B \cos^2 A\\ &= -\cos(A+B) \quad \cos(A-B).\end{aligned}$$

4. To prove $\cos^2 A - \sin^2 B = \cos(A+B) \,.\, \cos(A-B)$

$$\begin{aligned}\cos^2 A - \sin^2 B &= \cos^2 A - \cos^2 A \sin^2 B + \cos^2 A \sin^2 B - \sin^2 B\\ &= \cos^2 A\,(1-\sin^2 B) - \sin^2 B\,(1-\cos^2 A)\\ &= \cos^2 A \cos^2 B - \sin^2 A \sin^2 B\\ &= \cos(A+B) \,.\, \cos(A-B).\end{aligned}$$

NOTE 3.

In a spherical triangle given b, c, and the contained angle A, c, being very small compared with a and b; required an expression for the difference of a and b.

Let x = the required difference, so that $b - x = a$

Then $\cos a$ or $\cos(b-x) = \cos b \quad \cos c + \sin b \,.\, \sin c \,.\, \cos A \quad . \quad . \quad . \quad . \quad (1).$

But $\qquad \cos(b-x) = \cos b \,.\, \cos x + \sin b \quad \sin x,$

$$\therefore \cos b \,.\, \cos x + \sin b \,.\, \sin x = \cos b \,.\, \cos c + \sin b \,.\, \sin c \quad \cos A,$$

And transposing $\cos b \,.\, \cos x$, and dividing by $\sin b$,

$$\sin x = \sin c \,.\, \cos A - \cot b\,(\cos x - \cos c) \quad . \quad . \quad . \quad . \quad (2);$$

but $\qquad \cos x - \cos c = 2 \sin \tfrac{1}{2}(c+x) \,.\, \sin \tfrac{1}{2}(c-x) = \tfrac{1}{2}(c^2 - x^2) \,.\, \sin^2 1''$ nearly,

c and x being small quantities; writing also $x \quad \sin 1''$ and $c \,.\, \sin 1''$, for $\sin x$ and $\sin c$, and then dividing by $\sin 1''$,

$$x = c \,.\, \cos A - \tfrac{1}{2} \cot b\,(c^2 - x^2) \quad \sin 1''.$$

An approximate value of x is computed from the first term $c \,.\, \cos A$, and then this value is employed in ascertaining the value of the second term.

And $\qquad a = b - c \,.\, \cos A + \tfrac{1}{2} \cot b\,(c^2 - x^2) \,.\, \sin 1''$

It will be seen that the term $-c \,.\, \cos A$ will be *plus* when A exceeds 90°, and that the following term will be *minus* when b exceeds 90°

When no great precision is required, the value found from the term $c \,.\, \cos A$, may be considered as sufficiently accurate, and the next term may be neglected

Among the applications of this problem, may be mentioned the reduction of an observed zenith distance, to adapt it to a place differing slightly in position from that at which the observation was made. The triangle which is then to be considered is

that whose angular points are the place of the celestial object at the time of observation, the zenith of the place whence the observation was made, and zenith of the place arrived at

The given quantities are the distance between the zeniths, estimated by the run of the ship (c), the angle between the direction of the sun at the time of observation, and the direction in which the ship has sailed (A), and thirdly, the observed zenith distance.

Hence, the common rule—

$$\text{cor} = \text{distance} \cdot \cos A,$$

in which the term $\frac{1}{2} \cot b \cdot (c^2 - x^2) \sin 1''$ is neglected.

Another example of the application of these formulæ occurs in the method of finding the latitude by the altitude of the pole-star, in which it becomes necessary to compute the difference between P Z (the colatitude), and Z S (the star's zenith distance). The quantities known are the meridian distance h, the polar distance p, and the zenith distance z (or its complement, the altitude a), and thence the formula,

$$\text{lat} = a - p \cdot \cos h + \tfrac{1}{2} \cot z \cdot (p^2 - c^2) \cdot \sin 1'',$$

c representing the value of $p \cdot \cos h$, which is first computed.

The same problem occurs again in the investigation of the method of deducing the moon's right ascension from the occultation of a fixed star (page 291), when the difference between P S and P S′ is required

The little term $\frac{1}{2} \cot b \quad (c^2 - x^2) \cdot \sin 1''$, can be put into a form more convenient for calculation, as shown in the note appended to the discussion of the method of finding the latitude by the altitude of the pole-star, by expressing c and x in minutes, and observing that $30 \cdot \sin 1' = \frac{7}{800}$ very nearly, whence is obtained the value of this terms in *seconds*,

$$\frac{7}{800} \quad \cot b \cdot (c + x) \cdot (c - x).$$

As an example of the application of these formulæ, let $c = 1^\circ$, $b = 60^\circ$, and $\angle A = 114^\circ$, required the difference between a and b

The direct computation of a by the principles of spherical trigonometry may be made from these formulæ.

$$\left.\begin{aligned} \sin^2\theta &= \sin b \cdot \sin c \cdot \cos^2 \frac{A}{2} \\ \sin^2 \frac{a}{2} &= \sin\left(\frac{b+c}{2} + \theta\right) \cdot \sin\left(\frac{b+c}{2} - \theta\right) \end{aligned}\right\}$$

$\frac{1}{2}$ A = 57°				cos	9·736109
					2
					19 472218
	°	′	″		
b	60	0	0	sin	9 937531
c	1	0	0	sin	8 241855
$\frac{1}{2}(b+c)$	30	30	0	2)	17·651604
θ	3	50	21	sin	8·825802
Sum	34	20	21	sin	9·751349
Diff.	26	39	39	sin	9·651964
				2)	19 403313
$\frac{1}{2} a$	30	12	19·5	sin	9·701656
a	60	24	39		

And therefore the difference between a and b is 24′ 39″.

And next by the formula demonstrated in this note—

$$
\begin{array}{llll}
 & -\,C \cos A & & \\
C\ 60' & . \quad . & -\log & 1\cdot778151 \\
A\ 114^\circ & . \quad . & -\cos & 9\ 609313 \\
\hline
 & 24'\cdot40 & +\log & 1\ 387464 \\
\end{array}
$$

or $+\ 24'\ 24''$

This result differs already very little from the total difference obtained by the solution of the spherical triangle. The second term being required.

$$\frac{7}{800} \cdot (c + x) \cdot (c - x) \cdot \cot b$$

$$
\begin{array}{llll}
c & 60' & & \\
x & 24\cdot4 & & \\
\hline
\text{Sum} & 84\ 4 & \log & 1\ 9263 \\
\text{Diff} & 35\cdot6 \quad . \quad . & \log & 1\ 5515 \\
b & 60^\circ \quad . \quad . & \cot & 9\ 7614 \\
\hline
 & 1735 \quad\quad . & \log & 3\ 2392 \\
\end{array}
$$

And $\frac{7}{800}$ of $1735'' = \frac{7}{8}$ of $17''\ 35 = 15''\ 18$, and this correction is also additive, for b is less than 90°, the sum of the two corrections is, therefore, $24'\ 39''\cdot18$, agreeing exactly with the previous direct but more laborious computation

If when a celestial body has a small altitude, its distance from any terrestrial object in the horizon be measured, this distance will be very nearly equal to the difference between the bearings or the horizontal angle between the vertical planes passing through the objects observed If θ represent the observed distance, θ' the horizontal angle, and α the altitude, the angle A between the horizon and the vertical circle passing through the celestial object is a right angle, and the first term of the difference between θ and θ' vanishes, and the second term is

$$\tfrac{1}{2}\ \alpha^2 \quad \cot \theta \quad \sin 1''.$$

But this is only a particular case of the general problem in which both objects have a small elevation Let $s =$ sum of altitudes in minutes, and $d =$ their difference, and θ' the difference of bearings; the correction of the observed distance θ is

$$\theta - \theta' = \tfrac{1}{4}\left(d^2 \quad \cot\frac{\theta'}{2} - s^2 \cdot \tan\frac{\theta'}{2}\right) \cdot \sin 1''.$$

NOTE 4

The following remarks on "local deviation" are those published by Capt Matthew Flinders, R N., in August, 1812, and are so neat and comprehensive a description of the phenomena attending the effect of the iron of a ship upon the compasses, that this opportunity of presenting them to the reader is gladly embraced.

Magnetism of Ships

"During the course of my voyage in H M sloop 'Investigator,' for completing the discovery of New Holland and New South Wales, I remarked that the variation in the bearings of land taken with an azimuth compass upon the binnacle were very different when the ship's head was in different directions, and, at length, I found that the following circumstances obtained throughout the whole of the observations.

1. When the head was east, the variation differed from the truth; and always on the same side while the ship remained in the same hemisphere.

2. When the head was west, the differences were equally great, but a contrary way.

3. The head being north or south made no difference in the variation, and it was then a medium between what was found at east and at west.

4. At the intermediate points, between the magnetic meridian and east or west, the difference from the true variation bore a proportion to the angle made by the ship's head with the meridian. If the head were on the western side, the difference was of the same nature as that when the head was west, if on the east side, as at east

5. The proportion at the intermediate points obeys the following law.—

As radius
Is to the difference at east or west (for eight points),
So is the *sine* of the angle between the ship's head and the magnetic meridian
To the difference for that angle.*

Or if the number of points which the head was to the right or left of the meridian were taken as a *course*, and the difference for eight points, reduced to minutes, taken as a *distance*, then the difference for the number of points was found in the *departure* column of the traverse table.

6. These differences were of a directly contrary nature in the south to what they were in the northern hemisphere. In the English Channel the compass gave *too much* west variation when the head was west, but in the southern hemisphere it always gave too little, and the greatest west variations were found when the head was east.

7. The differences did not change suddenly on crossing the equator, but all the way from England they diminished gradually, and, to all appearance, as the dip of the needle diminished When the south end of the needle began to dip, the differences commenced the other way, and increased gradually as we advanced southward, until, having arrived in Bass's Strait, where the south dip is nearly as great as the north is in England, the differences became almost as great as when we first sailed, but, as I said before, of an opposite nature.

The experiments, lately made in England, prove that similar differences, obeying the same laws, take place in most or all ships of war, and perhaps they do in merchant ships also, for I have found them in vessels of 80 and 25 tons. Differences were also found in other parts, often greater than at the binnacles, but these being of less importance, the differences only which we observed at or above the binnacles, on changing the ships' heads *from east to west*, are here inserted.

		° ′	
Sheerness,	'Starling' gun-brig, difference of variation	7 29	greater
,,	'Helder' frigate	13 3	do.
,,	'Raisonnable' 64, armed *en flûte* . .	0 42	do.
Portsmouth,	'Loire' frigate	2 7	do.
,,	'Devastation' bomb	4 2	do.
Plymouth,	'Orestes' brig	Uncertain	
Channel,	'Investigator' armed ship . . .	8 4	greater

In Captain Cook's ships, the 'Endeavour' and 'Resolution,' and also in the 'Discovery,' commanded by Captain Vancouver, the differences appear to have been nearly the same as in the 'Investigator,' and also of a contrary nature in the two hemispheres."—M F.

* Maximum dev. × sin magnetic bearing of ship's head = dev. due to the ship's position.

Figures and Letters used in the Royal Navy to denote the Force of the Wind and the State of the Weather.

Figures to denote the Force of the Wind.

0 denotes	Calm		
1 „	Light Air . . .	just sufficient to give .	Steerage way.
2 „	Light Breeze .	with which a well-conditioned man-of-war, under all sail, and clean full, would go in smooth water from	1 to 2 knots
3 „	Gentle Breeze .		3 to 4 knots.
4 „	Moderate Breeze		5 to 6 knots.
5 „	Fresh Breeze .	in which the same ship could just carry close hauled	Royals, &c.
6 „	Strong Breeze .		Single-reefs and top-gallant sails
7 „	Moderate Gale .		Double-reefs, jibs, &c.
8 „	Fresh Gale . .		Triple-reefs, courses, &c.
9 „	Strong Gale . .		Close-reefs, and courses.
10 „	Whole Gale .	with which she could only bear	Close-reefed main top-sail and reefed foresail.
11 „	Storm	with which she would be reduced to . . .	Storm stay-sails.
12 „	Hurricane . . .	to which she could show	No canvas.

Letters to denote the State of the Weather.

b denotes Blue Sky—whether with clear or hazy atmosphere.
c „ Cloudy—*i.e.*, Detached opening clouds.
d „ Drizzling Rain.
f „ Fog—f Thick Fog.
g „ Gloomy Dark Weather.
h „ Hail.
l „ Lightning.
m „ Misty or hazy – so as to interrupt the view.
o „ Overcast—*i.e.*, The whole sky covered with one impervious cloud.
p „ Passing Showers.
q „ Squally.
r „ Rain—*i.e.*, Continuous rain.
s „ Snow.
t „ Thunder.
u „ Ugly threatening appearance of the Weather.
v „ Visibility of Distant Objects—whether the sky be cloudy or not.
w „ Wet Dew.
• „ Under any letter denotes an Extraordinary Degree.

By the combination of these letters all the ordinary phenomena of the weather may be recorded with certainty and brevity.

Examples.

b c m   Blue sky, with detached opening clouds, but hazy round the horizon.
g v   Gloomy dark weather, but distant objects *remarkably* visible.
q p d l t   Very hard squalls, and showers of drizzle, accompanied by lightning, with *very* heavy thunder

The above modes of expression are to be adopted in Her Majesty's ships and vessels; and, therefore, both the strength and direction of the wind, as well as the state of the weather, are to be regularly marked, every hour, in a narrow column on the log-board.

H.M.S. a. y. Black Eagle Wednesday, 8th day of March, 1854.

Hours	Knots	Tenths	Standard Compass Courses	Lee-way	Winds	Force	Weather — Barometer Thermometer	Deviation of Standard Compass	Remarks	Initials of Officer's Watch.
1	.	.	.	.	S W	4	*b c*	.	A M. A M	
2	At		.	.	.	.	.	.		
3			.	.	.	.	.	.		
4	anchor.		.	.	.	.	.	.	4h	
5			.	.	S.W.	4	*b c*	.	Weighed and proceeded down the Knab Channel.	
6	Steaming		.	.	.	.	.	.		J. E. P.
7	for the		.	.	.	.	30·50 41	.	Passed the Girdler Lt. V. „ Tongue Lt V „ N Foreland Lt. H.	
8			.	.	.	.	.	.	Showed No to H M Dockyard at Deal.	
9	Downs.		.	.	S.W	4	*b c*	.	Passed H M ship Melampus.	
10	Hove		.	.	.	.	*f*	.	Stopped and hove to for a dense fog.	
11	to.		.	.	.	.	*f*	.	Went on again	
12	10	.	W. by S ½ S	.	W S.W.	6	*b c*	.	Noon Folkestone N E by N.	

Course. Current.	Distance: Made good. Miles	Distance: Through the Water Miles.	Longitude: Dr Obs.	Latitude: Dr Chr	Variation allowed	Water Remaining: Daily Exp	True Bearing and Distance.	No. on Sick List.
								0

1	10	.	West	.	West	4	*c*	.	P.M. P M Passed Dungeness Lt H.	
2	10	.	.	.	.	.	.	.	Passed the Royal Sovereign Buoy	
3	10	.	.	.	.	.	.	.	Passed Beachy Head.	
4	10	.	.	.	.	.	.	.	Beltout Lt H. N N E. 2′.	
5	10	.	W by N. ½ N.	.	West	4	*c*	.	Several sail in sight Carried lights.	J. E. P.
6	10	.	.	.	.	.	30 50 41	.		
7	10	.	.	.	.	.	.	.	Stopped and sounded in 14 fm.	
8	10	.	.	.	W.S W.	4	*ch*	.	8′ Owers Lt. N N E 1′	
9	6	.	N W	.	,,	.	.	.	Saw the Nab N W by N.	
10	6	.	N W. by W.	.	,,	.	*f*	.	Stopped and anchored in 10 fms L V S by E 1′	
11	At		.	.	.	.	.	.	Carried away and lost overboard by fouling the wheel, lines, hand 2 in No , lead, hand one No.	
12	anchor.		.	.	W.S W	4	*f*	.	Mid.	

Signals, &c.

The 8th of March, 1854.

Hours	Height of Steam Gauge.	Revolutions per Minute	Consumption of Coal every Hour in Weight.	At each Period of Four Hours: Mean Height of Barometer: Starboard Engine	Port Engine.	Atmospheric or Marine Engine	Mean Pressure on Piston by Indicator.	State of Water of Boilers before blowing off: Temperature when Boiling	Density by Hydrometer.	Boiling Point of Sea Water	Every Twelve Hours: Average Temperature of: Sea Water	Atmosphere of Fore Stoke-hole.	Ditto, After Stoke-hole	Coal Boxes.	Every Twenty-four Hours: Total Expenditure of: Coals. For Engines: Tons	Cwts.	Ship: Tons	Cwts	Oil.	Tallow	Oakum.	Remarks
A.M.	In	No	lbs.	In	In	In	lbs	Deg	Deg	Deg	Deg	Deg	Deg	Deg	Tons	Cwts.	Tons	Cwts	Gall	lbs.	lbs	A.M
1	.	.	224	.	.	.	.	.	.	.	.	.	.	.	.	2	.	.	.	.		
2	.	.	224	.	.	.	.	.	.	.	.	.	.	.	.	2	.	.	.	.	.	Fires banked
3	.	.	224	.	.	.	.	.	.	.	.	.	.	.	.	2	.	.	.	.	.	
4	.	.	224	.		.	.	.	.	.	.	.	.	.	.	2	.	.	.	.	.	Drew fires forward
5	.	.	224	.	.	.	.	.	.	.	.	.	.	.	.	2	.	.	.	.	.	
6	11	22	Cwt. 25	26	26	30·50	.	.	$\frac{2}{32}$	213	.	90	92	78	1	5	.	.	.	.	.	Steam up
7	11	24	25	.	.	.	.	.	.	.	.	.	.	.	1	5	.	.	.	.	.	
8	11	23	25	26	26	.	.	.	$\frac{2}{32}$	213	.	.	.	.	1	5	.	.	.	.		Went on as ordered
9	11	23	25	.	.	.	.	.	.	.	.	.	.	.	1	5	.	.	.	.	.	Stopped
10	11	24	25	.	.	.	.	.	.	.	.	.	.	.	1	5	.	.	.	.	.	Went on as ordered
11	11	24	25	.	.	.	.	.	.	.	.	.	.	.	1	5	.	.	.	.		
12	11	24	25	26	26	30·50	.	.	$\frac{2}{31}$	213	.	93	96	81	1	5	.	.	.	.	.	Noon Engines working well
P.M. 1	11	23	2800	26	26	30·50	.	.	$\frac{2}{32}$	213	.	.	.	.	1	5	.	.	.	.	.	P.M
2	11	23	2800	.	.	.	.	.	.	.	.	.	.	.	1	5	.	.	.	.	.	
3	11	23	2800	.	.	.	.	.	.	.	.	.	.	.	1	5	.	.	.	.	.	Engines working well.
4	11	23	2800	26	26	30·50	.	.	$\frac{2}{31}$	213	.	95	97	84	1	5	.	.	.	.	.	
5	11	23	2800	.	.	.	.	.	.	.	.	.	.	.	1	5	.	.	.	.	.	
6	11	23	2800	.	.	.	.	.	.	.	.	.	.	.	1	5	.	.	.	.	.	
7	11	23	2800	.	.	.	.	.	.	.	.	.	.	.	1	5	.	.	.	.	.	Eased for a fog.
8	11	23	2800	26	26	30·50	.	.	$\frac{2}{32}$	213	.	82	89	71	1	5	.	.	.	.	.	Going easy
9	10	18	2800	.	.	.	.	.	.	.	.	.	.	.	1	5	.	.	.	.	.	Stopped and banked fire
10	10	18	2800	.	.	.	.	.	.	.	.	.	.	.	1	5	.	.	.	.	.	
11	.	.	224	.	.	.	.	.	.	.	.	.	.	.	.	2	.	.	.	.	.	
12	.	.	224	.	.	.	.	.	.	.	.	80	84	70	.	2	.	.	.	.	.	Mid

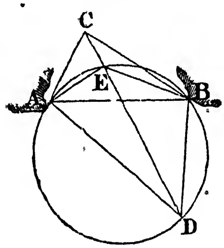
A Problem.

From a ship at D three objects on shore, A, C, and B, are observed, and with a sextant the angles A D C and B D C are measured, the relative positions of the objects A, C, B, are then found by the chart, so that A C, C B, and ∠ A C B, are also given

Find the position of the ship

This problem is usually solved by means of an instrument called a station pointer, which consists of three long brass arms, movable about a centre D, and which can be

Fig. 35.



opened and fixed so as to make with each other angles equal to those which are measured with the sextant, A D C and C D B. The instrument, being then laid upon the chart, is moved about till the bevelled edges of the arms pass through the three points A, C, B, and then the centre D is the place of the ship

The following is also an easy method of determining the position of D with respect to the objects which are observed.

Having transferred the triangle A C B from the chart to a clean sheet of paper, draw A E, making the angle B A E equal to ∠ C D B, and draw B E, making ∠ A B E equal to ∠ C D A, join C to E, the point where A E and B E intersect; and about the triangle A E B describe the circle A E B D.

Then D, where the line C E produced cuts the circle, is the place of the ship.

The trigonometrical solution of the problem presents no difficulty

The angles A C B and A D B being known, the sum of the remaining angles C A D and C B D of the quadrilateral A C B D is found by substracting the sum of A C B and A D B from four right angles.

Then in the triangle A C D, $CD \cdot \sin CDA = CA \cdot \sin CAD$ . . . (1),
and in the triangle B C D, $CD \cdot \sin CDB = CB \cdot \sin CBD$ . . . (2),

Therefore, by division, $\dfrac{\sin CDA}{\sin CDB} = \dfrac{CA}{CB} \cdot \dfrac{\sin CAD}{\sin CBD}$

Multiplying by $\dfrac{CB}{CA}$; $\dfrac{CB \cdot \sin CDA}{CA \cdot \sin CDB} = \dfrac{\sin CAD}{\sin CBD}$

Let $CB \cdot \sin CDA = m$, and $CA \cdot \sin CDB = n$, then

$$\frac{m}{n} = \frac{\sin CAD}{\sin CBD}.$$

whence
$$\frac{m+n}{m-n} = \frac{\tan \frac{1}{2}(CAD + CBD)}{\tan \frac{1}{2}(CAD - CBD)}$$

$$\therefore \tan \tfrac{1}{2}(CAD - CBD) = \tan \tfrac{1}{2}(CAD + CBD)\,\frac{m-n}{m+n} \quad . \quad . \quad . \quad (3).$$

C A D is the greater angle when $m - n$ and $\tan \frac{1}{2}(CAD + CBD)$ have the same sign, otherwise C B D is the greater.

By formula (3) half the difference between the angles C A D and C B D is to be computed, and half the difference added to half the sum gives the greater, and half the difference subtracted from half the sum gives the less of the two angles C A D and C B D.

These angles having been found, the distances A D, D C, and D B can be found by solving the plane triangles A C D and B C D

Example —Coming from sea, at the point D, I observed two headlands, A and B, and inland a steeple at C, which appeared between the headlands. I found from a map that the headlands were 5·35 miles from each other, that the distance of A from the steeple was 2·8 miles, and from B to the steeple 3·47 miles, and I found with a sextant that $\angle$ A D C was 12° 15′, and $\angle$ B D C 15° 30′ required my distance from each of the headlands and from the steeple?

A D 11·26, C D 12 46, B D 11·03 miles.

Problem.

The Lizard is in lat. 49° 58′ N, long 5° 11′ W, and the Land's End in lat 50° 4′ N, long. 5° 42′ W, and from a certain ship the Lizard bears N 45° 40′ E, and the Land's End N 23° 20′ E; required the latitude and longitude of the ship?

With the latitudes and longitudes it is found that the rhumb-line joining the Lizard and Land's End is N 73° 14′ W, distant 20 8 miles

The direction of the three sides of the triangle, whose angular points are at the ship, the Lizard, and the Land's End, are now all known, and hence the angles of the triangle are found to be, at the ship 22° 20′, at the Lizard 61° 6′ and at the Land's End 96° 34′, with these, and the distance 20 8 miles between the Lizard and Land's End, the distances of the ship from the Lizard and Land's End are found to be 54·0 and 47·58 miles.

With the course and distance to the Lizard, viz., N 45° 40′ E, and 54·0 miles, the difference of lat is found to be 39 miles, hence the *latitude in* is 49° 19′ N.

With this latitude of the ship, and the course and distance to the Lizard, the difference of longitude between the ship and the Lizard is found to be 60′ W, and therefore the *longitude in* is 6° 11′ W.

Problem.

Two points, A and B, are on the same parallel of latitude 47° N, but the true bearing of B from A is N 89° 30′ E, required the distance from each other, on the parallel and on a great circle?

On either the distance is 55·94 miles nearly.

Problem.

What is the greatest bearing of a star whose polar distance is 18° 45′, at a place in latitude 50°?

The greatest bearing occurs when the angle between the star's zenith distance and polar distance is a right angle, and $\sin p = \sin colat \cdot \sin a z$, the azimuth reckoned from the north or south according as the latitude is north or south.

$$\sin \text{azimuth} = \frac{\sin p}{\sin \text{colat.}} = \frac{\sin 18^\circ 45'}{\sin 40^\circ}$$

sin 18° 45′ . . .	9·507099
sin 40° 0′ . . .	9·808067
Azimuth 30° 1′	9·699032

EXAMINATION PAPER.—No. 1.

1. The light of a lighthouse, 150 feet high, is just visible from the top of the mast of a ship 54 feet in height; required the distance of the ship from the lighthouse?

2. What was the duration of twilight on January 22nd, 1855, at Greenwich?

3. If on February 1st, 1855, at apparent noon at the ship, in long. 5° 16′ 36″ W, a chronometer on board be 50 m. 18 s. fast for mean time at place, and at the next apparent noon the ship be in long. 3° 26′ 10″ W, what time will the chronometer show, its rate being 7 s. gaining daily?

4. February 1st, 1855, at noon, a point of land in lat. 35° 16′ N, long. 5° 25′ W, bore by compass W ¾ N (deviation ¾ pts. W), distant 16 miles, afterwards sailed as by the following log account; required the latitude and longitude in, on February 2nd, at noon?

H.	K.	1/10ths.	Courses.	Winds	Leeway	Dev.	REMARKS
1	5	8	ESE	NEbE	1	½ E	P.M.
2	5	4					
3	4	7					Variation of compass 1¾ pts. E.
4	5	0					
5	5	5					
6	5	1					
7	6	3					
8	4	5	SE ½ S	SW	1¼	0	
9	4	9					
10	4	3					
11	3	8					
12	5	6	S b W ½ W	W b S	½	½ W	
1	6	2					
2	6	0					A.M.
3	5	4					
4	4	7					A current set the ship from
5	5	6					9 P.M. till 2 A.M. NNW by
6	7	3	W b N ½ N	NNW	¼	¾ W	compass 3½ miles per hour
7	5	1					
8	4	5					
9	6	8					
10	4	6					
11	2	7	SW b W	S b E	½	¾ W	
12	4	4					

5. If on February 27th, 1855, in long. 16° 30′ W, the obs. mer. alt. of the sun's L.L. be 27° 15′ 10″ (Z.N.), index error + 2′ 50″, height of the eye 18 feet, required the latitude?

6. If on February 12th, 1855, the obs. mer. alt. of Spica be 20° 58′ 40″ (Z.N.), index error − 45″, height of the eye 25 feet; required the latitude?

7. If on February 26th, 1855, at 8 h. 40 m. P.M. mean time at place nearly, in long. 43° 26′ W, the obs. alt. of the moon's U.L. be 50° 20′ 30″ (Z.N.), index error + 2′ 30″, height of the eye 19 feet; required the latitude?

8. If on February 11th, 1855, at 10 h 16 m 45 s P M mean time at place, long 46° 25′ W, the obs alt of the Pole-star be 69° 29′ 54″, index error + 1′ 5″, height of the eye 13 feet, required the latitude?

9 February 24th, 1855, in lat by account 51° 40′ S, long 20° 17′ W, alts of the sun were taken near noon at the following times to determine the latitudes.—

Chron times H M S	Mean of Obs. alts sun's L.L.
10 28 13	
28 45	
29 14	47° 11′ 15″
29 50	
30 26	

Index error + 3′ 11″, height of the eye 21 feet, error of chronometer for mean time at place 1 h. 30 m. 27 s. slow.

10 February 20th, 1855, at about 7 hours A M. mean time at place nearly, lat. 50° 46′ N, long 15° 20′ W, the sun's rising amplitude was E 48° 20′ S by compass, (deviation 10° 55′ W); required the variation of the compass?

11. February 28th, 1855, at 3 h. 14 m. P M, mean time at place nearly, in lat. 38° 46′ N, long 97° 16′ W, the obs alt. of the sun's L L was 26° 54′ 29″, index error + 2′ 45″, height of the eye 17 feet, sun's bearing S 63° 26′ W (deviation 5° 55′ E); required the variation?

12. February 13th, 1855, at 8 h 13 m 20 s A M mean time at place, lat 32° 3′ S, long. 83° 20′ W, the sun's bearing by compass was N 49° 30′ E (deviation 10° 40′ E), required the variation?

13. February 22nd, 1855, at 8 h. 40 m A.M. mean time at place nearly, in lat. 23° 17′ 30″ N, long. by account 14° 30′ W, when a chronometer showed 9 h 47 m. 0 s, the obs. alt of the sun's L L. was 28° 52′ 21″·5, index error + 2′ 25″·5, height of the eye 21 feet, required the longitude?

On January 21st, at noon, the chronometer was fast on G. M. T. 5 m. 20 s., losing daily 5 s.

14. February 5th, 1855, at 8 h 25 m. P.M. mean time at place nearly, in lat. 47° 19′ N, long by account 33° 36′ E, when a chronometer showed 5 h 15 m 30 s, the obs alt of Pollux (E of mer) was 57° 43′ 20″, index error + 2′ 20″, height of the eye 19 feet; required the longitude?

On January 10th, 1855, at noon, the chronometer was slow on G M T. 47 m. 41 s, losing daily 6·5 s

15. February 9th, 1855, in lat 34° 12′ S, long 58° 16′ E, the sun had equal altitudes at 8 h. 47 m 13 s. and 2 h 58 m 47 s times by chronometer, required the error of chronometer at noon for mean time at the place of observation?

16. February 24th, 1855, in lat by account 52° 14′ S, long by account 38° 12′ E, the following observations were taken to determine the latitude and longitude.—

M T nearly	Chron times	Obs alt of ☉
9 h. 15 m. A M.	6 h. 27 m. 4 s.	33° 58′ 24″
2 14 P.M.	11 26 18	40 35 11

Index error + 2′ 10″, height of the eye 20 feet.

February 2nd, at noon, the chronometer was slow on G. M. T. 13 m. 15 s., losing daily 5 s.

17. January 4th, 1855, at about 10 h. 35 m. P M mean time at place, lat 33° 29′ 30″ N, long. by account 14° W, the following lunar was taken to determine the longitude:—

	Obs. alt α Tauri	Obs alt. ☽'s L L,	Obs dist N L.
	68° 18′ 40″	50° 13′ 24	55° 15′ 24″
Index error +	2 47	I. E. −3 32	I. E. + 1 45

Height of the eye 17 feet.

18. February 26th, 1855, lat. by account 46° 21′ S, long. by account 31° 30′ E, the following altitudes of stars were observed to determine the latitude and longitude:—

M T. nearly	Chron times.	Obs Alts.
7h 45 m. P M.	4h. 45 m 2 s	Aldebaran 23° 21′ 27″
9 20 P.M.	6 15 42	Canopus 75 48 16

Index error + 3′ 15″, height of the eye 16 feet

January 5th, 1855, at noon, my chronometer was slow on G. M. T. 56 m. 28·5 s, gaining daily 2·5 s.

Answers

1. 24 miles			
2. About 2 h 5 m.			
3 12h 57m. 2·3 s.			
4 Latitude	34° 20′·5 N.		Longitude 5° 29′ 5″ W.
5. ,,	54 6 40″ N.		
6. ,,	58 45 13 N.		
7 ,,	66 22 24 N.		
8 ,,	69 44 5 N.		
9. ,,	52 1 5 S		
10. Variation	19 45 W.		
11. ,,	16 55 W.		
12. ,,	25 50 E.		
13. Longitude	14 35 W.		
14. ,,	33 30 6″ E.		
15. Error 21 m. 22·4 s. slow.			
16. Latitude	52° 13′ 56″ S		Longitude 38° 11′ 54″ E.
17. Longitude	14 30 W.		
18. Latitude	46 25 23″ S.		Longitude 31° 27′ 8″ E

Examination Paper.—No. 2.

1. If a be the alt. of the sun on the prime vertical, and a' its alt on the 6 o'clock hour circle, find expressions for determining its declination and the latitude of the place of observation.

2. Required the distance on a great circle from lat. 35° 27′ 14″ N, long. 133° 27′ 18″ W, to lat. 29° 14′ 36″ N, long 125° 13′ 50″ E?

3. Required the course by compass and the distance from A to B,—

Lat. A 15° 55′ S.	Long. A 5° 45′ W.
,, B 23 56 S.	,, B 3 45 W.

Variation 20° 30′ W, local deviation 4° 35′ E.

4. March 18th, 1855, a point of land in lat. 36° 27′ S, long 131° 14′ W, bore by compass S b E (ship's head E, loc dev. ¼ pt E) distant 14 miles, afterwards sailed as by the following log account; required the latitude and longitude in, on March 19th, at noon?

H.	K.	$\frac{1}{10}$ths.	Courses	Winds	Lee-way	Dev	REMARKS
1	4	6	E b N ½ N	N b E	2	1 E	P.M.
2	5	7					
3	4	6					
4	6	1					Variation of compass 2¼ pts. W.
5	6	1					
6	5	5					
7	5	5	N b E	NW	1½	½ E	
8	4	7					
9	6	2					
10	7	1					
11	5	7	S	ESE	2½	½ W	
12	4	9					
1	3	9					A M
2	3	8	W b N ½ N	N b W	2	¾ W	
3	5	6					
4	4	7					A current set the ship from 5 A M till 10 A.M SE b S by compass, 3·9 miles per hour.
5	6	5					
6	7	4					
7	6	6	NNE	E	1¾	1 E	
8	5	8					
9	5	8					
10	5	9					
11	6	4					
12	6	3					

5. March 1st, 1855, in long 76° 19′ 10″ W, the obs mer alt of the sun's L.L. was 66° 10′ 15″ (Z S.), index error – 4′ 15″, height of the eye 14 feet, required the latitude?

6. March 2nd, 1855, the obs mer alt of Vega was 68° 13′ 5″ (Z N), index error + 2′ 10″, height of the eye 13 feet, required the latitude?

7. March 6th, 1855, at 1 h 40 m. A.M, in long. 39° 10′ 20″ E, the obs. mer alt. of the moon's L.L was 37° 10′ 30″ (Z N), index error – 3′ 10″, height of the eye 19 feet; required the latitude?

8. March 10th, 1855, at about 9 h. 14 m P M mean time at place, in long. 119° E, the obs. alt. of Polaris was 47° 4′ 36″, index error – 1′ 14″, height of the eye 26 feet; required the latitude?

9. March 9th, 1855, in lat by account 46° 23′ S, long 95° 37′ W, alts. of the sun were taken near noon to determine the latitude.

Chron H	M	S	
10	10	39	
	11	14	Mean of Obs alts sun's centre. 48° 10′ 15″
	11	59	
	12	17	
	12	48	

Index error + 3′ 14″, height of the eye 27 feet, chronometer slow for app T. at place 1 h. 52 m. 48·4 s.

10. March 20th, 1855, at 6 hours A M, in lat. 27° 39′ N, long. 109° 14′ W, the obs. rising amplitude of sun was E 17° 25′ S (deviation 4° 30′ E); required the variation of the compass?

11. March 2nd, 1855, at 9 h 14 m. A M mean time nearly, in lat. 34° 19′ S, long. 58° 20′ W, the obs alt. of the sun's L L. was 40° 46′ 53″, index error – 1′ 20″, height of the eye 25 feet, sun's bearing by compass N 49° 15′ E (deviation 8° 40′ W); required the variation?

12. March 3rd, 1855, at 9 h 17 m 14 s A.M., in lat. 47° 10′ N, long 136° 15′ 20″ W, the obs bearing of the sun was S 36° 10′ E (deviation 5° 4′ E), required the variation?

13 March 11th, 1855, at about 7 h 15 m A.M., in lat 18° 38′ S, and long about 74° W, when a chronometer showed 12 h. 1 m 9 s the obs alt. of the sun's L. L. was 16° 13′ 23″, index error − 2′ 10″, height of the eye 20 feet, required the longitude?

February 26th, the chronometer was slow on G M. T. 9 m. 10 s, losing daily 2·5 s.

14. March 12th, 1855, at about 7 h 36 m P.M., in lat 39° 18′ 27″ N, long. about 38° 10′ E, when a chronometer showed 5 h 20 m. 58 s., the obs. alt. of α Arietis (W of meridian) was 25° 52′ 7″, index error − 50″, height of the eye 15 feet; required the longitude?

February 19th, 1855, the chronometer was fast on G. M. T. 14 m. 21 s., gaining daily 3·6 s.

15 March 25th, 1855, in lat 56° 19′ S, long 25° 14′ W, when a chronometer showed 8 h 27 m 18 s and 4 h. 16 m. 19 s, the sun had equal altitudes. required its error for mean time at place?

16. March 30th, 1855, in lat by account 33° 10′ S, long by account 73° W, the following observations were taken to determine the latitude and longitude by Sumner's method —

M. T. nearly	Chron. times	Obs alt ☉'s L L.
2 h. 30 m P.M.	6 h 51 m. 41 s	39° 46′ 10″
4 15 P.M	8 40 1	20 3 40

Index error + 1′ 56″, height of the eye 25 feet; the sun's bearing at the first observation N 49° 35′ W, the run of the ship in the interval NNE 6 miles per hour.

On March 22nd, at noon, the chronometer was slow on G M T. 26 m. 33 s., losing daily 4·5 s.

N.B The answers given are found with assumed latitudes 33° 20′ S, and 32° 50′ S.

17. March 14th, 1855, at about 8 h 25 m. A.M mean time at place nearly, in lat. 24° 6′ 30″ N, long by account 15° 35′ W, the following lunar was taken; required the longitude.

Obs alt. ☉'s L L		Obs. alt. ☽'s L L.		Obs dist.
29° 4′ 7″		39° 42′ 30″		52° 32′ 59″
Index error + 2 25	I. E. −	3 15	I. E. −	1 10
Height of the eye 19 feet				

18 Required the true altitude of the sun's centre on March 5th, at 2 h. 30 m. P.M. mean time at place, in lat. 49° 20′ 10″ N, long. 35° 10′ 45″ W?

Answers.

1. $\begin{cases} \text{Sin dec.} = \sqrt{\sin a \cdot \sin a'} \\ \text{Sin lat} = \sqrt{\dfrac{\sin a'}{\sin a}} \end{cases}$
2. 4903·5 miles.
3. Course S 2° 44′ W. Distance 494 miles.
4. Latitude 35° 56′ S. Longitude 130° 54′·5 W.
5. „ 31° 17′ 48″ S.
6. „ 60 27 22 N.
7. „ 51 42 28 N.
8. „ 47 28 14 N
9. „ 46 21 13 S.
10. Variation 21 38 W.
11. „ 26 31 E.
12. „ 17 26 W
13. Longitude 74° 10′ 54″ W.
14. „ 38 14 17 E.
15. Fast 16 m 3·8 s.
16. Latitude 33° 2′ 9″ S. Longitude 72° 50′ 4″ W.
17. Longitude 15 42 W
18. True alt. 26 57 26″.

Questions in Trigonometry and Navigation.

1. Wanting to know the distance of an inaccessible object C, I measured a base A B of 486 yards. At A, I found the angle C A B subtended by the object, and the other end of the line, to be 88° 12′, and at B the angle C B A was observed to be 54° 48′, required the distance of the object from each of the stations A and B?

Ans. A C 659·6 yards, B C 807·2 yards

2. Being desirous of finding the distance between two distant objects, C and D, I measured a base A B of 384 yards, on the same horizontal plane with the objects C and D. At A, I found the angle D A B = 48° 12′, and C A B = 89° 18′, at B the angle A B C was 46° 14′, and A B D 87° 4′; it is required from these data to compute the distance between C and D? Ans C D 358·5 yards

3. Wanting to know the height of a steeple, I measured 210 yards from the bottom of it, and then found the elevation of its top above the level of my instrument to be 33° 28′ 40″; required its height, the instrument standing five feet above the ground?

Ans. 143·38 feet.

4. The elevation of a spire at one station was 23° 50′ 17″, and the horizontal angle at this station between the spire and another station was 93° 4′ 20″, the horizontal angle at the latter station between the spire and the first station was 54° 28′ 36″, and the distance between the two stations 416 feet, required the height of the spire?

Ans 278·8 feet.

5. Required the height of a wall, whose angle of elevation is observed, at the distance of 463 feet, to be 16° 21′? Ans. 135·8 feet

6. The angle of elevation of a hill is, near its bottom, 31° 18′, and 214 yards further off, 26° 18′; required the perpendicular height of the hill, and the distance of the perpendicular from the first station?

Ans. The height of the hill is 565·2, and the distance of the perpendicular from the first station is 929·6 yards

7. The wall of a tower which is 149·5 feet in height, makes, with a line drawn from the top of it to a distant object on the horizontal plane, an angle of 57° 21′, what is the distance of the object from the bottom of the tower?

Ans 233·3 feet

8. From the top of a tower, whose height was 138 feet, I took the angles of depression of two objects which stood in a direct line from the bottom of the tower, and upon the same horizontal plane with it. The depression of the nearer object was found to be 48° 10′, and that of the farther 18° 52′, what was the distance of each from the bottom of the tower?

Ans Distance of the nearest 123·5, and of the furthest 403·8 feet.

9. Being on the side of a river, and wishing to know the distance of a house on the other side, I measured 312 yards in a right line by the side of the river, and then found that the two angles, one at each end of this line, subtended by the other end and the house, were 30° 15′ and 86° 27′, what was the distance between each end of the line and the house? Ans 351·7 and 182·8 yards.

10. Having measured a base of 260 yards in a straight line, close by one side of a river, I found that the two angles, one at each end of the line, subtended by the other end and a tree close to the opposite bank, were 40° and 80°, what was the breadth of the river? Ans 190·1 yards

11. From a ship a headland was seen, bearing NE ½ N; the vessel then stood away NW ¾ W 20 miles, and the same headland was observed to bear from her E ¾ N, required the distance of the headland from the ship at each station?

Ans. Distance at the first station 19·09, and at the second 26·96 miles.

12. A cape was observed to bear from us NW, and another headland to bear NNE ½ E; standing away ENE ¾ E 23 miles, we found the first bore from us WNW, and the second N b W ½ W, required the bearing and distance of the cape from the headland? Ans S 87° 40′ W, 42·33 miles

13. From an eminence of 268 feet in perpendicular height, the angle of depression of the top of a steeple which stood on the same horizontal plane was found to be 40° 3′, and of the bottom 56° 18′; what was the height of the steeple?

Ans. 117·8 feet.

14. Wanting to know the distance between two objects which were separated by a morass, I measured the distance from each to a point where I could see them both, the distances were 1840 and 1428 yards, and the angle which, at that point, the objects subtended, was $36^\circ\ 18'\ 24''$; required their distance?

Ans 1090·85 yards.

15. From the top of a mountain, 3 miles in height, the visible horizon appeared depressed $2^\circ\ 13'\ 27''$, required the diameter of the earth, and the distance of the boundary of the visible horizon?

Ans. Diameter of the earth 7958 miles, distance of horizon 154·54 miles.

16. A statue 12 feet high stands on the top of a column, whose height is 48 feet above the level of the eye; at what distance from the base of the column, on the same horizontal plane, will the statue appear under the greatest possible visual angle, and what will that angle be?

Ans Distance 53·6656 feet, angle $6^\circ\ 22'\ 46''$.

17. A ship which can lie within 5 points of the wind is bound to a port bearing S ½ E 18 miles, the wind being at SE; required her course and distance on each tack to reach her port, close hauled, in two boards?

Ans. On the port tack S b W 19 39 miles, and on the starboard tack E b N 5·656 miles.

18. Wanting in two boards to reach the mouth of a river, which bore NW ½ W 10 miles, I found my ship could lie within 5½ points of the wind (then at NNW) on either tack, but that on the port tack she made ½ point leeway, while on the starboard tack she made 1½ points, required the course and distance on each tack?

Ans. On the starboard tack W b S 17·91 miles, and on the port tack NE 13·91 miles.

19. Wishing to go round a point, which bore NNW 15 miles, but the wind being at W b N, I was obliged to ply to windward; I found my ship would make way within 6 points of the wind; required the course and distance on each tack?

Ans. On the port tack N b W 17·65, and on the starboard SW by S 4·138 miles.

20. If a ship can lie within 6 points of the wind on the port tack, but within 5½ points on the starboard tack, required her course and distance on each tack to reach a port lying S b E 22 miles, the wind being at SW?

Ans. On the starboard tack S b E ½ E 23·66, and on the port WNW 2·79 miles.

21. From a ship, A, the mouth of a river bore NNE, and from another, B, it bore N b W, distant from each 18 miles. If each ship can lie within 5 points of the wind, and sail with the given wind, close hauled, 4 miles an hour, which will reach the harbour sooner, and how much, the wind being at N?

Ans. A will reach the river about 27 minutes 43 seconds before B.

22. The wind is at WSW, and a ship sailing within 5 points of it 2½ miles per hour, makes on each tack 2½ points leeway, in what time will she advance 30 miles directly to windward?

Ans. In 5 days 2 hours and 25 minutes.

23. If a ship sail E 7 miles an hour by the log, in a current setting ENE 2·5 miles per hour, required her true course and hourly rate of sailing?

Ans. Course N $84^\circ\ 8'$ E, and rate 9·358 miles per hour.

24. A ship has made by the reckoning N ½ W 20 miles, but by observation it is found that, owing to a current, she has actually gone NNE 28 miles, required the setting and drift of the current in the time which the ship has been running?

Ans Setting N $64^\circ\ 48'$ E, and drift 14·1 miles.

25. A ship's course to her port is WNW. and she is running by the log 8 miles an hour, but meeting with a current setting W ½ S 4 miles an hour, what course must she steer in the current that her true course may be WNW?

Ans Course N $53^\circ\ 51'$ W.

26. In a tide running NW b W, 3 miles an hour, I wished to weather a point of land which bore NE 14 miles, what course must I steer so as to clear the point, the ship going 7 miles an hour by the log, and what time shall I be in reaching the point?

Ans. Course N $69^\circ\ 51'$ E, and time 2 hours 25 minutes.

27. From a ship in a current steering WSW 6 miles an hour by the log, a rock was seen at 6 in the evening, bearing SW ½ S 20 miles The ship was lost on the rock at 11 P.M.; required the setting and drift of the current?

Ans Setting S 75° 10′ E, and drift 3·11 miles per hour.

28 If a ship sail due W, 8·5 miles per hour by the log, in a current setting SW b W 4 miles per hour, required her true course, and hourly rate of sailing?

Ans Course S 79° 21′ W, and rate 12·04 miles per hour

29. If a ship sail from lat 40° 5′ N, long 28° 14′ W, ENE ½ E till her difference of longitude is 320 miles, required her distance, latitude, and departure?

Ans Lat 41° 18′ N, dist 251·5 miles, and dep. 240 6 miles.

30 If a ship sail 272 miles northwards from the latitude 46° 20′ N till she has altered her longitude 4° W, it is required to find the course and latitude arrived at?

Ans Course N 36° 0′ W, latitude 50° 0′ N.

31. Given the sun's declination 3° 16′ 6″ S +, and right ascension 12 h. 30 m. 14·4 s., required his longitude and the obliquity of the ecliptic?

Ans Longitude 6s. 8° 13′ 57″, and obliquity 23° 27′ 46″.

32. In latitude 40° 48′ N, the sun bore S 79° 16′ W, at 3 h 37 m. 59 s. P.M.; required his altitude and declination?

Ans. Altitude 37° 24′, and declination 16° 32′ N.

33. In N latitude, when the sun's declination was 14° 20′ N, his altitudes, at two different times, on the same forenoon, were 43° 7′ +, and 67° 10′ +, and the change of his azimuth in the interval 45° 2′; required the latitude?

Ans 34° 20′ N.

34. In latitude 16° 4′ N, when the sun's declination is 23° 2′ N, required the time in the afternoon, and the sun's altitude and bearing, when his azimuth neither increases nor decreases?

Ans. Time 3 h. 9 m 26 s P M, altitude 45° 1′, and bearing S 73° 16′ W.

35. The sun set SW ½ S, when his declination was 16° 4′; required the latitude?

Ans. 69° 1′,

36. The altitude of the sun, when on the equator, was 14° 28′ +, bearing ESE; required the latitude and time?

Ans Latitude 56° 1′, and time 7 h. 46 m. 12 s. A.M.

37. The altitude of the sun was 20° 41′, at 2 h. 20 m. P M, when his declination was 10° 28′ S; required his azimuth and the latitude?

Ans. Azimuth S 37° 5′ W, latitude 51° 58′ N

38. If, on August 11th, 1840, Spica set 2 h 26 m 14 s before Arcturus, height of the eye 15 feet, required the north latitude?

Ans. 36° 46′ N.

39. If, on November 14th, 1829, Menkar rise 48 m 3 s. before Aldebaran height of the eye 17 feet, required the north latitude?

Ans 39° 43′ 30″ N.

40. If, on January 4th, 1825, Castor and Alphard be observed on the same vertical in the eastern hemisphere, at the same time that Betelgeuse and Rigel are on the same vertical in the western hemisphere, required the north latitude?

Ans 35° 12′ N.

41. In latitude 16° 40′ N, when the sun's declination was 23° 18′ N, I observed him twice, in the same forenoon, bearing N 68° 30′ E; required the times of observation, and his altitude at each time?

Ans. Times 6 h. 15 m. 40 s. A.M. and 10 h. 32 m. 48 s. A.M., altitudes 9° 59′ 36″ and 68° 29′ 42″.

42. The diff long. between two places, both in lat. 33° 51′ S, is 136° 10′; how much shorter is the distance between them on the arc of a great circle than on their common parallel, and what is the highest latitude attained by the ship in sailing from the one place to the other on the arc of a great circle?

Ans. Diff. of distances 637 geographical miles; highest latitude 60° 54′ S.

43. What is the highest latitude attained by a ship in sailing on a great circle from Port Jackson, lat. 33° 51′ S, to Cape Horn, lat. 55° 58′ S, the diff. of their longitudes being 140° 27′?

Ans. Lat. 72° 41′ S.

44. If the altitude of the sun when due W be $27^\circ\ 24'$, and on the 6 o'clock hour circle $14^\circ\ 43'\ 30''$; required the latitude and declination?

Ans. Lat. 48° N, decl. 20° N.

45. If the altitude of the sun on the 6 o'clock hour circle be $14^\circ\ 43'\ 30''$, and his amplitude W $30^\circ\ 44'\ 30''$; required the latitude and declination?

Ans. Lat 42° or 48°, decl. $22^\circ\ 20'$ or 20°.

46. If the sun's altitude on the 6 o'clock hour circle be $14^\circ\ 43'\ 30''$, and he set at 7 h. 35 m. 22 s. app. time; required the latitude and declination?

Ans. Lat. $48^\circ\ 1'$, and decl 20°.

47. The sun is W at 4h. 43 m. 28 s., and sets at 7 h. 35 m 22 s.; required the latitude and declination?

Ans. Lat 48° N, decl. 20°.

48. When the sun's declination was 20° N, he set 2 h 51 m. 54 s after he passed the prime vertical, required the latitude?

Ans. 48° N or 42° N.

49. Given the sun's meridian altitude 62°, and his altitude at 6 o'clock $14^\circ\ 43'\ 30''$, to find the latitude and declination?

Ans. Lat 48° N, decl. 20° N.

50. In latitude 45° N the meridian altitude of the sun was 30°; show that the tangent of quarter the length of the day $= \dfrac{1}{\sqrt[4]{3}}$.

51. The latitude of Dublin is $53^\circ\ 21'$ N, longitude $6^\circ\ 19'$ W, the latitude of Pernambuco $8^\circ\ 13'$ S, longitude $35^\circ\ 5'$ W; what is the sun's declination when he is on the horizon of both places at the same instant?

Ans. $18^\circ\ 6'$.

52. The latitude of a place A is 40° N, of B 50° N, and their distance from each other 20°: the longitude of A is 15° E; required the latitude and longitude of another place C to the north of and 20° distant both from A and B?

Ans. Lat $59^\circ\ 37'$ N, long $21^\circ\ 13'$ E, or $8^\circ\ 47'$ E.

53. If on April 28th, 1855, at about 6 h. 30 m. P M mean time at place, in latitude $44^\circ\ 45'$ N, longitude by account $165^\circ\ 15'$ E, the following lunar were taken; required the longitude?

	Obs alt. of Venus.	Obs. alt. ☽	Obs dist. nearest limb
	$33^\circ\ 20'\ 0$	$37^\circ\ 7'\ 40''$	$102^\circ\ 42'\ 37''$
Index error	$-\ 4\ 24$	$-\ 54{\cdot}5$	$+\ 1\ 26{\cdot}5$
Height of the eye 21 feet.			

On April 27th, the declination of Venus at mean noon at Greenwich is N $22^\circ\ 57'\ 39{\cdot}9''$ difference in 1 hour $+\ 34{\cdot}7''$, and its right ascension 4 h. 26 m. 56·29 s., difference in 1 hour + 12·78 s.

Date.	Semidiameter.		Hor. Parallax ☽		Date	Lunar Distances.			
	Noon.	Midnight.	Noon.	Midnight.		XVIII	P L. of Diff	XXI.	P. L. of Diff
	′ ″	′ ″	′ ″	′ ″		° ′ ″		° ′ ″	
April 27	..	14 59 0	..	54 52·6	April 27	101 42 11	3379	103 4 51	3368
„ 28	15 2·5	.	55 5 7	..					

Ans. Longitude $165^\circ\ 19'\ 55''$ E.

54. If on August 17th, 1855, at about 10 h. 15 m A M mean time at place, in latitude $21^\circ\ 30'$ N, longitude by account $41^\circ\ 10'$ W, the following lunar were taken; required the longitude?

	Obs. alt. ☉	Obs alt ☽	Obs dist
	$61^\circ\ 36'\ 10''$	$8^\circ\ 57'\ 40''$	$52^\circ\ 16\ 54{\cdot}6''$
Index error	$+\ 2\ 14$	$-\ 1\ 18$	$-\ 2\ 20{\cdot}0$
Height of the eye 25 feet.			

On August 17th, the declination of the sun at mean noon at Greenwich is N 13° 32′ 36″·6, difference in 1 hour — 48″ 08, and the equation of time at mean noon at Greenwich is 3 m. 56·74 s, difference in 1 hour — ·526 s. and additive to apparent time.

Date	Semidiameter.		Hor Parallax		Date	Lunar Distances.			
	Noon	Midnight	Noon	Midnight		Noon	P. L. of Diff	III.	P. L of Diff
August 17	′ ″ 14 56 1	′ ″ 14 59 7	′ ″ 54 42 0	′ ″ 54 55 2	August 17	° ′ ″ 51 31 28	3365	° ′ ″ 52 54 25	3355

Ans. Longitude 41° 12′ 50″ W.

55. If on March 21st, 1855, at about 4 h 10 m. P.M., mean time at place, in latitude 46° 50′ N, longitude by account 35° 20′ W, the following lunar were taken; required the longitude?

	Obs. alt ☉	Obs. alt ☽	Obs. dist.
	19° 36′ 38″	56° 53′ 43″	45° 54′ 21″
Index error	+ 1 16	+ 1 16	+ 4 0

Height of the eye 19 feet.

On March 21st, the declination of the sun at mean noon at Greenwich is N 0° 7′ 46″·8, difference in 1 hour 59″·22, and the equation of time at mean noon at Greenwich is 7 m. 26·6 s., difference in 1 hour — ·757 s. and additive to apparent time.

Date	Semidiameter		Hor. Parallax.		Date.	Lunar Distances.			
	Noon	Midnight.	Noon.	Midnight.		VI.	P. L. of Diff.	IX.	P. L. of Diff.
March 21	′ ″ 15 47·1	′ ″ 15 39·6	′ ″ 57 48 7	′ ″ 57 21·5	March 21.	° ′ ″ 46 34 58	2891	° ′ ″ 48 7 28	2909

Ans. Longitude 35° 25′ 30″ W.

56. On July 23rd, A.M., 1854, on board H.M.S. 'St. George,' Gulf of Finland, latitude by account 59° 37′ N, longitude 22° 0′ E, the following observations were taken; required the latitude?

Watch.	Alt. ☉.	Watch.	Alt. ☉.
9 h 43 m. 20·6 s	42° 45′ 20″	0 h. 27 m. 31·6 s.	50° 8′ 40″

Index error + 1′ 0″; height of the eye 40 feet.

The watch was fast on chronometer 1 h. 41 m. 27 s.; chronometer slow on G. M. T. 9 m. 17·2 s.

The run of the ship in the interval was E by N 3·6 miles per hour, the bearing of the sun at the first observation ESE.

Declination of the sun at mean noon July 22nd, 1854, 20° 19′ 4″·9, difference in 1 hour — 30″·19, semidiameter 15′ 46″·8; equation of time, sub. on mean time 6 m. 6·59 s., difference in 1 hour + 0·098 s.; right ascension of mean sun at noon, 7 h. 59 m. 38·67 s.

Ans. Latitude 59° 37′ 15″.

57. September 12th, 1854, at 11 h. 40 m. P.M. mean time, in long. 60° 40′ E, the observed alt. of Polaris was 54° 30′ 0″, index error — 1′ 10″, height of the eye 15 feet; required the latitude?

On September 12th, the declination of Polaris 88° 31′ 53″·6, the right ascension 1 h. 6 m. 48·57 s., and the right ascension of the mean sun 11 h. 25 m. 54·67 s.

Ans. Lat. 53° 8′ 36″ N.

58. On October 16th, 1854, at 4 h. 37 m. 11 s. A.M. mean time, in long. 157° 17′ W, the observed alt. of the pole-star was 39° 12′ 20″, index error + 15° 10″, height of the eye 37 feet; required the latitude?

On October 16th, 1854, the declination of Polaris 88° 32′ 6″·6, right ascension 1 h. 6 m. 55·25 s., and the right ascension of the mean sun 13 h. 39 m. 12·97 s.

Ans. Lat. 39° 2′ 0″ N.

59. On November 17th, 1854, at 11 h. 5 m. 54 s P.M. in mean time, in long. 13° 18′ W, the observed altitude of Polaris was 47° 13′ 50″, index error + 7′ 14″, height of the eye 23 feet; required the latitude?

On November 17th, 1854, the declination of Polaris 88° 32′ 18″ 4, the right ascension 1 h. 6 m. 47·92 s., and the right ascension of the mean sun 15 h. 46 m. 50·26 s.

Ans. Lat. 45° 57′ 15″ N.

60. May 18th, 1855, in lat by account 62° 20′ N, long 67° 13′ W, altitudes of the sun were taken, near noon, at the following times, to determine the latitude.

Chron. H.	M.	S.	
1	4	48	
	5	10	
	5	50	Mean of
	6	35	Obs. alts ☉'s L L.
	7	2	47° 28′ 52″·6
	7	51	
	8	15	
	8	47	

Index error + 1′ 47″, height of the eye 16 feet, error of chron. for M. T. at place 1 h. 13 m. 46 s. fast

May 18th, at mean noon, ☉'s declination N 19° 29′ 29″·0, diff. in 1 hour + 32″·85.

May 18th, at apparent noon, equation of time 3 m. 50 65 s. sub. from app time, diff. − ·091 s.

Lat. 61° 49′ 49″ N.

61. August 17th, 1855, in lat. by account 37° 20′ S, long 96° 15′ E, altitudes of the sun were taken, near noon, at the following times, to determine the latitude:

Chron. H.	M.	S.	
9	8	46	
	8	59	Mean of
	9	17	Obs alts. ☉'s U. L.
	9	42	39° 18′ 45″
	10	3	
	10	27	

Index error + 1′ 18″ height of the eye 17 feet, error of chron. for M. T. at place 2 h. 51 m. 13 s. slow.

Aug 16th, at mean noon, ☉'s declin N 13° 51′ 37″·6, diff. in 1 hour − 47″·54.

Aug. 16th, apparent noon, equation of time 4 m. 8·81 s. add to app. time, diff. in 1 hour − ·504 s.

Lat. 37° 22′ 51″·9 S.

62. October 19th, 1855, in lat. by account 48° 50′ N, long. 27° 56′ E, altitudes of the sun were taken, near noon, at the following times, to determine the latitude:

Chron. H.	M.	S.	
12	2	48	
	3	1	Mean of
	4	2	Obs. alts. ☉'s L L.
	5	3	31° 0′ 26″
	6	4	

Index error − 2′ 15″, height of the eye 17 feet, error of chron. for M. T. at Greenwich 1 h. 56 m. 17 s. fast.

Oct. 18th, at mean noon, ☉'s declin S 9° 31′ 22″·1, diff in 1 hour + 54″·57.

Oct. 18th, at apparent noon, equation of time 14 m. 41·01 s., sub. from app. time, diff. in 1 hour + ·469 s.

Lat. 48° 53′ 26″·4 N.

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